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Full Paper

An analogue of Eulerian polynomials related to L-type function

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Abstract: We introduce Dirichlet's type of twisted Eulerian polynomials by using p-adic fermionic q-invairant integral in the p-adic integer ring and obtain some new interesting identities. Using a complex contour integral representation based on the generating function of Dirichlet's type of twisted Eulerian polynomials, we get L-type function which interpolates for Dirichlet's type of Eulerian polynomials at negative integers.

Keywords: p-adic fermionic q-integral on Z_p, Eulerian polynomials, contour integral, L-function, Dirichlet character

INTRODUCTION

The Eulerian polynomials $A_n(x)$ are given by means of the following exponential generating series:

$$\sum_{n=0}^{\infty} A_n(x)\frac{t^n}{n!} = e^{A(x)t} = \frac{1-x}{e^{t(1-x)}-x}. \tag{1}$$

In fact, the Eulerian polynomials can be generated via the following recurrence relation:

$$\left(A(t)+(t-1)\right)^n - tA_n(t) = \begin{cases} 1-t & \text{if } n=0 \\ 0 & \text{if } n\neq 0, \end{cases} \tag{2}$$

where we have used $A^n(x) := A_n(x)$ symbolically [7, 22].

Let p be a fixed odd prime number. Throughout this paper we make use of the following

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notations: Z_p denotes the ring of p-adic rational integers, Q denotes the field of rational numbers, Q_p denotes the field of p-adic rational numbers and C_p denotes the completion of algebraic closure of Q_p. Let N be a set of natural numbers and $N^* = N \cup \{0\}$. The normalised p-adic absolute value is defined by

$$|p|_p = \frac{1}{p}.$$

When one talks of a q-extension, q can be variously considered as an indeterminate, a complex number $q \in C$, or a p-adic number $q \in C_p$. If $q \in C$, one normally assumes $|q| < 1$. If $q \in C_p$, one normally assumes $|q-1| < p^{-\frac{1}{p-1}}$. The q-integer $[x]_q$ with x in C is also defined by

$$[x]_q = \frac{1-q^x}{1-q} \text{ and } [x]_{-q} = \frac{1-(-q)^x}{1+q}.$$

It is easy to see that $\lim_{q\to 1}[x]_q = x$ [1-26].

For a positive integer d with $(d,p)=1$, set

$$X = X_d = \varprojlim_n Z/dp^nZ = \bigcup_{a=0}^{dp-1}(a + dpZ_p)$$

with

$$a + dp^nZ_p = \{x \in X \mid x \equiv a \,(\mathrm{mod}\, dp^n)\},$$

where $a \in Z$ satisfies the condition $0 \le a < dp^n$, and let $\sigma : X \to Z_p$ be the transformation introduced by the inverse limit of the natural transformation:

$$Z/dp^nZ \mapsto Z/p^nZ.$$

If f is a function of Z_p, then we will utilise the same notation to indicate the function $f \circ \sigma$.

Let $UD(Z_p)$ be the space of uniformly differentiable functions on Z_p. That is, for $f \in UD(Z_p)$, the p-adic q-integral on Z_p was originally defined by Kim [11-14] as follows:

$$I_q(f) = \int_X f(\upsilon)\,d\mu_q(\upsilon) = \int_{Z_p} f(\upsilon)\,d\mu_q(\upsilon) = \lim_{n\to\infty}\frac{1}{[p^n]_q}\sum_{\upsilon=0}^{p^n-1} q^\upsilon f(\upsilon) \qquad (3)$$

The bosonic integral is considered as the bosonic limit $q \to 1$, $I_1(f) = \lim_{q\to 1} I_q(f)$.

Similarly, the p-adic fermionic integration on Z_p was firstly defined by Kim [16, 18] as follows:

$$I_{-q}(f) = \lim_{q\to -q} I_q(f) = \int_{Z_p} f(\upsilon)\,d\mu_{-q}(\upsilon). \qquad (4)$$

The above integrals are powerful tools in the studying of p-adic analogue of some special polynomials such as Bernoulli polynomials, Euler polynomials, Genocchi polynomials, Frobenius-Euler polynomials, Eulerian polynomials, Boole polynomials and their various generalisations [1-33].

By (4), we have the following integral equation:

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$$q^n I_{-q}(f_n)+(-1)^{n-1} I_{-q}(f)=[2]_q \sum_{l=0}^{n-1}(-1)^{n-1-l} q^l f(l) \qquad (5)$$

where $f_n(x)$ is a translation given by $f_n(x) := f(x+n)$. It follows from (5) that

$$qI_{-q}(f_1)+I_{-q}(f)=[2]_q f(0). \qquad (6)$$

If we replace q by q^{-1} in (6), it becomes

$$I_{-q^{-1}}(f_1)+qI_{-q^{-1}}(f)=[2]_q f(0). \qquad (7)$$

Recently, Kim et al. [22] considered the case $f(x)=e^{-x(1+q)t}$ in (7) and derived the following Witt's formula of the Eulerian polynomials for $n \in N^*$:

$$I_{-q^{-1}}(x^n)=\frac{(-1)^n}{(1+q)^n} A_n(-q). \qquad (8)$$

Note that Kim and Kim [23, 24] also introduced the *q*-analogue of Eulerian polynomials and the *p*-adic integral representation of the Boole polynomials.

Based on the work of Kim et al. [22], Araci et al. [7] defined the generating function of Dirichlet's type of the Eulerian polynomials as follows:

$$\sum_{n=0}^{\infty} A_{n,\chi}(-q)\frac{t^n}{n!}=[2]_q \sum_{l=0}^{d-1}(-1)^l q^{d-l+1}\chi(l)\frac{e^{-l(1+q)t}}{e^{-d(1+q)t}+q^d}. \qquad (9)$$

Also, they gave the following Witt's formula for Dirichlet's type of the Eulerian polynomials:

$$I_{-q^{-1}}(\chi(x)x^n)=\frac{(-1)^n}{(1+q)^n} A_{n,\chi}(-q).$$

In this paper we construct Dirichlet's type of twisted Eulerian polynomials. By using a complex contour integral representation (or known as Mellin transformation) based on the generating function of Dirichlet type of twisted Eulerian polynomials, we define an *L*-type function which interpolates Dirichlet's type of twisted Eulerian polynomials at negative integers.

ON DIRICHLET'S TYPE OF TWISTED EULERIAN POLYNOMIALS

By using (5), we have

$$I_{-q^{-1}}(f_d)+q^d I_{-q^{-1}}(f)=[2]_q \sum_{l=0}^{d-1}(-1)^l q^{d-l+1} f(l). \qquad (10)$$

Let $C_{p^n}=\{\zeta \mid \zeta^{p^n}=1\}$ be the cyclic group of order p^n, and let

$$T_p=\lim_{n\to\infty} C_{p^n}=C_{p^\infty}=\cup_{n\geq 0} C_{p^n},$$

where T_p is locally constant space. Let χ be a Dirichlet character of conductor $d(=\text{odd})$ and $\zeta \in T_p$. Taking $f(x)=\zeta^x \chi(x) e^{-x(1+q)t}$ in (10) gives

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$$I_{-q^{-1}}\left(\zeta^{x+d}\chi(x+d)e^{-(x+d)(1+q)t}\right)+q^{d}I_{-q^{-1}}\left(\zeta^{x}\chi(x)e^{-x(1+q)t}\right)$$
$$=[2]_q\sum_{l=0}^{d-1}(-1)^l q^{d-l+1}\chi(l)e^{-l(1+q)t}\zeta^{l}.$$

From this, we derive

$$I_{-q^{-1}}\left(\zeta^{x}\chi(x)e^{-x(1+q)t}\right)=[2]_q\sum_{l=0}^{d-1}(-1)^l q^{d-l+1}\zeta^{l}\chi(l)\frac{e^{-l(1+q)t}}{\zeta^{d}e^{-d(1+q)t}+q^{d}}. \qquad (11)$$

Therefore, we give the following definition of the generating function for Dirichlet's type of twisted Eulerian polynomials.

Definition 1. *Let* $G_{q,\zeta}(t\mid\chi)=\sum_{n=0}^{\infty}A_{n,\chi,\zeta}(-q)\frac{t^n}{n!}$ and $\zeta\in T_p$. *Then we define the twisted Dirichlet's type of Eulerian polynomials by means of the following generating function:*

$$\sum_{n=0}^{\infty}A_{n,\chi,\zeta}(-q)\frac{t^n}{n!}=[2]_q\sum_{l=0}^{d-1}(-1)^l q^{d-l+1}\zeta^{l}\chi(l)\frac{e^{-l(1+q)t}}{\zeta^{d}e^{-d(1+q)t}+q^{d}}.$$

By this definition, we have the following corollary.

Corollary 1. *For any* $\zeta\in T_p$ *and* $n\in N^*$ *, we have*

$$A_{n,\chi,\zeta}(-q)=\left[\frac{t^n}{n!}\right][2]_q\sum_{l=0}^{d-1}(-1)^l q^{d-l+1}\zeta^{l}\chi(l)\frac{e^{-l(1+q)t}}{\zeta^{d}e^{-d(1+q)t}+q^{d}}$$

where $\left[t^n\right]f(t)$ *means the coefficient of* t^n *in* $f(t)$.

Now we give an integral representation of the twisted Dirichlet's type of Eulerian polynomials.

Theorem 1. *The following equalities hold*:

$$I_{-q^{-1}}\left(\zeta^{x}\chi(x)x^n\right)=\int_{Z_p}\zeta^{x}\chi(x)x^n d\mu_{-q^{-1}}(x)=\frac{(-1)^n}{(1+q)^n}A_{n,\chi,\zeta}(-q) \qquad (12)$$

Proof. It is evident from (11) and Definition 1.

By Definition 1, we have the following Theorem.

Theorem 2. *For* $\zeta\in T_p$ *, we have*

$$G_{q,\zeta}(t\mid\chi)=\sum_{n=0}^{\infty}A_{n,\chi,\zeta}(-q)\frac{t^n}{n!}=[2]_q\sum_{m=0}^{\infty}\frac{(-1)^m\zeta^m\chi(m)e^{-m(1+q)t}}{q^{m-1}}. \qquad (13)$$

Proof. From Definition 1, we discover:

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$$\sum_{n=0}^{\infty} A_{n,\chi,\zeta}(-q)\frac{t^n}{n!} = [2]_q \sum_{l=0}^{d-1}(-1)^l q^{d-l+1}\zeta^l \chi(l)\frac{e^{-l(1+q)t}}{\zeta^d e^{-d(1+q)t}+q^d}$$
$$= [2]_q \sum_{l=0}^{d-1}(-1)^l q^{-l+1}\zeta^l \chi(l) e^{-l(1+q)t}\sum_{m=0}^{\infty}(-1)^m \zeta^{md} q^{-md} e^{-md(1+q)t}$$
$$= q[2]_q \sum_{m=0}^{\infty}\sum_{l=0}^{d-1}(-1)^{l+md}\chi(l+md) q^{-(l+md)}\zeta^{l+md} e^{-(l+md)(1+q)t}$$
$$= q[2]_q \sum_{m=0}^{\infty}(-1)^m \zeta^m \chi(m) q^{-m} e^{-m(1+q)t}.$$

Thus, we arrive at the desired result.

Now we give the following Theorem, which is an explicit formula for twisted Dirichlet's type of Eulerian polynomials.

Theorem 3. *For* $\zeta \in T_p$ *, we have*

$$\frac{(-1)^n}{q(1+q)^{n+1}} A_{n,\chi,\zeta}(-q) = \sum_{m=1}^{\infty}\frac{(-1)^m \zeta^m \chi(m) m^n}{q^m}. \tag{14}$$

Proof. By using Taylor expansion of $e^{-m(1+q)t}$ in (13), we see that

$$\sum_{n=0}^{\infty} A_{n,\chi,\zeta}(-q)\frac{t^n}{n!} = [2]_q \sum_{n=0}^{\infty}\left(\sum_{m=0}^{\infty}\frac{(-1)^m \zeta^m \chi(m)(-m)^n (1+q)^n}{q^{m-1}}\right)\frac{t^n}{n!}$$

Comparing the coefficients of $\frac{t^n}{n!}$ on both sides of the above equation, we arrive at the desired result.

From (12) and (14), we have the following corollary.

Corollary 2. *For* $\zeta \in T_p$ *and* $n \in N$, *we have*

$$\lim_{n\to\infty}\sum_{x=0}^{p^n-1}\frac{(-1)^x \zeta^x \chi(x) x^n}{q^x} = 2q^2 \sum_{m=1}^{\infty}\frac{(-1)^m \zeta^m \chi(m) m^n}{q^m}.$$

Now also, we a give multiplication theorem for Dirichlet's type of twisted Eulerian polynomials.

Theorem 4. *The following is true*:

$$\frac{(-1)^n}{(1+q)^n} A_{n,\chi,\zeta}(-q) = \frac{d^n}{[d]_{-q^{-1}}}\sum_{a=0}^{d-1}(-1)^a \chi(a)\zeta^a q^{-a}\int_{Z_p}\left(\frac{a}{d}+x\right)^n \zeta^{dx} d\mu_{-q^{-d}}(x) \tag{15}$$

Proof. For each $\zeta \in T_p$ and $n \in N^*$ we see that

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$$
\begin{aligned}
& I_{-q^{-1}}\left(\zeta^x \chi(x) x^n\right) \\
& =\int_{Z_p} \zeta^x \chi(x) x^n d\mu_{-q^{-1}}(x) \\
& =\lim_{m\to\infty} \frac{1}{\left[dp^m\right]_{-q^{-1}}} \sum_{x=0}^{dp^m-1} (-1)^x \zeta^x \chi(x) x^n q^{-x} \\
& =\frac{d^n}{[d]_{-q^{-1}}} \sum_{a=0}^{d-1} (-1)^a \zeta^a \chi(a) q^{-a} \left(\lim_{m\to\infty} \frac{1}{\left[p^m\right]_{-q^{-d}}} \sum_{x=0}^{p^m-1} (-1)^x \zeta^{dx} \left(\frac{a}{d}+x\right)^n q^{-dx} \right) \\
& =\frac{d^n}{[d]_{-q^{-1}}} \sum_{a=0}^{d-1} (-1)^a \chi(a) \zeta^a q^{-a} \int_{Z_p} \left(\frac{a}{d}+x\right)^n \zeta^{dx} d\mu_{-q^{-d}}(x).
\end{aligned}
$$

So we complete the proof of the theorem.

Corollary 3. *For any* $\zeta \in T_p$ *and* $n \in \mathrm{N}^*$, *we have*

$$A_{n,\chi,\zeta}(-1)=(-2d)^n \sum_{a=0}^{d-1} (-1)^a \chi(a) \zeta^a E_{n,\zeta^d}\left(\frac{a}{d}\right).$$

Proof. It is interesting to point out that the case $q=1$ in (15) gives

$$\frac{(-1)^n}{2^n} A_{n,\chi,\zeta}(-1)=d^n \sum_{a=0}^{d-1} (-1)^a \chi(a) \zeta^a \int_{Z_p} \left(\frac{a}{d}+x\right)^n \zeta^{dx} d\mu_{-1}(x). \tag{16}$$

With the help of Rim and Kim's work [25], we get

$$E_{n,\zeta}(x)=\int_{Z_p} \zeta^y (x+y)^n d\mu_{-1}(y) \tag{17}$$

where $\zeta \in T_p$. Taking $x=0$ in the above equation, we have $E_{n,\zeta}(0) := E_{n,\zeta}$, which is defined by the following generating function:

$$\sum_{n=0}^{\infty} E_{n,\zeta} \frac{t^n}{n!}=\frac{2\sum_{l=0}^{d-1} (-1)^l \zeta^l e^{lt}}{\zeta^d e^{dt}+1}, \quad |t|<\frac{\pi}{d}. \tag{18}$$

Combining (16), (17) and (18), we end the proof of Corollary 3.

ON *L*-TYPE FUNCTION IN *C*

We describe *L*-type function by applying Mellin transformation to the generating function of Dirichlet's type of twisted Eulerian polynomials (stated in Definition 1), which is an interpolated function of Dirichlet's type of twisted Eulerian polynomials at negative integers for the corresponding Cauchy-Residue theorem. Further information about these topics can be obtained elsewhere [4, 6, 7, 15, 17, 19, 25, 31-33].

By (13), for $s \in C$, we consider the following complex contour integral representation (or known as Mellin transformation):

$$L_{E,\zeta}(s,\chi)=\frac{\int_0^{\infty} t^{s-1} G_{q,\zeta}(t \mid \chi) dt}{\int_0^{\infty} t^{s-1} e^{-t} dt}. \tag{19}$$

It follows from (19) that

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$$L_{E,\zeta}(s,\chi) = q[2]_q \sum_{m=0}^{\infty} (-1)^m \chi(m) \zeta^m q^{-m} \left\{ \frac{\int_0^{\infty} t^{s-1} e^{-m(1+q)t} dt}{\int_0^{\infty} t^{s-1} e^{-t} dt} \right\}$$

$$= \frac{q}{(1+q)^{s-1}} \sum_{m=1}^{\infty} \frac{(-1)^m \chi(m) \zeta^m}{q^m m^s}.$$

On the other hand, we see that

$$L_{E,\zeta}(s,\chi) = \sum_{n=0}^{\infty} \frac{A_{n,\chi,\zeta}(-q)}{n!} \left(\frac{\int_0^{\infty} t^{s-n+1} dt}{\int_0^{\infty} t^{s-1} e^{-t} dt} \right). \tag{20}$$

Therefore, we give the definition of twisted Eulerian L-function as follows:

Definition 2. *Let* $\zeta \in T_p$ *and* $s \in C$; *we define*

$$L_{E,\zeta}(s,\chi) = \frac{q}{(1+q)^{s-1}} \sum_{m=1}^{\infty} \frac{(-1)^m \chi(m) \zeta^m}{q^m m^s}.$$

By (14) and Definition 2, we derive the following Theorem.

Theorem 5. *The following equality holds:*

$$L_{E,\zeta}(-n,\chi) = (-1)^n A_{n,\chi,\zeta}(-q).$$

Proof. From (14):

$$\frac{(-1)^n}{q(1+q)^{n+1}} A_{n,\chi,\zeta}(-q) = \sum_{m=1}^{\infty} \frac{(-1)^m \zeta^m \chi(m) m^n}{q^m}.$$

Putting $s = -n$ in Definition 2 gives

$$L_{E,\zeta}(-n,\chi) = q(1+q)^{n+1} \sum_{m=1}^{\infty} \frac{(-1)^m \chi(m) \zeta^m m^n}{q^m}$$

from which we have

$$L_{E,\zeta}(-n,\chi) = (-1)^n A_{n,\chi,\zeta}(-q).$$

Thus, the proof of the theorem is completed.

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