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Technical Note

Development of a rehabilitation apparatus to actuate upper extremity passive motion

Chung-shun Hung [1], Ching-hua Chiu [2,*], Yi-hung Ho [1] and Wei-lin Cheng [1]

[1] Department of Bio-Industrial Mechatronics Engineering, The College of Agriculture and Natural Resources, National Chung Hsing University, 250 KuoKuang Road, Taichung, Taiwan 402, the R.O.C.
[2] Graduate Institute of Sports and Health Management, The College of Social Sciences and Management, National Chung Hsing University, 250 KuoKuang Road, Taichung, Taiwan 402, the R.O.C.

* Corresponding author, e-mail: chungoodman@yahoo.com.tw

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Abstract: An apparatus that can induce upper extremity passive motion in stroke patients was developed with the goal of providing rehabilitation for these individuals. The rehabilitation device consists of a robotic arm controlled by a computer interface and programmed to effect passive extension and flexion of the patient's elbow and fingers. The load imposed on the upper limb and fingers was analysed. A high-speed video camera captured the trajectory of the subject's arms and a kinetic model based on the arm structure was employed to analyse the trajectory. Results showed that the range of motion (ROM) of the subjects' elbows was between 77-158° (about 80° ROM) and the average range of movement from extension to flexion of the fingers was approximately 3 cm. The average loading moment on the shoulder and elbow was below 12 N-m. By varying the angular position and angular velocity of the robotic arm, the programme reproduced the motions of different arm functions. Thus, the present study shows that the apparatus provides safe and effective simultaneous rehabilitation of both the elbow and fingers, and that the use of this device may benefit patients undergoing rehabilitation in a clinical setting.

Keywords: rehabilitation apparatus, post-stroke patient, upper extremity passive motion

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INTRODUCTION

Stroke occurs when circulation of the blood is suddenly blocked, leading to oxygen deficiency in the brain. The effect of stroke is closely related to the distribution of the patients' brain vessels, the extent of concomitant disease processes, and the age of the patients at the onset of stroke. There are two types of stroke: hemorrhagic and nonhemorrhagic [1]. Stroke sufferers may experience partial paralysis of some limbs and in serious cases may totally lose function in affected body parts. One possible way to recover the original function in an affected limb is through targeted exercise with the assistance of an orthosis.

Jorgenson et al. [2] reported that two-thirds of stroke patients experienced paralysis of their upper limbs and were unable to move them freely. Even after 6 months of rehabilitation, more than 50% of those patients still felt no strength in their upper limbs and were incapable of moving them autonomously [3]. In a typical stroke case, the patient cannot maintain the flexion and extension of the elbow, wrist and fingers. Consequently, much research has focused on the use of gravity support to help patients effectively stretch their fingers and relax the finger flexor.

Housman et al. [4] observed that when a physiotherapist offered gravity assistance with a nonrobotic (despite the name) therapy Wilmington robotic exoskeleton (T-WREX), the patient's arm became more flexible, paralysis was relieved, and the arm's passive motion range significantly increased. In a study by Seo et al. [5], the grip strength of stroke patients trained with proper arm support was greatly enhanced. Meijer et al. [6] designed a device, the Handmaster orthosis, which was intended to help patients flex and extend their wrists. Their findings indicated that patients who used the device for only a short period experienced a remarkable increase in the passive range of motion (ROM). Similar results were also reported by Alon et al. [7], who found that the Handmaster system could improve the grip strength and the active ROM of patients' fingers.

After observing patients' conditions, doctors and physiotherapists tend to have patients engage in rehabilitation to improve muscle strength and ROM of joints. However, objective monitoring of rehabilitation progress is difficult, especially when the number of patients under assessment is very large. Therefore, robot-assisted rehabilitation has become increasingly popular, and research and development efforts in pursuit of such devices have rapidly expanded. For example, Mehrholz et al. [8] reviewed 11 trials in which electromechanical and robot-assisted arm training devices were used to improve arm function and activities of daily life. Their findings revealed that although the arm motor function and strength improved, significant improvement in the activities of daily life was lacking. Robots designed to assist stroke patients in recovering their arm muscle strength and autonomous motion have focused on activities that improve immobile strength and endurance, and results of clinical testing and follow-up diagnoses by physicians have confirmed previous conclusions that such rehabilitative efforts produced their intended effects [9-10].

From our literature review, numerous methods aimed at rehabilitating the upper extremities of patients have already been investigated. These approaches to rehabilitation concentrated on recovering patients' mobility, especially by improving the muscle strength and flexibility of the arms and fingers. However, most of the rehabilitation devices described were not versatile because they were designed to act on a single joint or body part. In view of this, the present study attempts to accomplish the following two tasks: (1) development of a convenient rehabilitation apparatus for the upper extremities that can produce passive flexion and extension of patients' elbows and fingers; and (2) examination of the load imposed on the patients' arms through analysis of the passive flexion and extension movements of the elbows and fingers.

METHODS

Abbreviations and Definitions

$\boldsymbol{\tau}_i$ = generalised force
q_i = generalised coordinate
$\dot{\boldsymbol{q}}_i$ = generalised velocity
$\ddot{\boldsymbol{q}}_i$ = generalised acceleration
${}^{i-1}E_i$ = homogeneous transformation matrix of ith coordinate frame relative to i-1th coordinate frame (see Appendix A)
${}^{0}E_j$ = coordinate transformation matrix from 0 coordinate frame to jth coordinate frame, and the equation could be written as ${}^{0}A_j = {}^{0}A_1{}^{1}A_2{}^{2}A_3 \ldots {}^{j-1}A_j$
$\boldsymbol{U}_{ij}$ is defined as $({}^{0}E_i/(q_j$ $(i,j = 1,2,\ldots,5)$
$\boldsymbol{U}_{ijk}$ is defined as $({}_0\boldsymbol{U}_{ij}/(q_k$ $(i,j,k = 1,2,\ldots,5)$
$\boldsymbol{J}_i$ = pseudo-inertia matrix 29–32
$G = [0,0,-|g|,0]$, $g = 9.8062\ \mathrm{ms}^{-2}$
$\boldsymbol{m}_j$ = mass of jth link
$\boldsymbol{r}_j = (\bar{x}_j, \bar{y}_j, \bar{z}_j, 1)^T$, position of centre of mass for jth link

Kinematic Model of Upper Extremity

The kinematic model of the human upper extremity used in this study is composed of five degrees of freedom (q_i; $i = 1, 2, \ldots, 5$), based on the structure of the arm joints. The shoulder joint (q_3) comprises three degrees of freedom (Figure 1). Two degrees of freedom describe the translation relative to the X-axis (q_1) and Y-axis (q_2), and the third is associated with the rotation of the shoulder joint. The fourth degree of freedom reflects the rotation of the elbow joint (q_4) and the fifth is associated with the rotation of the wrist joint (q_5). The symbol θ represents the joint angle of the elbow. The segment lengths of the upper arm, forearm and palm are represented by $\boldsymbol{l}_1$, $\boldsymbol{l}_2$ and $\boldsymbol{l}_3$ respectively. In Figure 1, $\boldsymbol{p}_i$ represents the translation from the origin of the coordinate frame of the ith segment to the coordinate frame of the i-1th link; $\boldsymbol{p}_i = [x_i, y_i, z_i]^T$, and r_i represents the position vectors of the centre of mass of segment i.

The five degrees of freedom serve as variables in this upper extremity model (Table 1). Each degree of freedom is assumed to be a coordinate frame. Consequently, according to the Lagrange-Euler equations of motion [11-16], this model can be represented as:

$$\tau_i = \sum_{j=i}^{n} \sum_{k=1}^{j} \mathrm{Trace}(\boldsymbol{U}_{jk}\boldsymbol{J}_j\boldsymbol{U}^T{}_{ji})\,\ddot{q}_k + \sum_{j=i}^{n} \sum_{k=i}^{j} \sum_{m=1}^{j} \mathrm{Trace}(\boldsymbol{U}_{jkm}\boldsymbol{J}_j\boldsymbol{U}^T{}_{ji})\,\dot{q}_k\,\dot{q}_m$$
$$-\sum_{j=i}^{n} (\boldsymbol{m}_j\boldsymbol{G}\boldsymbol{U}_{ji}\,r_j) \quad i,j,k = 1,2,3,\ldots,5. \qquad (1)$$

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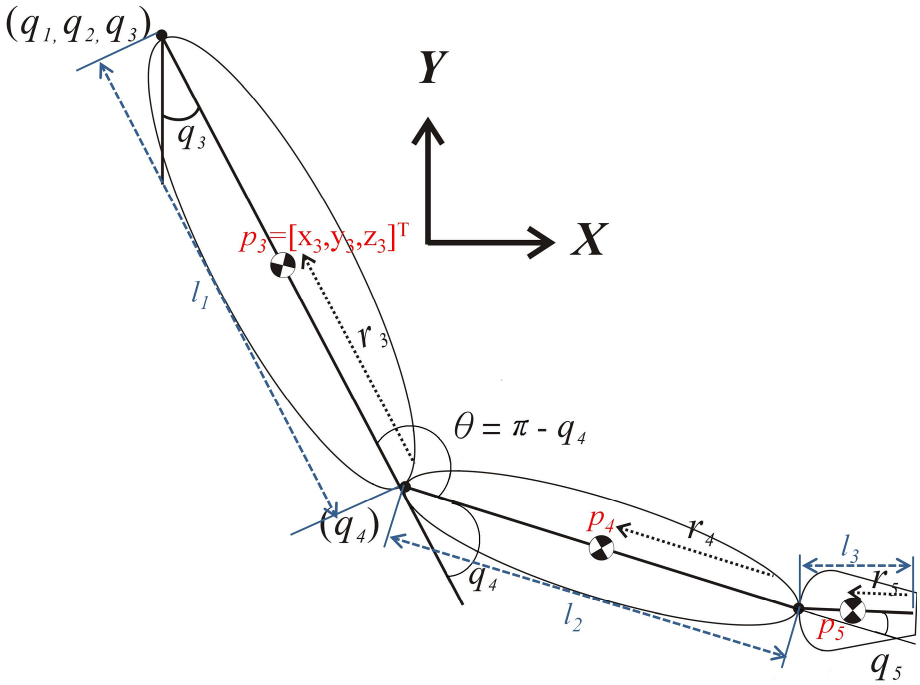

Figure 1. Upper extremity model

Table 1. Homogeneous transformation matrices and parameters for the arm model (see Appendix A)

Variable	${}^{i-1}E_i$ type	x_i	y_i	z_i	${}^{0}E_i$
q_1	${}^{0}E_1 = \boldsymbol{E}_1^{(x)}$	0	0	0	${}^{0}E_1 = \boldsymbol{E}_1^{(x)}$
q_2	${}^{1}E_2 = \boldsymbol{E}_2^{(y)}$	0	0	0	${}^{0}E_2 = \boldsymbol{E}_1^{(x)}\ \boldsymbol{E}_2^{(y)}$
q_3	${}^{2}E_3 = \boldsymbol{E}_3^{(R)}$	0	l_1	0	${}^{0}E_3 = \boldsymbol{E}_1^{(x)}\ \boldsymbol{E}_2^{(y)}\ \boldsymbol{E}_3^{(R)}$
q_4	${}^{3}E_4 = \boldsymbol{E}_4^{(R)}$	0	l_2	0	${}^{0}E_4 = \boldsymbol{E}_1^{(x)}\ \boldsymbol{E}_2^{(y)}\ \boldsymbol{E}_3^{(R)}\ \boldsymbol{E}_4^{(R)}$
q_5	${}^{4}E_5 = \boldsymbol{E}_5^{(R)}$	0	l_3	0	${}^{0}E_5 = \boldsymbol{E}_1^{(x)}\ \boldsymbol{E}_2^{(y)}\ \boldsymbol{E}_3^{(R)}\ \boldsymbol{E}_4^{(R)}\ \boldsymbol{E}_5^{(R)}$

The segment parameters required by the dynamic system include length, mass and inertia tensor. Zatsiorsky and Seluyanov [17] adopted a gamma-ray scanning technique in their study on human segment parameters. Compared with other studies, their data were more precise and complete. Therefore, the segment parameters used in the dynamic system of this study are based on the data established by Zatsiorsky and Seluyanov [17].

Subjects and Rehabilitation Apparatus

Fifteen stroke patients from China Medical University Hospital (age: 61.7 ± 11.5 years; height: 1.62 ± 0.08 m; weight: 64.6 ± 10.0 kg) participated in the study. All of the participants provided written consent to the experimental protocol approved by the institutional review board.

The rehabilitation apparatus used in the present study is designed for rehabilitation of the elbow and fingers. The mechanical components related to elbow rehabilitation include the axis of rotation, link bar, starting device and elbow bearer (Figure 2). To ensure the safety of the subject when using the apparatus, a limit-switch sensor is attached to the axis of rotation to prevent elbow injury due to over-bending.

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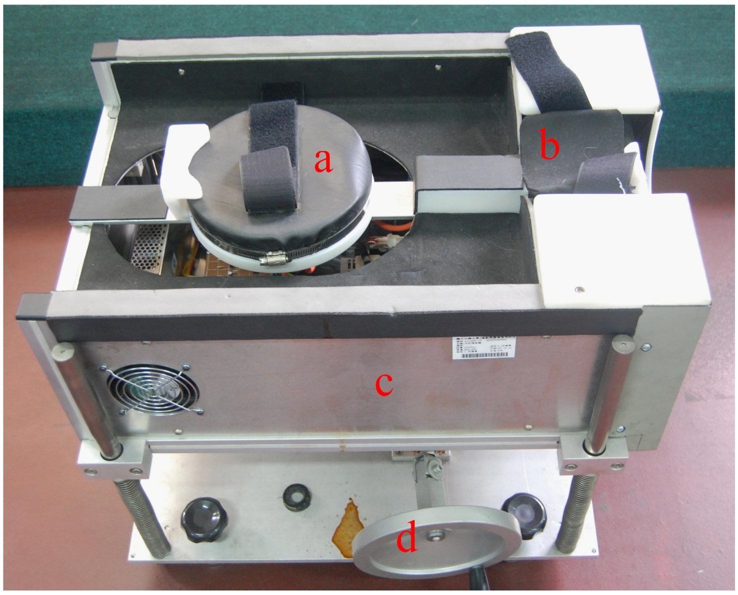

Figure 2. Rehabilitation apparatus: (a) filling solenoid, (b) elbow bearer, (c) axis of rotation, link bar and starting device in box, and (d) lifting mechanism

The components of the apparatus for elbow rehabilitation were carefully designed with comfort and safety in mind. Patients using this apparatus begin by resting the elbow on a plate (Figure 2, speed range = 0.1-0.8 ± 0.01 rad/s, range of joint = 0-90 ± 0.1°, and maximum joint moment = 45 ± 0.1 N-m for this apparatus). The height of the plate can be adjusted for maximal comfort.

The major mechanical components involved in fingers rehabilitation comprise a gas pocket and filling solenoid (Figure 2(a)). Patients or users wear velcro gloves and their hand and fingers are attached to the filling solenoid with velcro. The filling solenoid is connected to a gas pocket in order to inflate it at intervals, allowing repeated flexion of the patient's fingers.

The control interface of this apparatus was developed in Borland C^{++} programming language and the operating procedure for this apparatus is shown in Figure 3. The transmission of communication signals for motor control is via a universal serial bus (USB) interface. After starting the apparatus, the user enters the control parameters into the computer. These parameters comprise extension frequency, flexion frequency and joint angles. The input parameters function as the preconditions for controlling the elbow's initial angular position and angular velocity, and also serve as parameters for the motion procedures. The user then starts the motor to inflate the gas solenoid, compelling the fingers to extend and flex.

In these experiments, a high-speed video camera (120 Hz) was positioned at the side of the subject to capture upper extremity motions in two dimensions (for example, Figure 4). Five markers were placed at the following anatomical positions: right third phalanx, right styloid process of the radius, right lateral epicondyle of the humerus, right acromion joint, and the lateral midpoint of trunk. The markers' locations in the captured video were processed digitally. A programme developed in Borland C++ language was used to compute the angular position, angular velocity and passive loading moment of the elbow and shoulder joints.

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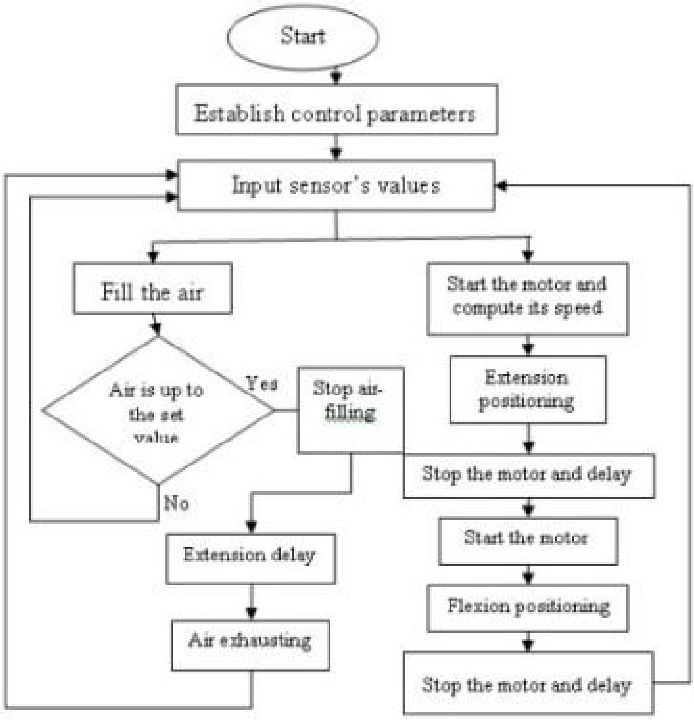
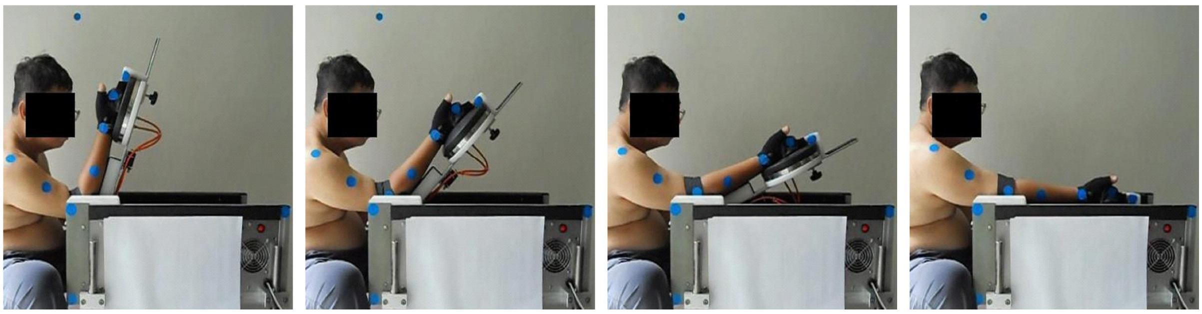


Figure 3. Procedure for using the apparatus



(a) (b) (c) (d)

Figure 4. Consecutive rehabilitation motions: (a) initial state, in which control parameters are input into computer; apparatus is then started; (b, c) extension of elbow joint; (d) extension of elbow to maximal angle. As elbow is extended, gas pocket is inflated to compel fingers to extend until maximal elbow extension angle is reached. The procedure is then reversed from (d) to (c), (b) and finally (a). The motion can be repeated to achieve continuous rehabilitation.

RESULTS AND DISCUSSION

When the subject's elbow was extended by the robotic arm, the gas pocket was inflated to compel passive plantarflexion of the subject's fingers. The markers' positions were captured by the video motion system and used to compute the height of gas pocket during filling and exhausting (Figure 5). The angular position, angular velocity and passive loading moment of the elbow and shoulder joints were calculated using programmes. A collected data sample from one patient is shown in Figure 6. The joint motions of shoulder and elbow for rehabilitation were observed and evaluated. Then, the experimental results of all subjects were calculated.

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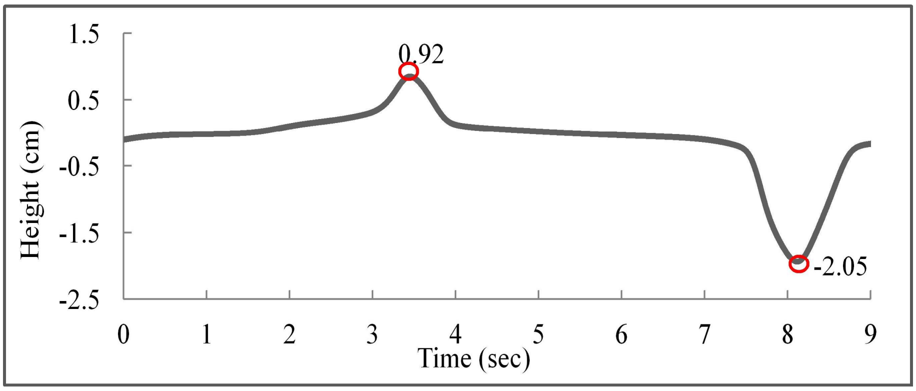

Figure 5. Average height of gas pocket during filling and exhausting (maximum: 0.92 ± 0.24 cm within 3.4 ± 0.11 s; minimum: –2.05 ± 0.46 cm within 8.2 ± 0.1 s).

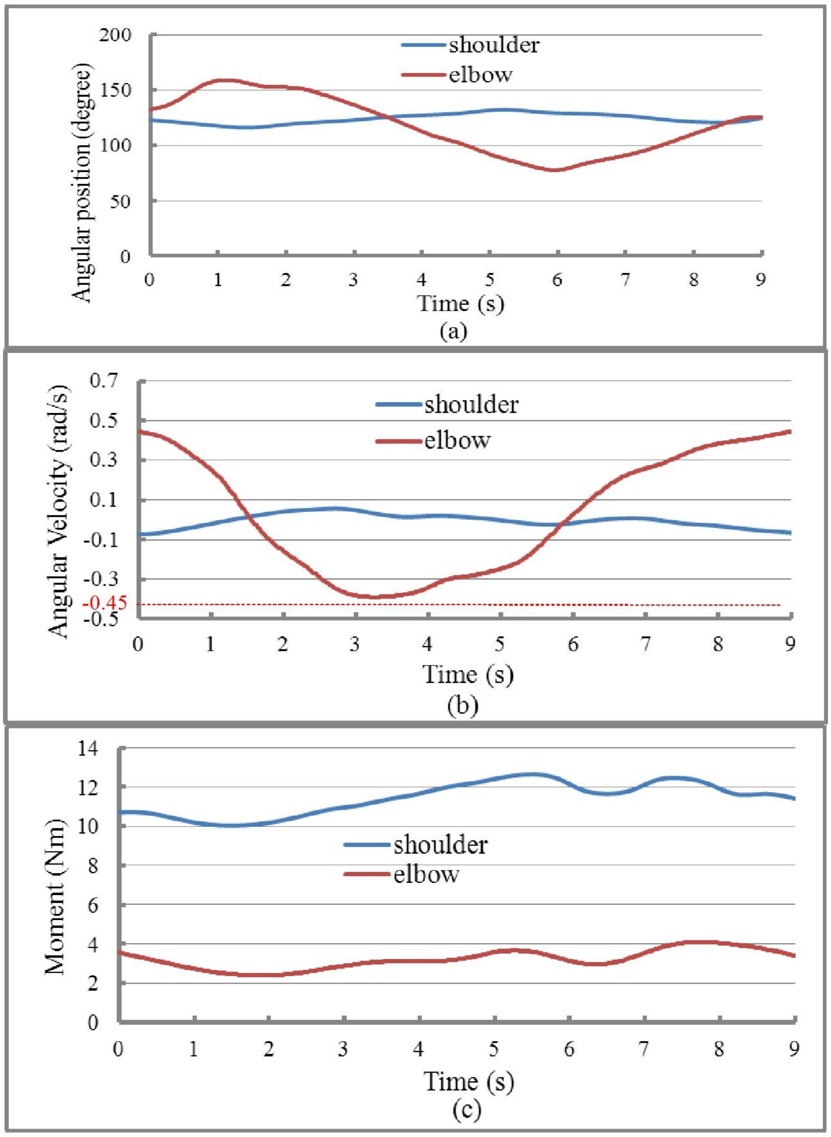

Figure 6. Sample results from one subject: (a) shoulder's angular position (q_3) and elbow's angular position (q_4); (b) angular velocities ($\dot{q}_3$, $\dot{q}_4$) of shoulder and elbow ; (c) passive moment of shoulder and elbow joints

According to the study by Wade et al. [18], stroke patients usually experience severe arm paralysis at the initial rehabilitation stage. To regain the level of function prior to the stroke, patients normally require at least 6 months of rehabilitation, during which time the arm needs to be constantly mobilised [19]. In the process of flexing and extending the shoulder and elbow joints provided by our apparatus (for example, Figure 6(a)), little variation was observed in the shoulder flexion angle whereas the elbow underwent more variation in flexion angle. The apparatus mainly provided more rehabilitation activity in the elbow than in the shoulder. The elbow activity could then provide muscle rehabilitation of the upper arm and forearm. In the present study the subjects' range of elbow motion, i.e. the average flexion angle (θ) for the elbow, was from $77.6 \pm 8.1°$ to $158.6 \pm 14.6°$ (about 80° ROM), which is within the range of ordinary elbow motions [14] and thus unlikely to cause injury.

The programmed shoulder and elbow angular velocities during extending were both below –0.45 rad/s (negative angular velocity) (as in Figure 6(b)), which are slower than the velocities during normal activity [14]. For rehabilitating stroke patients, sudden and dramatic motions are not appropriate. Therefore, the device should be stabilised when varying the ROM of the robotic arm [8].

We found that the profiles of the passive moment applied to the elbow and shoulder joints were similar (Figure 6(c)) because both joints were compelled to move by the robotic arm. The average moment of the subjects' shoulders was 11.2 ± 2.11 N-m (below 12 N-m), which is almost equivalent to the load induced by ordinary arm activity in daily life [12]. Because increased joint moment is helpful for neuromuscular strengthening, increasing joint moment is one of the rehabilitation device's functions. The apparatus compels the arm to perform motions in a repeated and fixed pattern, effectively stimulating muscles and cutaneous receptors, thereby allowing patients to gain better neuromuscular control. With the aid of this apparatus, therefore, the patients seemed to be able to increase their joint moment and recover their muscle strength.

The apparatus also provides rehabilitation motions for fingers (Figure 2(a)) to promote grip strength and increase their range of movement [7]. Periodic inflation of the gas pocket facilitates passive extension and flexion of the fingers (Figure 5). When the robotic arm returns to its initial position, the fingers are returned to passive dorsiflexion. In the present study, the range of extension and flexion of the fingers is controlled by the height of the gas pocket, which changes from 0.92 ± 0.24 cm to -2.05 ± 0.46 cm, the full average rehabilitation range from extension to flexion being approximately 3 cm. The range from extension to flexion is used to assess the motion activity of fingers and the progress of muscle rehabilitation.

There are some advantages to using a mechanical rehabilitation apparatus in place of manual passive motion by a therapist. In addition to the operational costs being reduced, rehabilitative motions will be more reproducible and technical errors will be less likely to occur [20]. The rehabilitation apparatus designed in the present study is intended for use with stroke patients. With this apparatus, motions of patients' upper extremities can be computer-controlled. Such controlled activities may help patients recover arm strength and active ROM by inducing motions of specified magnitude, duration and time [9-10]. The USB connection was adopted because of its 'plug and play' characteristics, which improve convenience and reduce operator errors. Our apparatus and its

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control programme allow users to easily monitor rehabilitation progress and adjust the control parameters and operating regime accordingly on the computer.

Most rehabilitation devices are intended to operate on a single body part. For example, Dobbe et al. [21] invented a finger rehabilitation device employing a constant-force spring motor. Our apparatus offers dual function, rehabilitating the elbow and fingers simultaneously. Efficiency is enhanced because less rehabilitation time is required. It can also offer different motion programmes with variation in ROM and angular velocity without risking injury to the patients. These motion programmes and operational functions can be applied in clinical practice because they are safe and convenient to use.

CONCLUSIONS

The apparatus described in this report is safe and helpful for rehabilitating patients. With this apparatus, rehabilitation courses can be more diverse and thus more appealing. Future research should explore how the factors affecting the speed and range of the mechanical device optimise rehabilitation of the upper extremities. Forthcoming efforts should also include promotion of the device's efficacy and comfort when used in clinical practice, leading to enhanced rehabilitative effects.

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APPENDIX A

Chiu [22] defined seven types (*CH-7T*) of homogeneous transformation matrices [11-15]. This study adopted three of the seven types to design the LE equations. For example, in the basic homogeneous rotation matrix (Table 1) using equations (2–4), the symbol q_i represents a generalised coordinate as a joint variable associated with the ith link. The translation from the origin of the ith link coordinate frame relative to the i-1th link coordinate frame is represented by p_i, where $p_i = [\, x_i, y_i, 0\,]^T$.

$$E_i^{(x)} = \begin{bmatrix} 1 & 0 & 0 & q_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$E_i^{(y)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$E_i^{(R)} = \begin{bmatrix} \cos q_i & -\sin q_i & 0 & 0 \\ \sin q_i & \cos q_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & x_i \\ 0 & 1 & 0 & y_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

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