

Full Paper

## **Determination of proportionality constants from cutting force modelling experiments during broaching operation**

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**Abstract:** Mechanistic model assumes that the types of cutting force acting on the broach teeth, namely axial force, normal force and lateral force, are proportional to chip-thickness area. In this paper, the proportionality constants related to the cutting force and chip-thickness area were obtained through experimentation. The shaping process was used to determine the proportionality constants in terms of specific cutting energy constants. The paper also includes static force modelling for broaching operation and graphical presentation of the experimental and simulated results.

**Keywords:** metal cutting, cutting force, broaching operation

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### **Introduction**

Metal cutting is one of the basic operations in manufacturing industries to produce the parts of desired dimensions and shape. Metal cutting constitutes a complex process involving the diversity of physical phenomena such as large plastic deformation, frictional contact, thermo-chemical coupling, and chip and burr formation mechanisms. A great deal of research [1-9] has been devoted to understanding the mechanism of machining with the objective of obtaining more effective tools and manufacturing operations.

Over the last several years, research has been carried out to develop mechanistic force models for a variety of machining processes, including end milling, face milling, boring, turning and broaching. These models have been employed in a number of designs, operation plannings and process control settings to predict both the cutting force and the resulting machine surface error.

In the literature, two different approaches have been adopted for the prediction of the cutting force system. The first method is based on the work done by Merchant [1] and involves a study of the cutting mechanics and the prediction of the shear angle in metal cutting. Both analytical and empirical models for shear angle prediction have been attempted. Lee and Shaffer [6] applied slip line theory to machining to develop the equation for predicting the shear angle. Usui et al. [10] developed a model that was based on the minimum energy criterion for predicting the chip flow angle and empirical models were used for predicting both the friction angle and the shear angle. This approach generally requires experimentation of a more fundamental nature of cutting mechanics to achieve the measurement and prediction of the shear angle.

In this paper, an attempt has been made to determine the specific energy constants experimentally for mild steel, aluminium and cast iron. Shaping process was used for the experimental purpose of determining specific energy constants and proportionality constants which are used in mechanistic modelling during broaching operation. A mechanistic model has been developed to compute static forces. The experimental results and simulated results by mechanistic modelling are presented graphically.

## Methods

### *Computation of specific cutting energy constant for predicting the cutting forces—mechanistic modelling approach*

In the mechanistic modelling approach, for any machining process the basic equations that relate the axial force  $F_a$ , normal force  $F_n$  and lateral force  $F_t$  to the chip-thickness area are given by [10]:

$$\begin{aligned} F_a &= K_a A_c \\ F_n &= K_n A_c \\ F_t &= K_t A_c \end{aligned} \quad (1)$$

where  $F_a$ ,  $F_n$  and  $F_t$  are the three-dimensional forces acting on the tool tip.  $A_c$  is the chip-thickness area and  $K_a$ ,  $K_n$  and  $K_t$  are the proportionality constants corresponding to three-directional cutting forces.

The proportionality constants depend on the chip thickness  $t_c$ , cutting velocity  $v_c$  and rake angle  $\gamma_a$  of the cutting tool. Mathematically [10],

$$\begin{aligned} K_a &= e^{a_0 + a_1 \log t_c + a_2 \log v_c + a_3 \log t_c \log v_c + a_4 \log \gamma_a} \\ K_n &= e^{b_0 + b_1 \log t_c + b_2 \log v_c + b_3 \log t_c \log v_c + b_4 \log \gamma_a} \\ K_t &= e^{c_0 + c_1 \log t_c + c_2 \log v_c + c_3 \log t_c \log v_c + c_4 \log \gamma_a} \end{aligned} \quad (2)$$

The coefficients  $a_i$ ,  $b_i$  and  $c_i$  ( $i = 1, 2, 3, 4$ ) are called the specific cutting energy constants. These constants depend upon the tool, work piece material, range of cutting speed and chip thickness. They are independent of the machining process. These constants are determined from calibration test for a given tool and work piece combination and a given range of cutting conditions.

Keeping the rake angle and the velocity of the tool movement constant corresponding to the broaching tool, the equations (2) reduce to:

$$\begin{aligned}
 K_a &= e^{a_0 + a_1 \log t_c} \\
 K_n &= e^{b_0 + b_1 \log t_c} \\
 K_t &= e^{c_0 + c_1 \log t_c}
 \end{aligned}
 \tag{3}$$

The values of specific cutting energy constants can be determined using a simple calibration experiment. The experiment was conducted on a shaping machine and the specific cutting energy constants were determined.

### *Experimental investigation*

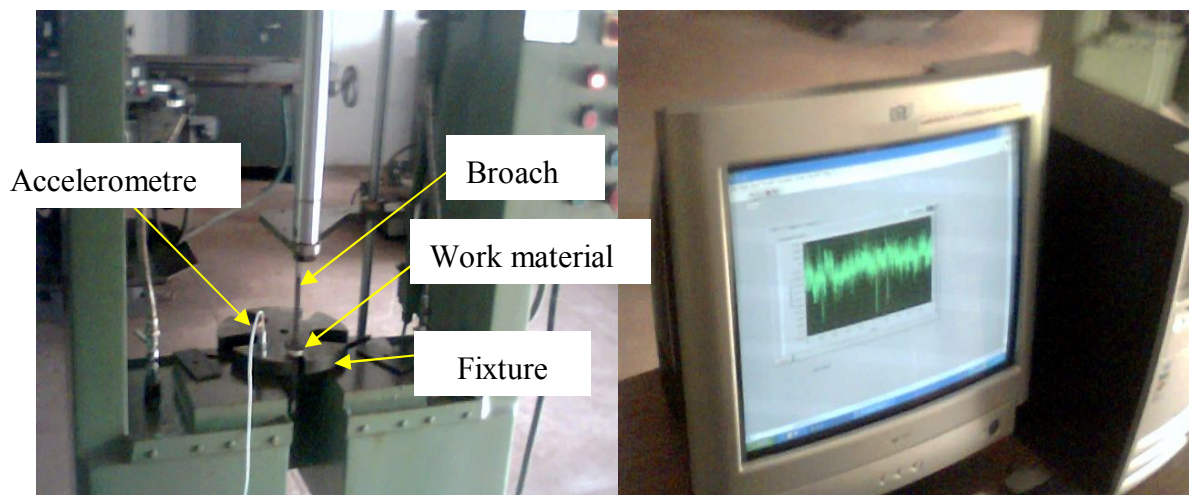
Several sets of shaping operation experiments were conducted in order to determine the specific cutting energy constants. Three sets of experiments were conducted on each set of processes. The first set of experiments was used to ascertain which tool and cutter geometry variables affected the proportionality constants. The second set of experiments was used to develop an adequate model for proportionality constants based on important tool and cutter geometry variables determined from the first set of experiments. The third set of experiments was used to determine the specific cutting energy constants. Experiments were repeated for aluminum, cast iron and mild steel at different depths of cut. Chip-thickness area for calibration purpose was measured from the chip curl. The volume of the chip was measured using water displacement method. After that the chip curl was heated and elongated. The chip width ( $b$ ) and length ( $\ell$ ) were measured using a micrometer and vernier calliper respectively. Then the actual chip thickness ( $t_c$ ) was obtained by dividing the volume by the product of length and width, i.e.  $t_c = v / b \ell$ , where  $v$  is the volume of chip curl. Knowing the chip thickness and width, the chip-thickness area could be computed as the product of  $t_c$  and  $b$ .

### *Static model*

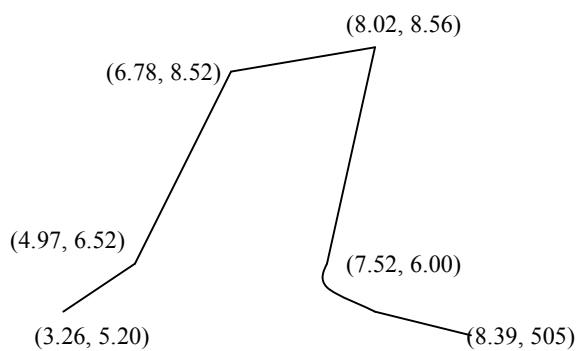
Figure 1 illustrates the static force measurement set-up during the broaching operation. The cutting forces in three directions were measured using an accelerometer and the root mean square (RMS) value was taken as the static cutting force. A static analysis calculated the effects of steady loading conditions on a structure while ignoring inertia and damping effects such as those caused by time-varying loads. A static analysis can, however, include steady inertia loads (such as gravity and rotational velocity) and time-varying loads that can be approximated as static equivalent loads.

### *Chip-thickness area measurement*

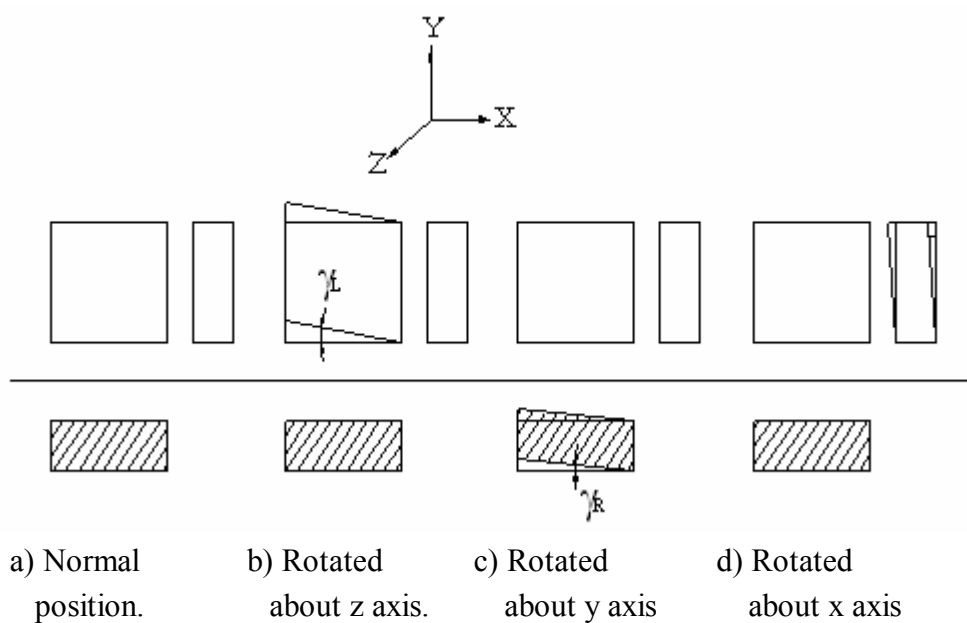
A tool-maker's microscope was used to obtain the coordinates of the tooth profile of a broaching tool. AutoCAD drawing of the broaching tool was drawn to procure the chip load area and the dimensions of the tooth profile. Figure 2 shows the coordinates of a single tooth of the broach obtained through the tool-maker's microscope and AutoCAD drawing. Figure 3 shows three planar angles of the cutting edge of a broach tooth where  $\gamma_R$  is the rake angle and  $\gamma_L$  is the release angle. Figure 4 shows the coordinates of the full broach, which gives pitch = 6.78 mm, rake angle =  $30^\circ$ , rise per tooth = 0.05 mm and width = 8 mm; hence, chip-thickness area = width x rise/tooth =  $0.4 \text{ mm}^2$ .



**Figure 1.** Experimental set-up to obtain the force pattern during the broaching process



**Figure 2.** Broach tooth profile as obtained by tool maker's microscope



**Figure 3.** Planar angles of the cutting edge of broach tooth

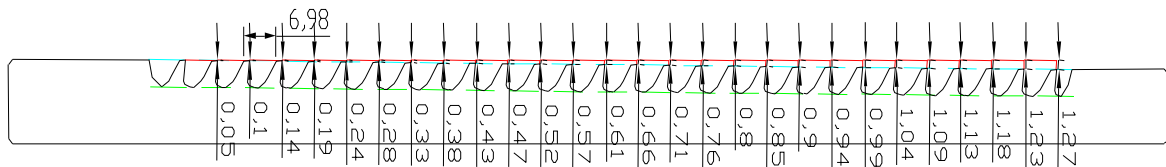


Figure.4. AutoCAD drawing of a broach tool

Results

Figures 5, 6 and 7 illustrate the cutting force versus chip thickness during shaping operation for mild steel, aluminium and cast iron respectively. In the plot, the x-axis is the logarithm of chip thickness (marked as d) and y-axis is the logarithm of the cutting forces in Newton (marked as Kx, Ky and Kz) along x, y and z directions respectively. Chip thickness corresponds to depth of cut which varied from 0.04 mm to 0.16 mm. The calibration test was performed for a small depth of cut to avoid error due to impulsive cutting force coming on the work piece at greater depth of cut. A linear curve fitting was made using Matlab software to determine the specific energy constants. Negative-slope linear curves were obtained and the coefficients of the linear curve fitting gave the specific energy constants. The proportionality constants  $K_a$ ,  $K_n$  and  $K_t$  were determined using equation (3). Tables 1 and 2 give the values of specific cutting energy constants and proportionality constants respectively.

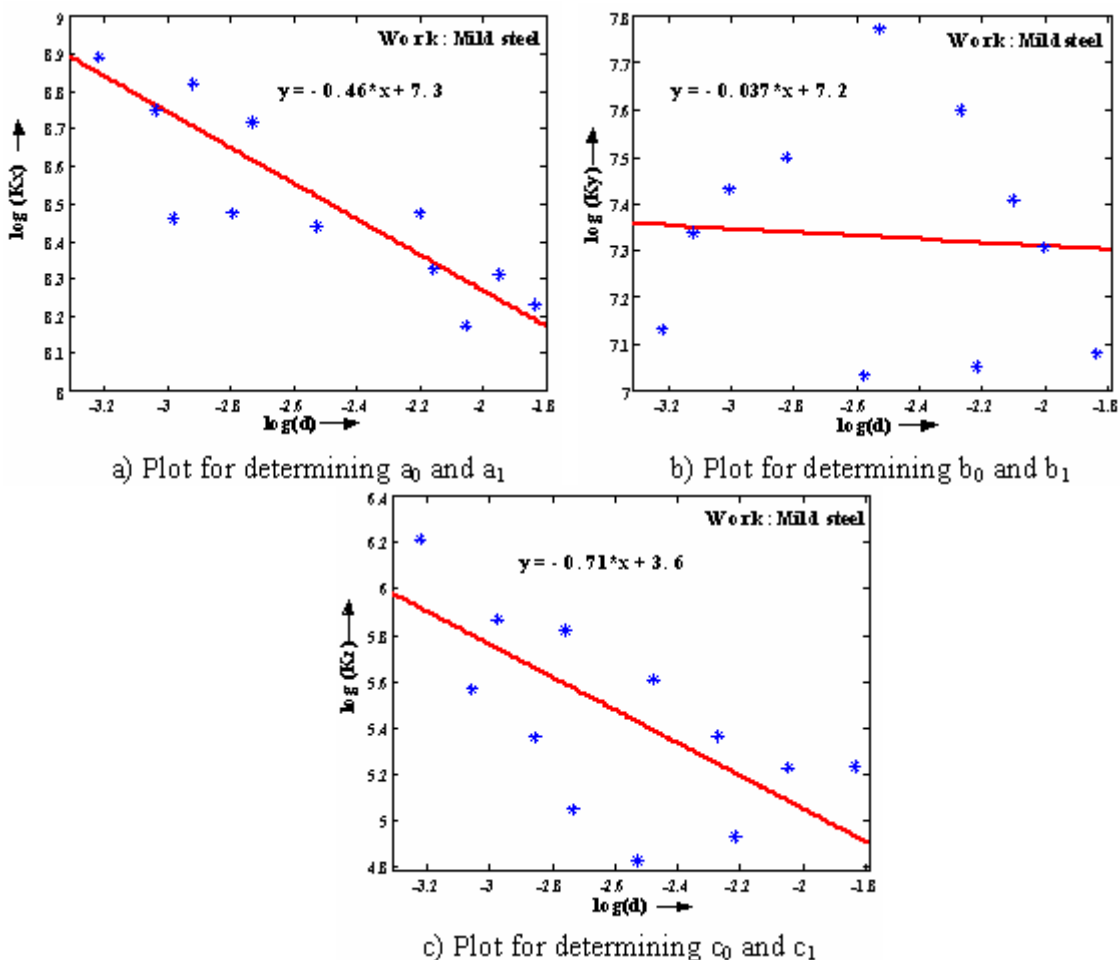


Figure 5. Specific cutting energy constants for mild steel work material during shaping operation

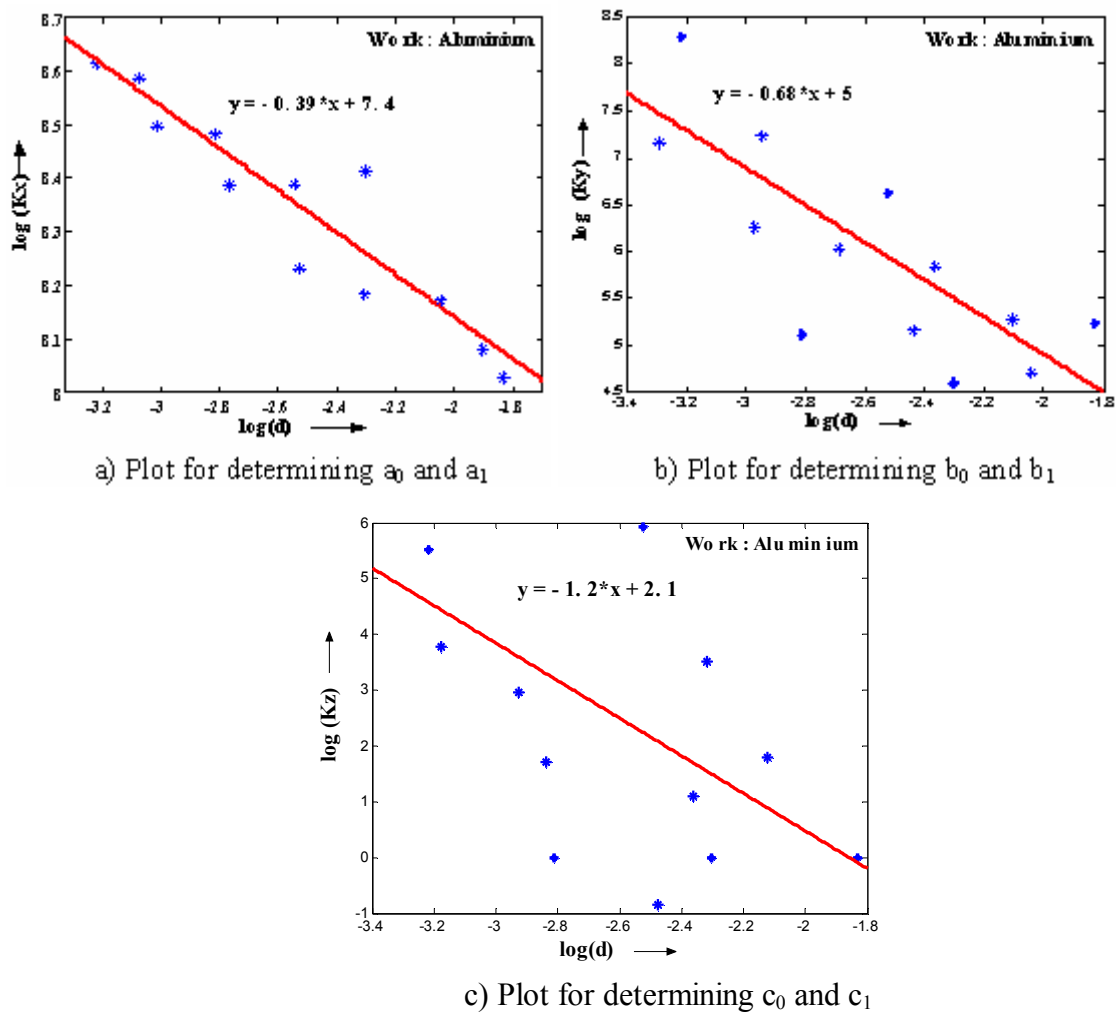


Figure 6. Specific cutting energy constants for aluminium work material during shaping operation

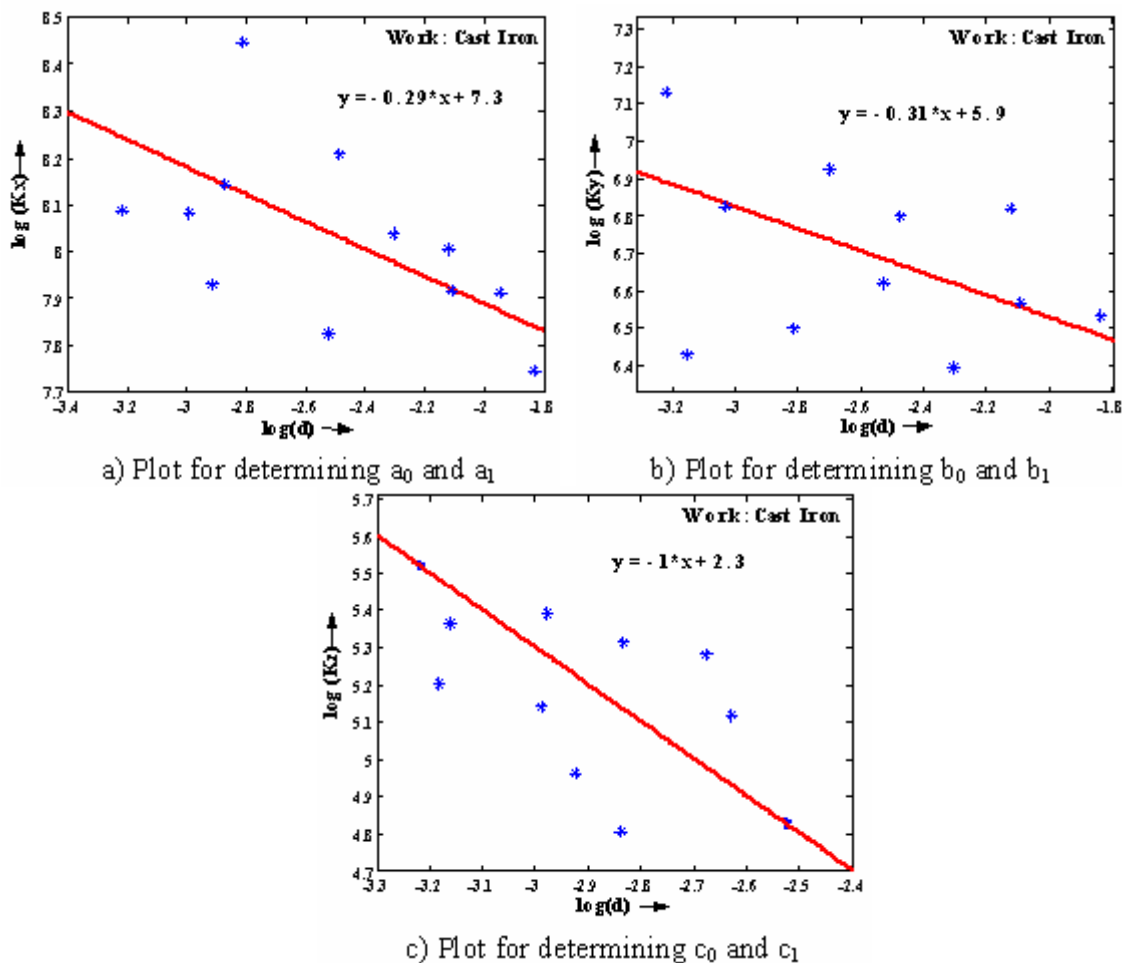


Figure 7. Specific cutting energy constants for cast iron work material during shaping operation

Table 1. Specific cutting energy constants obtained during shaping operation

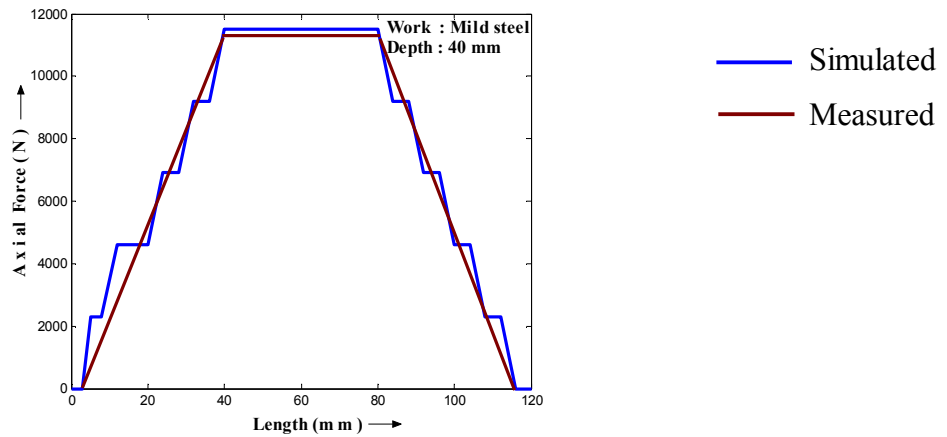
Material	$a_0$	$a_1$	$b_0$	$b_1$	$c_0$	$c_1$
Mild steel	7.3	-0.46	7.2	-0.037	3.6	-0.71
Aluminium	7.4	-0.39	5.0	-0.68	2.1	-1.2
Cast iron	7.3	-0.29	5.9	-0.31	2.3	-1.0

Table 2. Proportionality constants determined from specific cutting energy constants

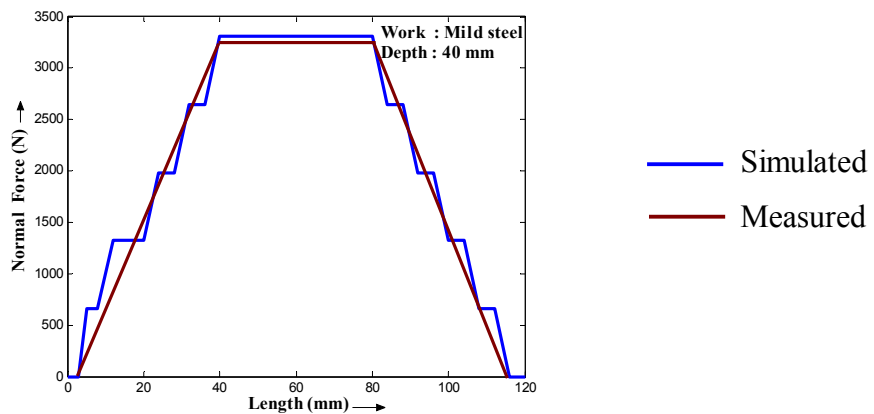
Material	$K_a$ (N/mm <sup>2</sup> )	$K_n$ (N/mm <sup>2</sup> )	$K_t$ (N/mm <sup>2</sup> )
Mild steel	5732.3	1654.0	307.1
Aluminium	5370.0	1088.0	279.4
Cast iron	3562.0	924.4	199.5

Static forces during broaching operation

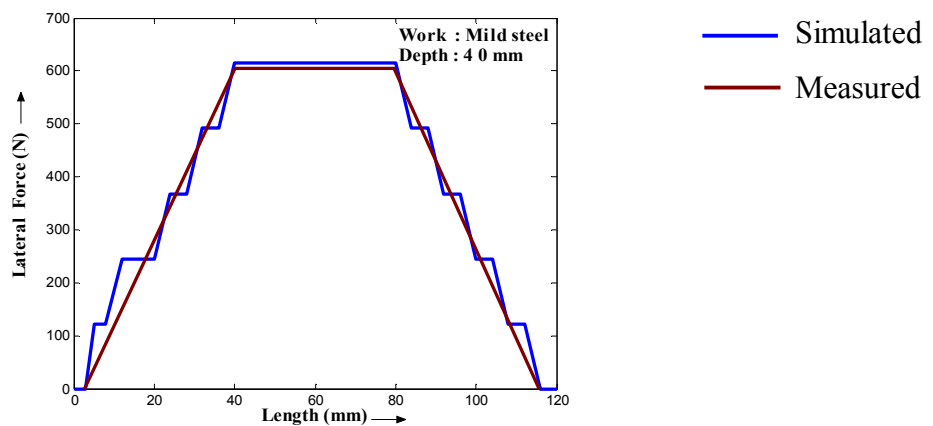
A MatLab program was written to simulate the static forces during broaching for different materials at different depths. The simulation results were plotted as shown in Figure 8 for mild steel work material at 40-mm depth. The cutting force progressively increased to a steady-state force when all the teeth were engaged in the work piece.



a) Axial force acting on the broach



b) Normal force acting on the broach



c) Lateral force acting on the broach

**Figure 8.** Comparison of measured and simulated static forces for mild-steel work material at 40-mm depth



## Conclusions

This paper presents the determination of proportionality constants, which are a determining factor in cutting force modelling. These proportionality constants were determined experimentally using shaping as a fundamental calibration process. The experiment was repeated for different materials, i.e. mild steel, aluminium and cast iron, and results are presented graphically. After determining the proportionality constants, mechanistic modelling for static force computation was carried out for the broaching operation. The comparison of the experimental and simulated results for static forces are presented graphically and are closely in agreement.

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