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Full Paper

Reachability analysis of a class of Petri nets using place invariants and siphons

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Abstract: This paper proposes a novel and computationally efficient approach to deal with the reachability problem by using place invariants and strict minimal siphons for a class of Petri nets called pipe-line nets (PLNs). First, in a PLN with an appropriate initial marking, the set of invariant markings and the set of strict minimal siphons are enumerated. Then a sufficient and necessary condition is developed to decide whether a marking is spurious by analysing the number of tokens in operation places of any strict minimal siphon and their bounds. Furthermore, an algorithm that generates the reachable markings by removing all the spurious markings from the set of invariant markings is proposed. Finally, experimental results show the efficiency of the proposed method.

Keywords: Petri nets, strict minimal siphons, place invariants, reachability analysis, flexible manufacturing system

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INTRODUCTION

A flexible manufacturing system (FMS) is an automatically running system that consists of resources such as machines, robots, buffers, and conveyors. In an FMS, part processing sequences are executed concurrently, which have to compete for the limited system resources. This competition can cause deadlocks when some processes keep waiting indefinitely for other processes to release resources [1]. Deadlocks must be considered in FMSs since they may offset the advantages of these systems and even lead to catastrophic results such as long downtime and low use of some critical and expensive resources. Therefore, it is necessary to ensure that deadlocks will never occur in such a system.

To deal with deadlock problems in FMSs, Petri nets [2-6], automata [7-8], and graph theory [1] are major mathematical tools. Many researchers use Petri nets as a formalism to deal with deadlock problems [9-14]. There are mainly three approaches: deadlock detection and recovery [15-16], deadlock avoidance [17-19] and deadlock prevention [1, 9, 20, 21].

For Petri nets, there are two widely used analysis techniques for deadlock prevention in FMSs: structure analysis [4, 10, 11, 15, 22, 23] and reachability graph analysis [24-27]. The former always derives a deadlock prevention policy by structural objects of Petri nets, such as siphons and resource-transition circuits. The policy is often simple but always restricts the behaviour of a system in the sense that a part of permissive behaviour is excluded. Therefore, it is suboptimal in general. The latter, the reachability graph analysis, can obtain a liveness-enforcing supervisor with highly permissive or even maximally permissive behaviour. However, its computation is always expensive, which always suffers from a state explosion problem since it requires an enumeration of all or a part of reachable markings. Thus, to tackle this problem, it is necessary to explore more efficient approaches to compute reachable markings.

This paper proposes a novel approach to compute the set of reachable markings using P-invariants and strict minimal siphons in a class of Petri nets called PLNs. First, the set of invariant markings and the set of strict minimal siphons of a PLN are enumerated. As known, the set of invariant markings include spurious markings. Then a sufficient and necessary condition to identify the spurious markings is established. Finally the reachability set of the net is generated by removing all the spurious markings from the set of invariant markings.

PRELIMINARIES

Basics of Petri nets

A Petri net [2] is a four-tuple $N=(P,T,F,W)$ where P and T are finite and non-empty sets. P is a set of places and T is a set of transitions with $P \cap T = \phi$. $F \subseteq (P\times T)\cup(T\times P)$ is called the flow relation of the net, represented by arcs with arrows from places to transitions or from transitions to places. $W:(P\times T)\cup(T\times P)\rightarrow \mathrm{IN}$ is a mapping that assigns a weight to an arc: $W(x,y)>0$ if $(x,y)\in F$, and $W(x,y)=0$ otherwise, where $x,y\in P\cup T$ and $\mathrm{IN}=\{0,1,2,...\}$ is the set of non-negative integers. $N=(P,T,F,W)$ is said to be ordinary if $\forall(x,y)\in F$, $W(x,y)=1$. A net $N=(P,T,F,W)$ is pure (self-loop free) if $\forall x,y\in P\cup T$, $W(x,y)>0$ implies $W(y,x)=0$. A pure net $N=(P,T,F,W)$ can be represented by its incidence matrix $[N]$, where $[N]$ is a $|P|\times|T|$ integer matrix with $[N](p,t)=W(t,p)-W(p,t)$.

A marking M of a Petri net N is a mapping from P to IN. $M(p)$ denotes the number of tokens in place p. A place p is marked at M if $M(p)>0$. A subset $S\subseteq P$ is marked at M if at least one place in S is marked at M. The sum of tokens in all places in S is denoted by $M(S)$, i.e. $M(S)=\sum_{p\in S}M(p)$. S is said to be empty or unmarked at M if $M(S)=0$. M_0 is called an initial marking of N and (N,M_0) is called a net system or marked net.

Let $x\in P\cup T$ be a node of net N. The preset of node x is defined as ${}^\bullet x=\{y\in P\cup T \mid (y,x)\in F\}$, while the postset of x is defined as $x^\bullet=\{y\in P\cup T\mid (x,y)\in F\}$. These notations can be extended to a set of nodes as follows: given $X\subseteq P\cup T$, ${}^\bullet X=\bigcup_{x\in X}{}^\bullet x$ and $X^\bullet=\bigcup_{x\in X}x^\bullet$. For $t\in T$, $p\in{}^\bullet t$ is called an input place of t and $p\in t^\bullet$ is called an output place of

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t. For $p \in P$, $t \in {}^\bullet p$ is called an input transition of p and $t \in p^\bullet$ is called an output transition of p. A state machine is an ordinary Petri net satisfying $|{}^\bullet t| = |t^\bullet| = 1$, $\forall t \in T$. A marked graph is an ordinary Petri net satisfying $|{}^\bullet p| = |p^\bullet| = 1$, $\forall p \in P$. A sequence of nodes $x_1 \ldots x_i \ldots x_n$ is called a path of N if $\forall i \in \{1,2,\ldots,n-1\}$, $x_{i+1} \in x_i^\bullet$, where $x_i \in P \cup T$. An elementary path from x_1 to x_n is a path whose nodes are all different (perhaps, except for x_1 and x_n). It is called a circuit if $x_1 = x_n$. A Petri net N is said to be strongly connected if there is a sequence of nodes $x,a,b,\ldots,c,y$ in N for $\forall x,y \in P \cup T$ such that (x,a), (a,b), ..., $(c,y) \in F$, where $\{a,b,\ldots,c\} \subseteq P \cup T$.

A transition $t \in T$ is enabled at M if $\forall p \in {}^\bullet t$, $M(p) \geq W(p,t)$. This fact is denoted as $M[t\rangle$. Firing it yields a new marking M' such that $\forall p \in P$, $M'(p) = M(p) - W(p,t) + W(t,p)$, denoted as $M[t\rangle M'$. M' is called an immediately reachable marking from M. The marking M'' is said to be reachable from the marking M if there exists a sequence of enabled transitions $\sigma = t_0 t_1 \ldots t_n$ and markings M_1, M_2, ..., and M_n such that $M[t_0\rangle M_1[t_1\rangle M_2 \ldots M_n[t_n\rangle M''$ holds, which is denoted as $M[\sigma\rangle M''$. The set of markings reachable from M_0 by firing any possible sequence of transitions in N is called the reachability set of Petri net (N, M_0) and is denoted by $R(N, M_0)$. A reachability graph is a directed graph whose nodes are markings in $R(N, M_0)$ and arcs are labelled by the transitions of N. An arc from M_1 to M_2 is labelled by t if $M_1[t\rangle M_2$. N' is the reverse net of N obtained by reversing the direction of all arcs in N with the initial marking unchanged.

A P-vector is a column vector $I: P \to \mathbf{Z}$ indexed by P and a T-vector is a column vector $J: T \to \mathbf{Z}$ indexed by T, where $\mathbf{Z}$ is the set of integers. P-vector I is called a P-invariant (place invariant) if $I \neq \mathbf{0}$ and $I^T[N] = \mathbf{0}^T$. T-vector J is called a T-invariant (transition invariant) if $J \neq \mathbf{0}$ and $[N]J = \mathbf{0}$. A P-invariant I is said to be a P-semiflow if every element of I is non-negative. $\|I\| = \{p \mid I(p) \neq 0\}$ is called the support of *I*. I is called a minimal P-invariant if $\|I\|$ is not a superset of the support of any other one and its components are mutually prime. Let I be a P-invariant of (N, M_0) and M be a reachable marking from M_0. Then $I^T M = I^T M_0$.

Let X be a matrix where each column is a P-semiflow of the net (N, M_0) and $I_X(N, M_0) = \{M \in \mathrm{IN}^{|P|} \mid X^T M = X^T M_0\}$ denotes the set of invariant markings, where $\mathrm{IN}^{|P|}$ is a set of non-negative vectors, each of which has a length of $|P|$. It can be noted that $R(N, M_0) \subseteq I_X(N, M_0)$.

A non-empty set $S \subseteq P$ is a siphon if ${}^\bullet S \subseteq S^\bullet$. $S \subseteq P$ is a trap if $S^\bullet \subseteq {}^\bullet S$. A siphon is minimal if there is no siphon contained in it as a proper subset. A minimal siphon is said to be strict if ${}^\bullet S \subset S^\bullet$.

S^{3}PR Nets

Definition 1 [9]. A simple sequential process (S^{2}P) is a Petri net $N = (P_A \cup \{p^0\}, T, F)$, where the following statements are true: (1) $P_A \neq \phi$ is called a set of operation places; (2) $p^0 \notin P_A$ is called the process idle place; (3) N is a strongly connected state machine; and (4) every circuit of N contains place p^0.

Let $N=(P,T,F)$ be an S²P with idle process place p^0. Let C be a circuit of N, and x and y be two nodes of C. Node x is said to be previous to y if there exists a path in C from x to y, the length of which is greater than one and does not pass over the idle place p^0. This fact is denoted by $x <_C y$. Let x and y be two nodes in N. Node x is said to be previous to y in N if there exists a circuit C such that $x <_C y$. This fact is denoted by $x <_N y$.

Definition 2 [9]. A system of simple sequential processes with resources (S³PR) $N=(P^0 \cup P_A \cup P_R, T, F)$ is defined as the union of a set of nets $N_i=(\{p_i^0\} \cup P_{A_i} \cup P_{R_i}, T_i, F_i)$ sharing common places, where the following statements are true:

(1) p_i^0 is called the process idle places of N_i. Elements in P_{A_i} and P_{R_i} are called operation places and resource places respectively;

(2) $P_{R_i} \neq \phi$; $P_{A_i} \neq \phi$; $p_i^0 \notin P_{A_i}$; $(P_{A_i} \cup \{p_i^0\}) \cap P_{R_i} = \phi$;

(3) $\forall p \in P_{A_i}$, $\forall t \in {}^\bullet p$, $\forall t' \in p^\bullet$, $\exists r_p \in P_{R_i}$, ${}^\bullet t \cap P_{R_i} = t'^\bullet \cap P_{R_i} = \{r_p\}$;

(4) $\forall r \in P_{R_i}$, ${}^{\bullet\bullet} r \cap P_{A_i} = r^{\bullet\bullet} \cap P_{A_i} \neq \phi$ and ${}^\bullet r \cap r^\bullet = \phi$;

(5) ${}^{\bullet\bullet}(p_i^0) \cap P_{R_i} = (p_i^0)^{\bullet\bullet} \cap P_{R_i} = \phi$;

(6) N_i' is a strongly connected state machine, where $N_i' = (P_{A_i} \cup \{p_i^0\}, T_i, F_i)$ is the resulting net after the places in P_{R_i} and related arcs are removed from N_i. Every circuit of N_i' contains place p_i^0;

(7) any two N_i's are composable when they share a set of common places. Every shared place must be a resource place; and

(8) transitions in $(p_i^0)^\bullet$ and ${}^\bullet(p_i^0)$ are called source and sink transitions of an S³PR respectively.

In an S³PR, P^0 is called the set of process idle places, P_A is called the set of operation places and P_R is called the set of resource places.

LS³PR Nets

Definition 3 [28]. An S³PR $N=(P,T,F)$ is called a linear S³PR (LS³PR) if

(1) $P = P^0 \cup P_A \cup P_R$ is a partition of places, where

(1.a) $P^0 = \{p_1^0, p_2^0, \ldots, p_k^0\}$, $k>0$,

(1.b) $P_A = \bigcup_{i=1}^k P_{A_i}$, where $P_{A_i} \cap P_{A_j} = \phi$, for all $i \neq j$,

(1.c) $P_R = \{r_1, r_2, \ldots, r_n\}$, $n>0$;

(2) $T = \bigcup_{i=1}^k T_i$, where $T_{A_i} \cap T_{A_j} = \phi$, for all $i \neq j$;

(3) $\forall i \in \{1,2,\ldots,k\}$, the subnet N_i generated by $\{p_i^0\} \cup P_{A_i} \cup T_i$, is a strongly connected state machine such that every cycle of N_i contains place $\{p_i^0\}$ and $\forall p \in P_{A_i}$, $|p^\bullet| = 1$;

(4) $\forall i \in \{1,2,\ldots,k\}$, $\forall p \in P_{A_i}$, ${}^{\bullet\bullet} p \cap P_R = p^{\bullet\bullet} \cap P_R$ and $|{}^{\bullet\bullet} p \cap P_R| = 1$; and

(5) N is strongly connected.

Definition 4 [28]. Let $N=(P^0 \cup P_A \cup P_R, T, F)$ be an LS³PR. Given $p \in P_A$, if ${}^{\bullet\bullet} p \cap P_R = \{r_p\}$, r_p is called the resource used by p. For $r \in P_R$, $H(r) = {}^{\bullet\bullet} r \cap P_A$ is called the set of holders of r.

Definition 5 [28]. Let $N=(P^0 \cup P_A \cup P_R, T, F)$ be an LS³PR. An initial marking M_0 is called an admissible initial marking for N if

(1) $M_0(p^0) \geq 1, \forall p^0 \in P^0$;

(2) $M_0(p) = 0, \forall p \in P_A$; and

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(3) $M_0(r) \geq 1, \forall r \in P_R$.

REACHABILITY ANALYSIS FOR A PIPE-LINE NET (PLN)

Definition of a PLN

Definition 6. Let $N = (P^0 \cup P_A \cup P_R, T, F)$ be an LS[3]PR and N_i be the subnet generated by $\{p_i^0\} \cup P_{A_i} \cup T_i$. The two subnets N_i and N_j are said to be mutually reversed if $\exists r, r_1 \in P_R$ $(r \neq r_1)$ such that one of the two following statements holds: 1) $p_1 <_{N_i} p$ and $q <_{N_j} q_1$; and 2) $p <_{N_i} p_1$ and $q_1 <_{N_j} q$, where $p, p_1 \in P_{A_i}$, $q, q_1 \in P_{A_j}$, $i \neq j$, $p, q \in H(r)$, and $p_1, q_1 \in H(r_1)$.

The net shown in Figure 1 is an LS[3]PR with $P_0 = \{p_1, p_7, p_{11}\}$, $P_A = \{p_2, p_3, p_4, p_5, p_6, p_8, p_9, p_{10}, p_{11}, p_{12}, p_{13}\}$ and $P_R = \{p_{14}, p_{15}, p_{16}, p_{17}, p_{18}\}$. It consists of three subnets: N_1 generated by $\{p_1\} \cup \{p_2, p_3, p_4, p_5, p_6\} \cup \{t_1, t_2, t_3, t_4, t_5, t_6\}$, N_2 generated by $\{p_7\} \cup \{p_8, p_9, p_{10}\} \cup \{t_7, t_8, t_9, t_{10}\}$ and N_3 generated by $\{p_{11}\} \cup \{p_{12}, p_{13}\} \cup \{t_{11}, t_{12}, t_{13}\}$. In the net, $p_2, p_3, p_4, p_5, p_6 \in P_{A_1}$, $p_8, p_9, p_{10} \in P_{A_2}$, $p_{12}, p_{13} \in P_{A_3}$, $p_2, p_8 \in H(p_{14})$, $p_3, p_9 \in H(p_{15})$, $p_4, p_{10} \in H(p_{16})$, $p_5, p_{12} \in H(p_{17})$ and $p_6, p_{13} \in H(p_{18})$. Since $p_2 <_{N_1} p_3$ and $p_9 <_{N_2} p_8$; $p_3 <_{N_1} p_4$ and $p_{10} <_{N_2} p_9$; and $p_2 <_{N_1} p_4$ and $p_{10} <_{N_2} p_8$, the two subnets N_1 and N_2 are mutually reversed. Since $p_5 <_{N_1} p_6$ and $p_{12} <_{N_3} p_{13}$, the two subnets N_1 and N_3 are not mutually reversed.

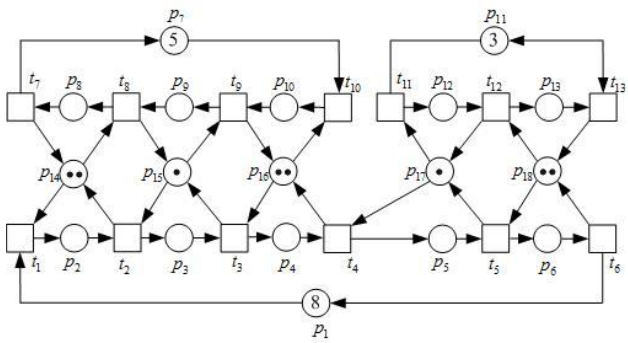

Figure 1. A Petri net model [28]

Definition 7. An LS[3]PR $N = (P^0 \cup P_A \cup P_R, T, F)$ is called a PLN if

(1) $\forall r \in P_R$, $\forall i \in \{1, 2, \ldots, k\}$, $|H(r)| = 2$ and $|H(r) \cap P_{A_i}| \leq 1$; and

(2) $q_1 \in q^{\bullet\bullet} \cap P_{A_j}$ holds if the two subnets N_i and N_j are mutually reversed with $p, p_1 \in P_{A_i}$, $q, q_1 \in P_{A_j}$, $p_1 \in^{\bullet\bullet} p \cap P_{A_i}$ $p, q \in H(r)$, $p_1, q_1 \in H(r_1)$, $r, r_1 \in P_R$ and $r \neq r_1$.

Let $N = (P^0 \cup P_A \cup P_R, T, F)$ be a PLN with $t \in T$. ${}^{(p)}t$ and $t^{(p)}$ denote the sets of input and output operation places of t respectively, and ${}^{(r)}t$ and $t^{(r)}$ denote the sets of input and output resources of t respectively. Hence ${}^{\bullet}t = {}^{(p)}t \cup {}^{(r)}t$ and $t^{\bullet} = t^{(p)} \cup t^{(r)}$.

As shown in Figure 1, $|H(p_{14})| = 2$, $|H(p_{15})| = 2$, $|H(p_{16})| = 2$, $|H(p_{17})| = 2$ and $|H(p_{18})| = 2$. Also we have $|H(p_{14}) \cap P_{A_1}| = 1$, $|H(p_{14}) \cap P_{A_2}| = 1$, $|H(p_{14}) \cap P_{A_3}| = 0$, $|H(p_{15}) \cap P_{A_1}| = 1$, $|H(p_{15}) \cap P_{A_2}| = 1$, $|H(p_{15}) \cap P_{A_3}| = 0$, $|H(p_{16}) \cap P_{A_1}| = 1$, $|H(p_{16}) \cap P_{A_2}| = 1$, $|H(p_{16}) \cap P_{A_3}| = 0$, $|H(p_{17}) \cap P_{A_1}| = 1$, $|H(p_{17}) \cap P_{A_2}| = 0$, $|H(p_{17}) \cap P_{A_3}| = 1$, $|H(p_{18}) \cap P_{A_1}| = 1$, $|H(p_{18}) \cap P_{A_2}| = 0$ and $|H(p_{18}) \cap P_{A_3}| = 1$. Moreover, in the two subnets N_1 and N_2,

$p_2, p_8 \in H(p_{14})$, $p_3, p_9 \in H(p_{15})$, $p_2 \in^{\bullet\bullet} p_3 \cap P_{A_1}$, $p_8 \in p_9^{\bullet\bullet} \cap P_{A_2}$, $p_3, p_9 \in H(p_{15})$, $p_4, p_{10} \in H(p_{16})$, $p_3 \in^{\bullet\bullet} p_4 \cap P_{A_1}$ and $p_9 \in p_{10}^{\bullet\bullet} \cap P_{A_2}$. Therefore, the net in Figure 1 is a PLN.

A Sufficient and Necessary Condition for Reachability in a PLN

Definition 8. Let $N = (P^0 \cup P_A \cup P_R, T, F)$ be a PLN with an admissible initial marking M_0. The maximum number of tokens in place p is called the bound of place p, denoted by b_p. That is to say, $b_p = \max\{M(p) \mid M \in R(N, M_0)\}$.

Lemma 1 [28]**.** Let $N = (P^0 \cup P_A \cup P_R, T, F)$ be an LS^{3}PR. The set of minimal P-semiflows of N is $I = I_R \cup I_{SM}$, where $I_R = \bigcup_{r \in P_R} H(r) \cup \{r\}$ and $I_{SM} = \bigcup_{i \in \{1,\ldots,k\}} P_{A_i} \cup \{p_i^0\}$.

Lemma 2. Let $N = (P^0 \cup P_A \cup P_R, T, F)$ be a PLN with an admissible initial marking M_0. Then $\forall p^0 \in P^0$, $b_{p^0} = M_0(p^0)$; $\forall r \in P_R$, $b_r = M_0(r)$; and $\forall p \in P_A$, $b_p = M_0(r_p)$.

Proof. From Lemma 1, the set of minimal P-semiflows of a PLN consists of two subsets. The first corresponds to the token conservation law associated with resources. $\forall r \in P_R$, the P-semiflow $I_r = H(r) \cup \{r\}$, states that for each reachable marking M , the token conservation law $\sum_{p \in H(r)} M(p) + M(r) = M_0(r)$ is true. The second subset is associated with the token conservation law for each state machine (in the sense of processes). $\forall i \in \{1,\ldots,k\}$, $p_i^0 \in P^0$, the P-semiflow $I_{SM_i} = P_{A_i} \cup \{p_i^0\}$, establishes the invariant relation $\sum_{p \in P_{A_i}} M(p) + M(p_i^0) = M_0(p_i^0)$ for each reachable marking M . Taking into account of $M \geq \mathbf{0}$, it is easy to see that $\forall p^0 \in P^0$, $b_{p^0} = M_0(p^0)$; $\forall r \in P_R$, $b_r = M_0(r)$; and $\forall p \in P_A$, $b_p = M_0(r_p)$.

Definition 9. Let $N = (P^0 \cup P_A \cup P_R, T, F)$ be a PLN. A circuit C that contains resources and transitions only is called a resource-transition circuit if ${}^{(r)}C_T = C_T{}^{(r)} = C_R$, where C_R and C_T denote the sets of all resources and transitions of C respectively.

Definition 10. Let $C(r_1, t_1, r_2, t_2, \ldots, r_m, t_m)$ be a resource-transition circuit in a PLN, where 1) $m \geq 2$; 2) $\forall i \in \{1,2,\ldots,m\}$, $r_i \in^{\bullet} t_i$; 3) $\forall i \in \{2,\ldots,m\}$, $r_i \in t_{i-1}{}^{\bullet}$; and 4) $r_1 \in t_m{}^{\bullet}$. r_i is called a connected resource if there exists $r_i = r_j$ in $C(r_1, t_1, r_2, t_2, \ldots, r_m, t_m)$, where $i, j \in \{1,2,\ldots,m\}$ and $i \neq j$.

As shown in Figure 1, there are three resource-transition circuits in the net: $C_1 = C(p_{14}, t_8, p_{15}, t_2)$, $C_2 = C(p_{15}, t_9, p_{16}, t_3)$ and $C_3 = C(p_{14}, t_8, p_{15}, t_9, p_{16}, t_3, p_{15}, t_2)$ with ${}^{(r)}C_{1_T} = C_{1_T}{}^{(r)} = \{p_{14}, p_{15}\} = C_{1_R}$, ${}^{(r)}C_{2_T} = C_{2_T}{}^{(r)} = \{p_{15}, p_{16}\} = C_{2_R}$ and ${}^{(r)}C_{3_T} = C_{3_T}{}^{(r)} = \{p_{14}, p_{15}, p_{16}\} = C_{3_R}$. p_{15} appears twice in C_3. Hence, p_{15} is a connected resource.

Theorem 1 [11]**.** Let $N = (P^0 \cup P_A \cup P_R, T, F)$ be a PLN with an admissible initial marking M_0. $S = S_A \cup S_R$ is a strict minimal siphon of N if

1) $S_A \neq \phi$, $S_R \neq \phi$;
2) $S_R = C_R$, where C_R is the set of all resources of a resource-transition circuit C in N; and
3) $S_A = \{p \mid p \in \bigcup_{r \in C_R} H(r) \wedge (p^{\bullet\bullet} \cap (P_A \cup P^0)) \text{Ø} \bigcup_{r \in C_R} H(r)\}$.

Corollary 1. Let $N = (P^0 \cup P_A \cup P_R, T, F)$ be a PLN and $S = S_A \cup S_R$ be a strict minimal siphon of N. Then $|S_A| = 2$.

Proof. From Theorem 1, we have $S_R = C_R$ and $S_A = \{p \mid p \in \bigcup_{r\in C_R} H(r) \wedge (p^{\bullet\bullet} \cap (P_A \cup P^0)) Ø \bigcup_{r\in C_R} H(r)\}$. We accordingly have the following two cases:

(1) $|S_R| = 2$. By the definition of resource-transition circuits, there necessarily exist two transitions in C_T that is the set of all the transitions of resource-transition circuit C associated with S. By the definition of a PLN, the two transitions in C_T necessarily belong to two subnets that are mutually reversed. Since $S_A = \{p \mid p \in \bigcup_{r\in C_R} H(r) \wedge (p^{\bullet\bullet} \cap (P_A \cup P^0)) Ø \bigcup_{r\in C_R} H(r)\}$, $\forall p \in S_A$, $p \in C_T^{\bullet}$ holds. Therefore, $|S_A| = 2$ is true.

(2) $|S_R| > 2$. By the definition of resource-transition circuits, there necessarily exist more than two transitions in C_T. By the definition of a PLN, the transitions in C_T necessarily belong to two subnets that are mutually reversed. Since $S_A = \{p \mid p \in \bigcup_{r\in C_R} H(r) \wedge (p^{\bullet\bullet} \cap (P_A \cup P^0)) Ø \bigcup_{r\in C_R} H(r)\}$, $\forall p \in S_A$, $p \in C_T^{\bullet}$ and $p \notin H(r_c)$ hold, where r_c is a connected resource in S_R. Therefore, $|S_A| = 2$ is true.

Definition 11. Let $N = (P^0 \cup P_A \cup P_R, T, F)$ be a PLN. An initial marking M_0 is called an appropriate initial marking of N if

(1) $M_0(p^0) \geq 1, \forall p^0 \in P^0$;
(2) $M_0(p) = 0, \forall p \in P_A$; and
(3) $\forall r \in P_R$; if r is a connected resource, $M_0(r) = 1$, otherwise $M_0(r) \geq 1$.

Definition 12. The markings in the set of invariant markings $I_X(N, M_0)$ that are not in the reachability set $R(N, M_0)$ are called spurious markings.

A backward firing in N is equivalent to a forward firing in the reverse net N'[30]. This implies that the directed path in the reachability graph of N' from M' to M is just the reverse path in the reachability graph of N from M to M'. Similarly, a spurious marking in N does not have directed paths from reachable markings and the corresponding marking in N' does not have directed paths to reachable markings.

Theorem 2. Let $N = (P^0 \cup P_A \cup P_R, T, F)$ be a PLN with an appropriate initial marking M_0, Π be the set of strict minimal siphons, and S be a strict minimal siphon in N. A marking M is spurious in the set of invariant markings of N if $\forall S \subseteq \Pi$, $M(S_A) = \sum_{p\in S_A} b_p$, where S_A is the set of operation places of S and b_p is the bound of place p.

Proof. 1. We first prove the sufficiency. By Definition 8, $\forall p \in S_A$, $b_p = \max\{M(p) \mid M \in R(N, M_0)\}$ holds. $\forall S \subseteq \Pi$, $M(S_A) = \sum_{p\in S_A} b_p$ means that the number of tokens in each operation place of any strict minimal siphon S reaches its bound at marking M. We have to prove that M is a spurious marking. That is to say, we need to show that there exist no directed paths from initial marking M_0 to M in N. By Corollary 1, $|S_A| = 2$. Without loss of

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generality, let $S_A = \{p_1, p_2\}$. From Theorem 1, $|S_R| \geq 2$ holds. We accordingly have the following two cases.

(1) $|S_R| = 2$. From Theorem 1, $S_R = \{r_{p_1}, r_{p_2}\}$ holds. From the proof of Corollary 1, there necessarily exist two transitions in C_T that is the set of all transitions of resource-transition circuit C associated with S, and $\forall p \in S_A$, $p \in C_T^\bullet$ holds. Without loss of generality, let $C_T = \{t_1, t_2\}$, $t_1 \in^\bullet p_1$, and $t_2 \in^\bullet p_2$.

By contradiction, suppose that M is reachable from M_0 in N with $M(p_1) = M_0(r_{p_1})$ and $M(p_2) = M_0(r_{p_2})$. By Definition 11, $M_0(p_1) = 0$ and $M_0(p_2) = 0$ hold. According to the token conservation law, $M(r_{p_1}) = 0$ and $M(r_{p_2}) = 0$ hold. Since M is reachable from M_0 in N, there necessarily exists a reachable marking M' in the reachability graph of N such that $M'[t_1\rangle M$ or $M'[t_2\rangle M$ holds. This implies that t_1 or t_2 must be enabled at M in the reverse net N' such that $M[t_1\rangle M'$ or $M[t_2\rangle M'$. By the definition of a PLN and Definition 10, $r_{p_1} \in t_2^\bullet$ and $r_{p_2} \in t_1^\bullet$ hold in N. This implies that $r_{p_1} \in^\bullet t_2$ and $r_{p_2} \in^\bullet t_1$ hold in the reverse net N'. Therefore, both t_1 and t_2 are disabled at M in the reverse net N', which contradicts that t_1 or t_2 is enabled at M in the reverse net N'. Thus, M is a spurious marking in N.

(2) $|S_R| > 2$. From Theorem 1, there necessarily exist connected resources in S_R. We denote R_C as the set of connected resources in S_R. From the proof of Corollary 1, there necessarily exist more than two transitions in C_T, and $\forall p \in S_A$, $p \in C_T^\bullet$ and $p \notin H(r_c)$ hold, where $r_c \in R_C$. Let $t_1 \in^\bullet p_1$ and $t_2 \in^\bullet p_2$. From Theorem 1, $r_{p_1} \notin R_C$ and $r_{p_2} \notin R_C$ hold. Since M_0 is an appropriate initial marking, $\forall r_c \in R_C$, $M_0(r_c) = 1$ holds.

By contradiction, suppose that M is reachable from M_0 in N with $M(p_1) = M_0(r_{p_1})$ and $M(p_2) = M_0(r_{p_2})$. From the proof of case (1), since there exist connected resources in S_R, $\forall r_c \in R_C$, $M(r_c) = 1$ may hold according to the token conservation law. Therefore, t_1 or t_2 may be enabled at M in the reverse net N'. Similarly, there exist a sequence of transitions σ in $R_C^\bullet$, which may be enabled from M in N' such that $M[\sigma\rangle M''$ with $M''(r_{p_2}) = 0$. Therefore, the transition $t \in r_{p_2}^\bullet$ must be disabled at M''. That is to say, M does not have directed path to M_0 in N'. This implies M_0 does not have directed path to M in N, which contradicts that M is reachable in N. Therefore, we can conclude that M is a spurious marking.

2. We prove the necessity. By contradiction, suppose that $\forall S \subseteq \Pi$, $M(S_A) \neq \sum_{p \in S_A} b_p$. Since M is a marking in the set of invariant markings of N, the token conservation law $\sum_{p \in H(r)} M(p) + M(r) = M_0(r)$ is true. From Lemma 2, $\forall p \in S_A$, $b_p = M_0(r_p)$ holds. Note that $M \geq \mathbf{0}$. We can conclude that $\forall p \in S_A$, $M(p) \leq b_p$. Therefore, $M(S_A) < \sum_{p \in S_A} b_p$ holds.

$M(S_A) < \sum_{p \in S_A} b_p$ means that at marking M the number of tokens in each operation place of S does not reach its bound at the same time. From the proof of the sufficiency, we can similarly prove that M is reachable from M_0, which contradicts that M is a spurious marking. Therefore, if M is a spurious marking in the set of invariant markings of N, $\forall S \subseteq \Pi$, $M(S_A) = \sum_{p \in S_A} b_p$ holds.

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An Algorithm Computing the Set of Reachable Markings

From Theorem 2, given a PLN with an appropriate initial marking M_0, all the spurious markings can be identified from the set of invariant markings. Then the set of reachable markings can be calculated by removing all the spurious markings from the set of invariant markings. An algorithm to compute the set of markings reachable from M_0 is presented as follows:

Algorithm 1. Computation of reachable markings

Input: a PLN model (N, M_0).

Output: The reachability set $R(N, M_0)$.

1) Check if N is a PLN and M_0 is an appropriate initial marking. If not, exit.
2) Compute the set of minimal P-semiflows by Lemma 1.
3) Compute the bounds of all the places by Lemma 2. According to the token conservation law, enumerate the set of invariant markings $I_X(N,M_0)=\{M \in \mathrm{IN}^{|P|} \mid X^T M = X^T M_0\}$, where X is a matrix whose column each is a P-semiflow of the net (N, M_0), and $\mathrm{IN}^{|P|}$ is a set of non-negative vectors with a length of $|P|$.
4) Compute the set C of resource-transition circuits by Definition 9.
5) Compute the set of strict minimal siphons $\prod$ due to Theorem 1.
6) **if** $\{\prod = \phi\}$ **then** $R(N,M_0)=I_X(N,M_0)$.
   else $R(N,M_0)=I_X(N,M_0)\setminus\{M \mid \forall S \subseteq \Pi, M(S_A)=\sum_{p\in S_A} b_p\}$.
7) Output $R(N, M_0)$.
8) End.

AN EXAMPLE

To practically test the efficiency of the proposed method, a C program has been developed, which implements the algorithm and runs on a Windows XP operating system with Intel CPU Core 2.60 GHz and 3 GB memory.

Take the net in Figure 1 as an example. There are 18 places and 13 transitions. It is a PLN and M_0 is an appropriate initial marking. By Lemma 1, the net has eight minimal P-semiflows as follows: $I_{SM_1}=\{p_1,p_2,p_3,p_4,p_5,p_6\}$, $I_{SM_2}=\{p_7,p_8,p_9,p_{10}\}$, $I_{SM_3}=\{p_{11},p_{12},p_{13}\}$, $I_{r_1}=\{p_2,p_8,p_{14}\}$, $I_{r_2}=\{p_3,p_9,p_{15}\}$, $I_{r_3}=\{p_4,p_{10},p_{16}\}$, $I_{r_4}=\{p_5,p_{12},p_{17}\}$ and $I_{r_5}=\{p_6,p_{13},p_{18}\}$.

By Lemma 2, the bounds of all the places are as follows: $b_{p_1}=8$, $b_{p_2}=2$, $b_{p_3}=1$, $b_{p_4}=2$, $b_{p_5}=1$, $b_{p_6}=2$, $b_{p_7}=5$, $b_{p_8}=2$, $b_{p_9}=1$, $b_{p_{10}}=2$, $b_{p_{11}}=3$, $b_{p_{12}}=1$, $b_{p_{13}}=2$, $b_{p_{14}}=2$, $b_{p_{15}}=1$, $b_{p_{16}}=2$, $b_{p_{17}}=1$ and $b_{p_{18}}=2$. By the definition of the set invariant markings $I_X(N,M_0)$, we can obtain $I_X(N,M_0)$ that has 1944 markings.

By Definition 8, the net has three resource-transition circuits: $C_1=C(p_{14},t_8,p_{15},t_2)$, $C_2=C(p_{15},t_9,p_{16},t_3)$ and $C_3=C(p_{14},t_8,p_{15},t_9,p_{16},t_3,p_{15},t_2)$. By Theorem 1 we can find that the net correspondingly has three strict minimal siphons: $S_1=\{p_3,p_8,p_{14},p_{15}\}$, $S_2=\{p_4,p_9,p_{15},p_{16}\}$ and $S_3=\{p_4,p_8,p_{14},p_{15},p_{16}\}$.

By Theorem 2, a marking M with $M(p_3)=2$ and $M(p_8)=1$ is spurious in $I_X(N,M_0)$, a marking M' with $M'(p_4)=2$ and $M'(p_9)=1$ is spurious, and a marking M'' with $M''(p_4)=2$ and $M''(p_8)=2$ is also spurious. We impose the constraints on $I_X(N,M_0)$ as follows:

$M(p_4)+M(p_9)<3$, $M(p_4)+M(p_9)<3$ and $M(p_4)+M(p_8)<4$, where $M \in I_X(N,M_0)$. Then the reachability set $R(N,M_0)$ that has 1710 markings is generated by removing 234 spurious ones from $I_X(N,M_0)$.

The software package INA2003 [30] can also compute the reachability set. For comparison, reachability analysis of the Petri net in Figure 1 is conducted by the use of INA. The tool generates the reachability set consisting of 1710 markings, which are in agreement with the markings in the reachability set $R(N,M_0)$ generated by the proposed method, validating the correctness of the proposed algorithm.

TINA [31] is a toolbox for editing and analyzing Petri nets, which can also compute the reachability set. For comparison, reachability analysis of the Petri net in Figure 1 is conducted by the use of TINA. The toolbox generates a reachability set consisting of 1710 markings, which are also in agreement with the markings in the reachability set $R(N,M_0)$ generated by the proposed method.

EXPERIMENTAL RESULTS

The net structure in Figure 1 is selected for experimental studies. We vary the initial markings of resource places p_{14}, p_{15}, p_{16}, p_{17} and p_{18}, and idle places p_1, p_7 and p_{12}. Table 1 shows various parameters in the net, where the first column represents the initial tokens in places p_1, p_7, p_{12}, p_{14}, p_{15}, p_{16}, p_{17} and p_{18}. N_I, N_S and N_R indicate the numbers of invariant markings, spurious markings and reachable markings respectively. The fifth column shows the total CPU time for computing $R(N,M_0)$ by using the proposed method. The sixth and the last columns show the total CPU time for computing $R(N,M_0)$ by using INA and the total CPU time for computing $R(N,M_0)$ by using TINA for comparison purpose respectively.

Table 1. Parameters in the model depicted in Figure 1 with varying markings

p_1, p_7, p_{12}, p_{14}, p_{15}, p_{16}, p_{17}, p_{18}	N_I	N_S	N_R	CPU time (s)	INA time (s)	TINA time (s)
8, 5, 3, 2, 1, 2, 1,2	1,944	234	1,710	<1	<1	<1
15, 9, 6, 4, 1, 4, 2, 4	60,750	2,790	57,960	<1	216	3
29, 17, 12, 8, 1, 8, 4, 8	4,100,625	61,425	4,039,200	22	>7200	—
36, 21, 15, 10, 1, 10, 5, 10	18,112,248	184,338	17,927,910	398	—	—
43, 25, 18, 12, 1, 12, 6, 12	63,299,964	466,284	62,833,680	1432	—	—
50, 29, 21, 14, 1, 14, 7, 14	186,624,000	1,041,120	185,582,880	5365	—	—

As shown in Table 1, we can see that the proposed method becomes more efficient with the increase of the initial markings. Note that "—" in Table 1 means that the computation cannot be finished with a reasonable time or memory is overflowed.

CONCLUSIONS

The set of reachable markings play an important role in the deadlock control in Petri nets. This paper presents a novel approach in computing the set of reachable markings using P-invariants and strict minimal siphons without the construction of reachability graph that often makes the analysis intractable. The method is applied to a small class of Petri nets called PLNs that are a subclass of LS^3PR. Experimental results show its efficiency via studying a number of examples. Future work should extend the method in this paper to more general classes of Petri nets.

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