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Full Paper

Application of a modified Monte Carlo method for the simulation of heat conduction in a rectangular slab

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Abstract: Monte Carlo method has been used to study heat conduction problems. It is grid-free in implementation, unlike the conventional Finite Element and Finite Difference Method. However for Monte Carlo method, solutions of desired field of interest can only be obtained one after the other, unlike the others that can be obtained simultaneously. Therefore a modified method has been developed which benefits from the simplicity of the Monte Carlo method and also provides full field description of temperature at one computer run.

The Modified Monte Carlo first obtains sample values of temperature in the domain of interest using the conventional Monte Carlo technique. Thereafter a histogram table is constructed which is then used to predict values for the other parts of the field unknown. The number of clusters obtained from the prediction is noted. The prediction that has minimal clusters is adjudged the best. The technique was tested on grid configurations such as 3 x 3 and 4 x 4 grids. Only isothermal steady-state 2-D cases were considered.

When compared with the results of ordinary Monte Carlo method, the modified technique incurred a maximum error of 4% for 4 x 4 grids. Larger grids were obtained by seamless stitching of smaller grids and as such no growth in errors was noticed. For the 3 x 3 grids, the sample size was about 55% while only 25% of the domain was sampled using 4 x 4 grids.

Using the modified technique, the errors incurred even with the 4 x 4 grids was only 4%. The technique can therefore be used for simulation of steady-state heat conduction.

Keywords: heat conduction, simulation , probability method, Modified Monte Carlo

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Introduction

Various heat transfer processes require the knowledge of temperature distribution over the surface where the heat transfer is taking place. This in essence provides for better engineering decision and design. However, because of the mathematical rigour that has to be performed, the governing equations derived are often non-linear. Various numerical methods such as Finite Difference and Finite Element Method have been used extensively to tackle them. Many of such are well documented by Incroprera and DeWitt [1] as well as by Welty et al.[2] and Eckert and Drake [3]. Some recent applications of Finite Element Method and Finite Difference Method include the work of Jing Zhang et al. [4] and Tasarkuyu and Akinoglu [5]. These methods generally require simultaneous solution of domain solution points.

However, a probability method was applied by Haji-Sheikh and Sparrow [6] to obtain temperature of isolated points during heat conduction in a domain. A more recent effort was reported by Grigorin [7]. Leveque and Rezzong [8] carried out thermal studies of a superconductivity current limiter using Monte Carlo method.

Generally, probability methods can be used to obtain a single-point solution in a domain. They are therefore simpler to implement and less demanding on the computer memory. They may however be slower in run time. When a full description of the temperature history of the entire domain is needed, more computational time may be required.

The objective of this work is to preserve the simplicity of the Monte Carlo method by developing a Modified Monte Carlo approach which will make it possible to fully characterise the domain without excessive computational burden. This paper discusses the typical Monte Carlo solution for temperature in a domain and thereafter the Modified Monte Carlo technique is discussed.

Simple Monte Carlo Method

Monte Carlo application to thermal conduction was used by Haji-Sheikh and Sparrow [6] for obtaining temperature history in a rectangular slab. Both unsteady and steady-state cases were treated. In this method a probabilistic approach was adopted which makes single point solution achievable. An abridged description of the method is hereby presented.

Given any domain, the solution of a point in it (e.g. A) is obtained by commencing many random walks from the point of interest. Such walks are terminated whenever an absorbing boundary (a boundary with constant temperature) is encountered. Whenever the walk is terminated, the boundary value is scored. The average of all the scores at the absorbing boundary(ies) gives the solution of the point of interest.

Thus,

$$T_A = \frac{\sum_i^n T_i}{n}$$

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where,

n = number of walks

T_i = temperature score at walk i

T_A= temperature of the point of interest in the domain

The procedure is illustrated in Figure 1.

Ogundare [9] used the Monte Carlo technique to solve for temperature distribution in arbitrary surfaces. He reported good agreement with the Finite Difference solution for isothermal cases, but for adiabatic case an error of about 5% was noticed.

Modified Monte Carlo Method

Modified Monte Carlo method is developed to enable full characterisation of a domain, since the Monte Carlo solution can only provide single-point solution at any computational run. An algorithm of this technique is hereby presented.

(1) First divide the entire domain into the desired grid sizes such as 3 x 3 internal grid as in Figure 2.

(2) Select some nodes such as A,B,C, D, E (the selected points are block centred) and designate them as sample points.

(3) Determine the solution values of sample points using the normal Monte Carlo method. For instance, the assumed estimated values of the sample points are shown in Figure 3.

(4) Using the sample values, a histogram is then constructed as shown below.

x	f
1	2
2	3

(5) With the histogram, guess values for the remaining portion of the domain by making use of random numbers between 0 and 1. In the case being considered, if the random number is less than 2/5, a guess value 1 is assigned (x=1 at f =2), otherwise a guess value 2 is assigned.

(6) By comparing guess values , a set of estimated values that minimises cluster formation is chosen. For instance Guess 1 of Figure 4 is chosen.

(7) Steps 1-5 are repeated several times. Thereafter average values at the various grid points are estimated.

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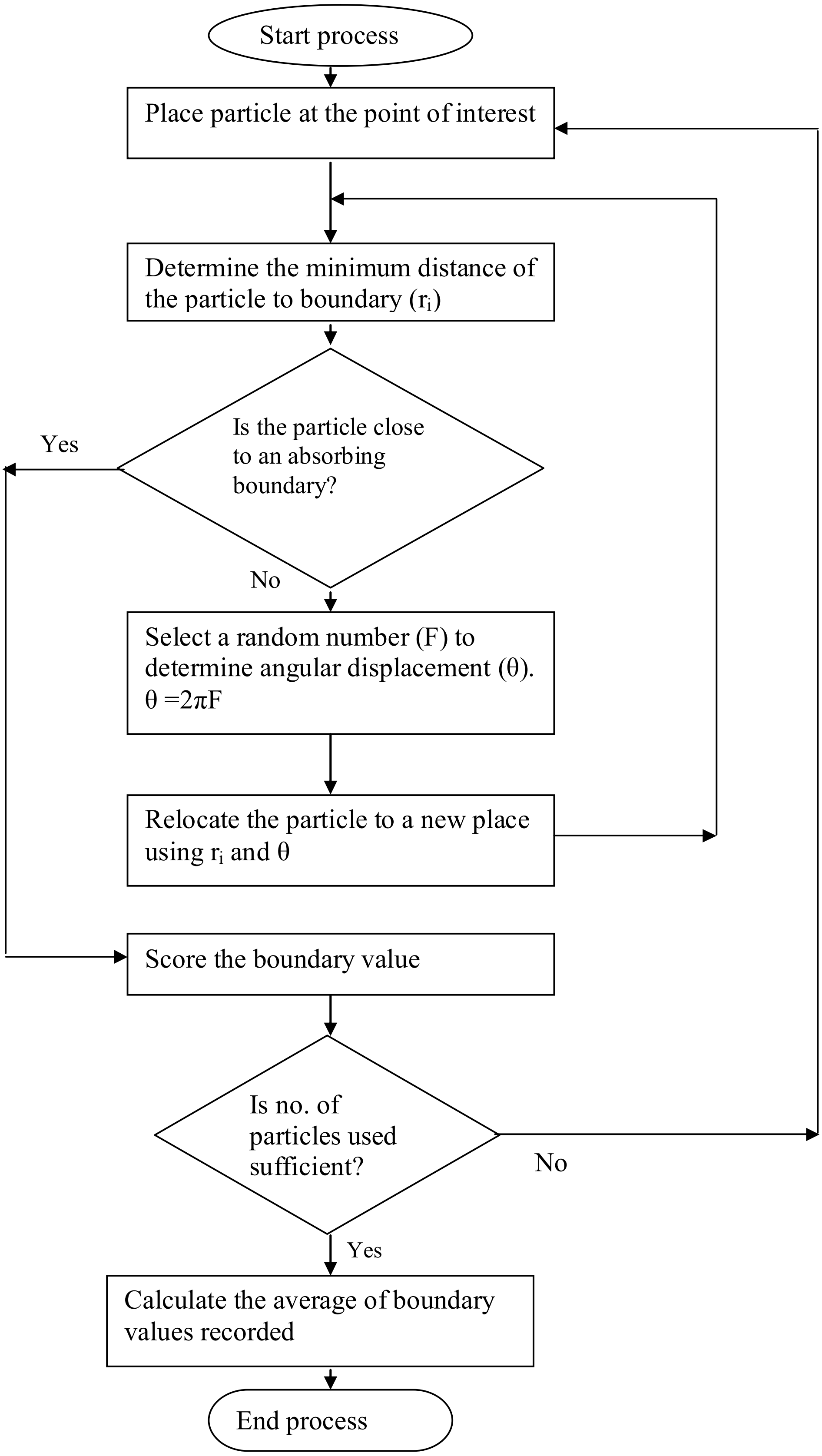

Figure 1. Simple Monte Carlo flow chart

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B		**C**
	D	
A		**E**

Figure 2. Domain with sample points

2		1
	2	
2		1

Figure 3. Domain with sample values

Guess 1

2	2	1
2	2	1
2	2	1

Number of clusters = 2

Guess 2

2	2	1
1	2	2
2	1	1

Number of cluster =5

Figure 4. Domain with predicted values

Larger grid solutions are obtained by seamless stitching of smaller grids. For instance, in Figure 5, the 8 x 8 internal grid solutions are obtained as follows:

(i) Divide the 8 x 8 grid into four 4 x 4 internal 'sub-grids.'

(ii) Identify sample points in each of the sub-grids and obtain their values using the simple Monte Carlo technique. Note that sample points solutions must be obtained using the global domain. This enables seamless stitching of the grids.

(iii) Each of the 'sub-grids' is now separately considered. Unknown values in each 'sub-grid' are then obtained using the Modified Monte Carlo procedure.

(iv) All the 'sub-grids' with all the values now known are joined together. No further processing of the results at the grid borders is needed and it is therefore regarded as seamless stitching. Further clarification can be obtained from the accompanying Figure 5.

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A			A	C			C
B			B	D			D
E			E	G			G
F			F	H			H

(a) Domain with sample points in each sub-grid identified and their values obtained

A	X	X	A
X	X	X	X
X	X	X	X
B	X	X	B

(b) A sub-grid separately treated with values obtained globally unaltered

Figure 5. Stitching process for 8 x 8 grid (Note: Values of points marked with 'X' in Fig. 5b are predicted using Modified Monte Carlo technique. These values are retained at their corresponding positions in the global domain.)

Application of the Modified Monte Carlo Method to Simple Thermal Conduction Problems

The Modified Monte Carlo method was used to simulate heat conduction in a rectangular slab with thermal conductivity of 0.035 W/m^2 K. This was carried out using different kinds of grid configurations. The results for such grids are hereby presented.

Simulation using 3 x 3 domain

The results for this case is presented in Table 1, which shows no noticeable discrepancies. The sample values are underlined implying that only four other values are simulated using the Modified Monte Carlo technique. In a way this may imply a saving of 45% in computational run when compared to ordinary Monte Carlo technique.

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Table 1. Simulation temperature results (K) for 3 x 3 grid using Modified Monte Carlo technique

400 K

400 K

385	377.6	*373*
378	*351*	320
376	324	*262*

200 K

200 K

(Note : All simulated values are for internal grids. Only four new points were predicted using the Modified Monte Carlo technique. The samples (in underlined form) is thus about 55%.)

Simulation using a 4 x 4 grid

The simulated values using the same material properties as for the 3 x 3 grid are presented for the ordinary and Modified Monte Carlo method in Tables 2 and 3 respectively. The sample values are shown underlined for Modified Monte Carlo case. There are thus 12 simulated values obtained using the Modified Monte Carlo technique. The sample size in this case is 25%.

Table 2. Simulated temperature results (K) for 4 x 4 grid using normal Monte Carlo technique

773 K

373 K

557.8	**773**	**773**	**568.2**
373	**538.6**	***525.8***	**373**
373	**417**	**418**	**373**
373	**373**	**373**	**373**

373 K

373 K

Table 3. Simulated temperature results (K) for 4 x 4 grid using Modified Monte Carlo method

773 K

373 K

557.8	**773**	**773**	**555.4**
373	**538.6**	*503.4*	**373**
373	**416.2**	**418.6**	**373**
373	**373**	**373**	**373**

373 K

373 K

(Notes: 1. Sample values are shown underlined.

2. Except for boundary values, all others are for internal grids.)

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Comparison of simulated results of modified and ordinary Monte Carlo method

Inspection of the 3 x 3 grid solution revealed no discrepancies while point-by-point solution comparison for the 4 x 4 grid showed a discrepancy of 4% (discrepancy points in italics).

Simulation results for 12 x 12 grid

The solutions using a 12 x 12 grid were obtained by seamlessly joining solutions for four 4 x 4 sub-grids as earlier highlighted. A sample solution using the same material properties as for the earlier-mentioned cases and with the boundary condition boldly highlighted is presented in Table 4.

Table 4. Simulated temperature (K) results for 12 x 12 grid using Modified Monte Carlo method

591.67	**773**	**773**	**773**	**773**	**773**	**773**	**773**	**773**	**773**	**773**	**579.67**
373	577	626.33	695.67	730.33	725	729	727.67	690.33	594.33	597	**373**
373	531.67	567.67	615.67	671.67	663.67	670.33	663.67	621	542.33	541	**373**
373	438.33	499.67	549	582.33	591.67	587.67	581	555.67	511.67	455.67	**373**
373	399.67	467.67	530.33	531.67	525	526.33	550.33	507.67	458.33	399.67	**373**
373	393	449	486.33	487.67	494.33	510.33	507.67	471.67	442.33	386.33	**373**
373	390.33	431.67	454.33	462.33	487.67	495.67	474.33	427.67	406.33	387.67	**373**
373	391.67	411.67	434.33	438.33	458.33	450.33	439.67	425	411.67	386.33	**373**
373	381	393	403.67	419.67	407.67	401	418.33	405	391.67	378.33	**373**
373	381	382.33	385	405	401	399.67	401	391.67	382.33	374.33	**373**
373	373	374.33	378.33	383.67	379.67	383.67	386.33	379.67	374.33	373	**373**
373	**373**	**373**	**373**	**373**	**373**	**373**	**373**	**373**	**373**	**373**	**373**

(Notes: 1. Boundary values are boldly highlighted.
2. Other values are for internal grids.)

Conclusions

This paper has established the possibility of using a Modified Monte Carlo method for solving heat conduction problems. It shows that by seamless stitching of 4 x 4 grids, a full temperature characterisation of the domain of interest can be obtained. In view of its likely saving in computational time, the method may therefore be explored for numerical studies.

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