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Full Paper

Elementary theory of quantum Hall effect

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Abstract: The Hall effect is the generation of a current perpendicular to both the direction of the applied electric as well as magnetic field in a metal or in a semiconductor. It is used to determine the concentration of electrons. The quantum Hall effect with integer quantization was discovered by von Klitzing and fractionally charged states were found by Tsui, Stormer and Gossard. Robert Laughlin explained the quantization of Hall current by using "flux quantization" and introduced incompressibility to obtain the fractional charge. We have developed the theory of the quantum Hall effect by using the theory of angular momentum. Our predicted fractions are in accord with those measured. We emphasize our explanation of the observed phenomena. We use spin to explain the fractional charge and hence we discover spin-charge locking.

Keywords: Hall effect, Dirac equation, magnetic moments, effective charge

Introduction

The ordinary Hall effect was discovered by Edwin Hall in 1879 [1]. In 1978 von Klitzing and Englert found a plateau in the Hall effect [2]. In 1980 von Klitzing et al. found the value of h/e^2 from the plateau in the Hall effect [3]. In 1982, Tsui et al. discovered the steps at fractional numbers [4]. The force is $\mathbf{F}= e\, \mathbf{v}\mathrm{x}\mathbf{B}$ so that the Hall voltage is, V= IB/necd where I is the Hall current, B is the magnetic induction, n is the electron concentration, e is the electron charge, c is the velocity of light and d is the thickness of the sample as shown in Figure 1.

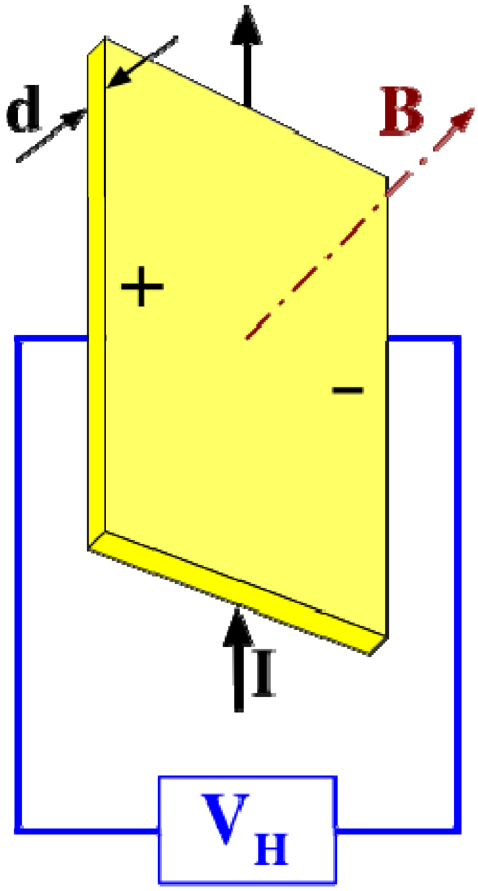

Figure 1. The Hall voltage is measured orthogonal to both the electric as well as the applied magnetic field.

Two Dimensional Electron Systems

A two dimensional electron system is formed in a heterostructure which has layers of GaAs over AlGaAs. The energy gap of GaAs increases upon Al doping. When GaAs is doped with donors at zero temperature the Fermi level lies higher than the bottom of the conduction band. The electrons bound to donors move into GaAs conduction band and the process stops when some proportion of electrons have moved. The electrons in the inversion layer are two dimensional as shown in Figure 2.

The average drift velocity of the electron subjected to the electric field is,

$$V_d = -eE\tau/m \qquad (1)$$

where E is the electric field, m is the electron mass and τ is the mean life time so that the current density is,

$$j = -neV_d = \sigma_o E \qquad (2)$$

where

$$\sigma_o = ne^2\tau/m \qquad (3)$$

where n is the electron density. In the presence of a steady magnetic field, the conductivity and resistivity become matrices,

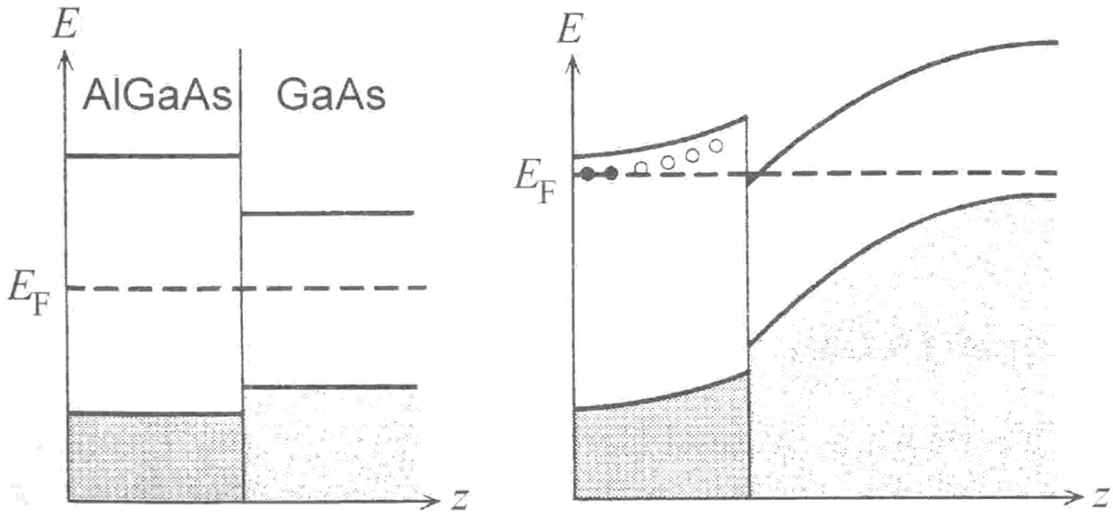

Figure 2. The two dimensional electrons are formed in between AlGaAs and GaAs.

$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}, \quad \rho = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix}. \qquad (4)$$

We take x and y axes in the 2-dimensional plane, to obtain,

$$i_x = \sigma_{xx}E_x + \sigma_{xy}E_y,$$
$$i_y = \sigma_{yy}E_x + \sigma_{yx}E_y. \qquad (5)$$

Owing to the isotropy, $\sigma_{xx}=\sigma_{yy}$ and $\sigma_{xy}=-\sigma_{yx}$. The first one is called the diagonal conductivity and the second one is called the Hall conductivity. The relation between conductivity and resistivity is,

$$\rho_{xx} = \rho_{yy} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} \qquad (6)$$

for the diagonal resistivity and

$$\rho_{xy} = -\rho_{yx} = -\frac{\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2} \qquad (7)$$

for the Hall resistivity. We measure these quantities by connecting various leads as given in Figure 3. In the case of homogeneous current in the y direction, $i_x=I/W$ and $i_y=0$. The electric field is given by, $E_x=V_{12}/L$ and $E_y=V_{13}/W$ so we have, $\rho_{xx}=V_{12}W/IL$ and $\rho_{yx}= V_{13}/I=R_H$ where R_H is the Hall coefficient. According to the Drude (ordinary non-interacting metals) theory, $\rho_{xx}=1/\sigma_o=m/ne^2\tau$ and $\sigma_{xy}=B/nec$ in a weak magnetic field. The Hall resistivity is inversely proportional to the electron density and independent of mean scattering life time. In a strong magnetic field, there are new phenomena. Von Klitzing et al. [3] found the plateau in the Hall resistivity which gave the correct value of h/e^2. One of the plateaus is given in Figure 4. We see from the plot that (i) there is a plateau region in which the Hall resistivity remains constant. As the electron density is varied in this region the diagonal resistivity is almost zero, and (ii) the value of the Hall resistivity in the plateau regions is exactly equal to h/e^2 divided by an integer. Therefore, the Hall conductivity σ_{xy} in the plateau region is "quantised" into integer multiples of e^2/h. This phenomenon was called the "integer quantised Hall effect" (IQHE).

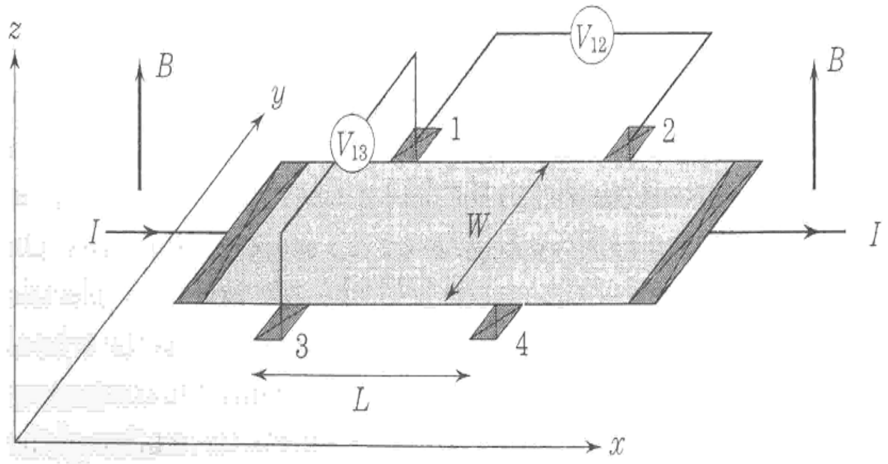

Figure 3. The sample with the magnetic field B and the current I perpendicular to it. The wires are connected to measure Hall voltage perpendicular to both I and B in various directions. W is the width of the sample and L is the length.

Flux Quantisation and the Hall Effect

Introduction of the flux quantisation immediately explains the integer quantised Hall effect. The Hall resistivity is,

$$\rho = B/nec \tag{8}$$

where B is the magnetic field. According to the flux quantisation, the field in a certain area, A, is quantised,

$$B.A = m\varphi_o \tag{9}$$

where the magnetic flux quantum is, φ_o=hc/e. Substitution of (9) into (8) gives the integer quantised Hall effect as,

$$\rho = \frac{\frac{mhc}{Ae}}{\frac{N_o}{A}ec} = \frac{m}{N_o}\frac{h}{e^2}. \tag{10}$$

The quantum Hall effect thus is the quantisation of Hall resistivity as,

$$\rho = \frac{h}{ie^2}. \tag{11}$$

Hence the charge of the quasiparticle is ie . Here i = integer. The charge thus becomes 1e, 2e, 3e, …, ie, … . Von Klitzing obtained the correct value of the charge for i=1. So the value of $h/1e^2$ became "one von Klitzing". When this experiment was repeated with cleaner samples with higher electron mobility, with higher magnetic fields and lower temperatures, it led to the discovery of plateau at i =1/3 which gave birth to the fractional charge. We show the data in Figure 4 with plateau at 1/3. Since this fractional value occurred in the middle of various highly degenerate Landau levels, where no gap is apparent, the observation could not be explained by the experimentalists by using the non-interacting quantum mechanical theory. It was thought that the observation is a result of the many-body effects of

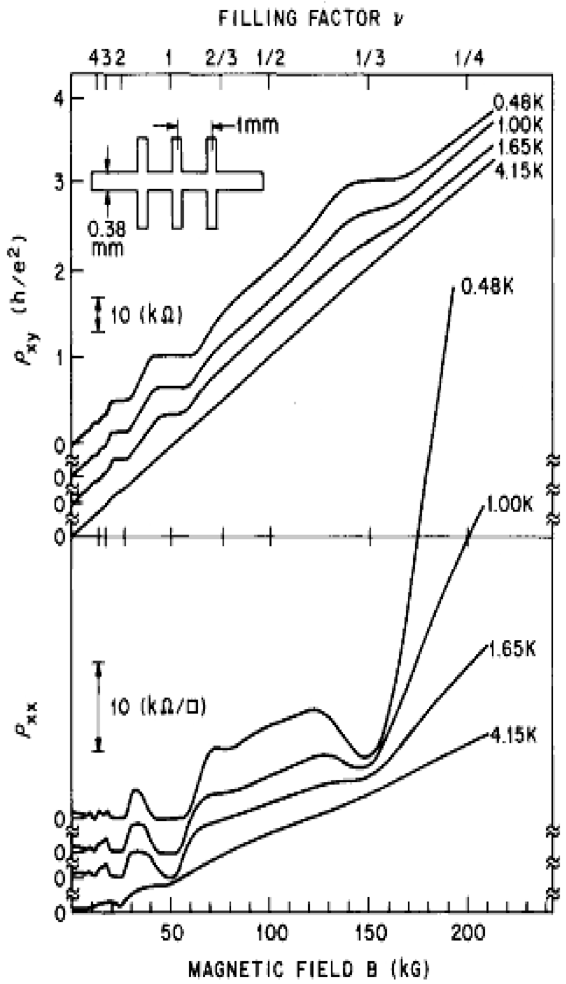

Figure 4. The data of experimentally measured Hall resistivity showing the plateau at i = 1/3

electron interactions. Subsequent experimental work showed a lot many more plateaus which are displayed in Figure 5. We will see that there is no need of any interaction to explain the plateau at 1/3.

Laughlin's Theory

Laughlin made the first efforts to explain the quantum Hall effect [5]. The flux quantisation was immediately found to explain at least the integer quantised Hall effect. Subsequently, Laughlin started from first principles using the Hamiltonian,

$$H = \sum_j \{ \left| (\hbar / i)\nabla_j - (e/c)\vec{A}_j \right|^2 + V(z_j)\} + \sum_{j>k} e^2 / \left| z_j - z_k \right| \qquad (12)$$

where j and k sum over N particles and V is the potential due to nuclei. The repulsive Coulomb interactions can produce the plateaus only when flux quantisation is considered. A trial wave function of the form given below,

$$\left| \Psi(z_1,\ldots,z_N) \right|^2 = e^{-\beta(\phi+\gamma)} \qquad (13)$$

with β=1/m ,

$$\phi = -2m^2 \sum_{j<k} \ell n \left| z_j - z_k \right| + m/2 \sum_{\ell}^{N} \left| z_\ell \right|^2 \qquad (14)$$

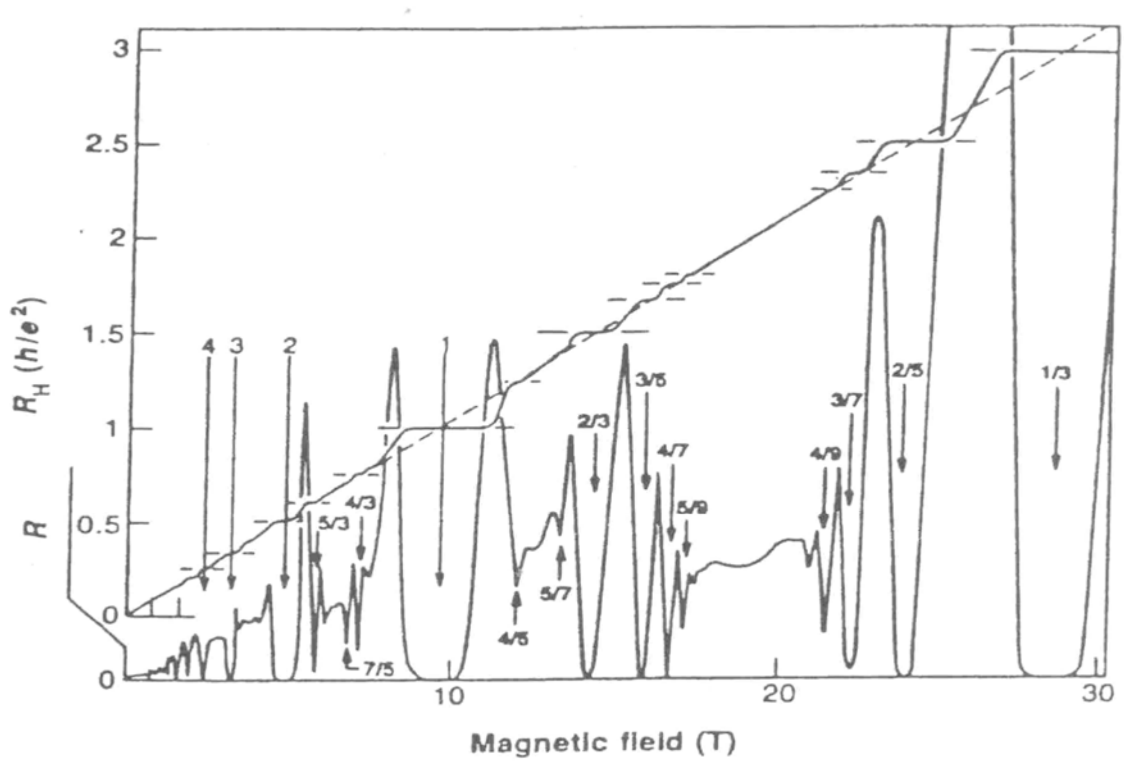

Figure 5. The data of quantum Hall effect showing many plateaus including the one at 1/3

is used to solve the Schrodinger equation to find the ground state. Laughlin obtained the approximate energy expression in terms of m as well as computed the energy by using the exact density of states. By using "incompressibility" the charge of the particles is fixed at 1/3 and 1/5. Unfortunately there is area in the flux quantisation which must also be fixed, otherwise the charge will leak. Laughlin thus laid the foundation for the study of fractional charges from the first principles by using only the Coulomb interactions. The Hamiltonian then consists of the kinetic energies and the Coulomb potentials and the correlations produce the fractional charges. Unfortunately the problem requires incompressibility due to the area in flux quantisation.

Shrivastava's Theory [6-17]

We consider that electrons have spin as well as the orbital angular momentum so that,

$$g_j j = g_s\vec{s} + g_l\vec{l} = \frac{1}{2}(g_l + g_s)\vec{j} + \frac{1}{2}(g_l - g_s)(\vec{l} - \vec{s}) \ . \quad (15)$$

Multiplying both sides by j = *l* +s and taking eigen values,

$$g_j j(j+1) = (1/2)(g_l + g_s)j(j+1) + (1/2)(g_l - g_s)[(\vec{l}(l+1) - s(s+1)]. \quad (16)$$

Substituting s=1/2 and j = *l* ± (1/2) we get,

$$g_j = g_l \pm \frac{g_s - g_l}{2l+1} \ . \quad (17)$$

For g_s=2, g_l = 1, we find,

$$g_{\pm} = 1 \pm \frac{1}{2l+1} \quad . \qquad (18)$$

The equation (17) has both the signs for the spin. The cyclotron frequency is,

$$\omega = \frac{eB}{mc} \quad . \qquad (19)$$

From the charge, e in the cyclotron frequency, we generate the charge of a particle. It is also possible to obtain the charge from the e in Bohr magneton. Corresponding to the cyclotron frequency, the voltage along y direction is,

$$\hbar\omega = eV_y \qquad (20)$$

or,

$$\hbar \frac{eB}{mc} = eV_y.$$

Multiplying this expression by e/h, we get,

$$\frac{e^2 B}{2\pi mc} = \frac{e^2}{h} V_y \qquad (21)$$

which is the current in x direction, so that the resistivity,

$$\rho_{xy} = \frac{h}{e^2} \quad . \qquad (22)$$

The S_z=1/2, and the energy in a magnetic field is $g\mu_B H.S$, so correcting B in the cyclotron frequency we find,

$$I_x = \frac{1}{2} g \frac{e^2 B}{2\pi mc} = \frac{1}{2} g \frac{e^2 V_y}{h} . \qquad (23)$$

For l =0, g=2,

$$I_x = \frac{e^2}{h} V_y \qquad (24)$$

which describes the quantised current correctly for ν=1. From the above equations we have $\nu = \frac{1}{2} g_{\pm}$ which gives the filling factor, one for + sign and the other for – sign as in (18). For l =0, we obtain $(1/2)g_+$=1 and $(1/2)g_-$=0 and other values as given in Table 1. The Landau levels are introduced by multiplying the above values by n so that

$$\nu = n(\frac{1}{2} g_{\pm})$$

so we can multiply the tabulated values by an integer when needed. The values of ρ_{xx} and ρ_{xy} for a single interface of GaAs/AlGaAs have been measured at 150 mK. The values predicted in Table 1 are exactly the same as in the experimental data shown in Figure 5. The values shown in the figure occur in two sets: 2/5, 3/7, 4/9, 5/11, 6/13, … etc. and 2/3, 3/5, 4/7, 5/9, 6/11 etc. Using the Table 1, when we multiply the values by n, we can interpret all of the experimentally measured values correctly. The columns of Table 1 belong to two Kramers conjugates, one belonging to +1/2 and the other one to -1/2 spin and the experimental data also display them in two sets.

For the cyclotron frequency $\hbar\omega_c = g\mu_B B$ where $\mu_B = e\hbar/2mc$ is the Bohr magneton. Therefore, $(1/2)g_{\pm}$ can be considered to be the effective charge, $e_{eff} = \frac{1}{2} ge = \nu e$. In Table 1, we see two series,

$\nu_- = \frac{l}{2l+1}$ and $\nu_+ = \frac{l+1}{2l+1}$ which can be used to explain the high Landau levels easily.

Table 1. Predicted values of the fractional charge

l	(1/2)g- = $\frac{l}{2l+1}$	(1/2)g+ = $\frac{l+1}{2l+1}$
0	0	1
1	1/3	2/3
2	2/5	3/5
3	3/7	4/7
4	4/9	5/9
5	5/11	6/11
6	6/13	7/13
∞	1/2	1/2

For the higher values of the Landau level quantum number, n, the number of fractions observed are much less than at the lowest Landau level. At the magnetic field of 4 or 5 Tesla only a small number of fractions are observed, the strongest ones being at: 8/3, 5/2 and 7/3. The series $l/(2l+1)$ is the particle-hole conjugate of $(l+1)/(2l+1)$. For l=7, two values, 7/15 and 8/15, are predicted and for l=∞ the value is ½. When the same particle occurs in different levels its charge remains unchanged. We can multiply the values by n=5 so that the predicted values of ½, 7/15 and 8/15 become 5/2, 7/3 and 8/3. These predicted values are exactly the same as those observed experimentally. Thus, 7/3 is the particle-hole conjugate of 8/3 as seen in Table 2 for n=5. For a long time only odd denominators were reported which show that even denominators are weak. After that even denominators as well as even numerators with odd denominators are found. We go back to the same formula, $\frac{l+\frac{1}{2}\pm s}{2l+1}$. When s=1, l = 0, (1/2)+1=3/2 has even denominator.

Table 2. The fractions produced for high Landau levels

l	l/(2l+1)	(l +1)/(2l+1)	n l/(2l+1)	n(l+1)/(2l+1)
∞	1/2	1/2	5/2	5/2
7	7/15	8/15	7/3	8/3

The predicted fraction for l=0 is 3/2 and for l=1 it is 5/2. Hence for electron clusters or pairs the fraction has even denominator. Since the number of particles is larger than one its probability becomes small so these plateaus are weak but the same theory explains the even denominators. There is a limiting value of the series which also gives ½. One can introduce a Fermi surface at n/2, thus ½, 2/2, 3/2, 4/2, 5/2, 6/2 and 7/2 are predicted which are the same values as observed. The effective mass and g factor of some of the fractions are equal to those of others. We have shown that the effective mass can be equal only when the two quasiparticles are particle-hole conjugates. The particle-hole conjugates should obey the following relation,

$$\nu_p+\nu_h = 1 \qquad (25)$$

The values given in Table 1 always obey this relation [8].

Half-filled Landau Level

The $l = \infty$ in the two series produces,

$$\lim_{l\to\infty} \nu_+ = \frac{1}{2}(+)$$
$$\lim_{l\to\infty} \nu_- = \frac{1}{2}(-) \qquad (26)$$

One ½ comes from the right and the other from the left when magnetic field is varied, i.e. one while increasing the field and the other while reducing. One can go from 1/3 to ½ by reducing the field while from 2/3 to ½ is obtained by increasing the field. We can go from 1/3 to 3/5 by reversing the spin and increasing *l*, and similarly from 2/3 to 2/5 by reducing the spin and increasing *l*. In this way angular momentum is conserved. One of the ½ values is like an electron (1/2)A and the other is like a hole, (1/2)B, (A for + series and B for – series). Since the electron and the hole are separated by a distance, this state is compressible. When we multiply this result by n, the Landau level quantum number, we obtain: ½,2/2,3/2,4/2,5/2, …, which are in agreement with data.

Effective Charge

We discussed the Laughlin's wave function earlier. Here the repulsive Coulomb interactions cannot give rise to a fractional charge of 1/3. It is first assumed on the basis of experimental data and then substituted in the theory. Laughlin's charge is independent of spin but in our theory it depends. In Laughlin's theory a particle of charge 1/3 is produced but in our theory splitting occurs in fractions of 1/3 and 2/3, etc. In Laughlin's theory, 1/3 charge arises due to incompressibility and the Hamiltonian first principles. In our theory 1/3 charge can arise only when 2/3 also arises as depicted in Figure 6. There is a difference between Laughlin's and our theory. The fractionalisation in Laughlin's theory is independent of spin whereas in our theory spin plays an important role in determining the fraction.


Laughlin's 1/3 charge

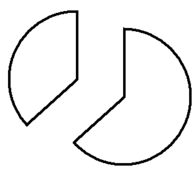
Shrivastava's theory. (1/3)e+(2/3)e =1e, (2/5)e+(3/5)e=1e, etc.

Figure 6. Pictorial display of the difference between the two theories

Dirac Equation

The basic idea of Dirac equation is to have space-time symmetry and the constancy of velocity of light, so that instead of $p^2/2m$, the kinetic energy appears as $c\boldsymbol{\alpha}.\mathbf{p}$ and the wave equation becomes,

$$(c\alpha.p+\beta mc^{2})\Psi(x,t)=i\hbar\frac{\partial}{\partial t}\Psi(x,t)\ , \qquad (27)$$

the free particle solutions of which are,

$$E_{\pm}=\pm(c^{2}p^{2}+m^{2}c^{4})^{1/2}\ . \qquad (28)$$

This equation gives the correct magnetic moments for the proton as well as for the neutron, subject to using the mass of the respective particle and an appropriate g value. In the case of electron Lande's formula,

$$g=1+\frac{j(j+1)-l(l+1)+s(s+1)}{2j(j+1)} \qquad (29)$$

with positive spin is used. In our case, the effective charge and hence the magnetic moment is determined by the g values. Hence, our method of defining the charge is the same as that for the magnetic moments of proton and neutron.

Shubnikov-de Haas Effect

At low temperatures, the integration over the Fermi distributon leads to x/sinh x type expression which is called Dingle's formula. The spin symmetry is found to modify this formula which determines the oscillation amplitude of resistivity as a function of magnetic field, called the Shubnikov-de Haas effect. Our theory introduces the effective charge so that the cyclotron frequency gets fractionalised resulting in $m/\nu_{\pm}$ which for $\nu_{\pm}=1$ becomes m, the electron mass. Thus, we have taken into account the spin-charge fractions to obtain the correct mass. For example, at certain magnetic field 1.5 m is found instead of m. The mass of the electron relative to band value as a function of carrier density has been deduced from Shubnikov-de Haas effect in GaAs/AlGaAs heterostructures. The factors 1 and 2/3 are found to arise from the spin-charge effect of Shrivastava. The experimental data are obtained from Tan et al.[18]. The dashed line is found by Kwon et al. [19] on the basis of small self energy corrections due to many body perturbative interactions. The factors of 1 and 2/3 in the mass are found by us. The mass of the free electron is m_e and the screening radius is deduced from the density, n, per unit area. We find that $m/\nu_{\pm}$ occurs in place of m for the mass of the electron in the Shubnikov-de Haas (SdH) effect. In fact, many other fractions of the mass of the electron given in Table 1, become allowed so that the electron really "falls apart". The oscillations due to flux quantisation allow the measurement of m/h^2. The flux quantisation in the Shubnikov-de Haas effect leads to "quantised Shubnikov-de Haas effect". Therefore, we observe consequences of the effect of flux quantisation on the Shubnikov-de Haas oscillations. There are zeroes in the resistivity at certain fields. There is a spin-charge effect so that the spin flip corresponds to a change in the charge. The Shubnikov-de Haas effect uses quantisation of Landau levels but not the flux quantisation. Hence, we find that there is a "quantised Shubnikov-de Haas effect" which measures the m/h^2. We find that when fractional values of $\nu_{\pm}$ are taken into account, the mass of the electron, equal to band mass in GaAs/AlGaAs, is obtained. When the magnetic field is varied, the different values of n cross the Fermi energy at different fields resulting in oscillations in the resistivity as a function of magnetic field. The oscillating resistivity is given by,

$$\delta\rho_{xx}(\varepsilon)_{\pm} = \rho_o \sum_p \gamma_{th} c_{\alpha,\beta} \exp\left(-\frac{p\pi}{v_{\pm}\omega_c\tau_q}\right)\cos\left[2\pi p\left(\frac{\varepsilon}{v_{\pm}\hbar\omega_c}-\frac{1}{2}\right)\right] \tag{30a}$$

where

$$\gamma_{th} = \frac{2\pi^2 pk_B T/(\hbar\omega_c v_{\pm})}{\sinh\left[2\pi^2 pk_B T/(\hbar\omega_c v_{\pm})\right]} \tag{30b}$$

The cosine factor also leads to zero resistivity whenever,

$$2\pi p\left(\frac{\varepsilon}{v_{\pm}\hbar\omega_c}-\frac{1}{2}\right)=\frac{\pi}{2} \tag{31}$$

so that the resistivity vanishes when B satisfies the above formula. We introduce the flux quantisation so that the exponential factor in (30) becomes,

$$\exp\left(-\frac{P\pi\ mc}{v_{\pm}\tau_c eB}\right)=\exp\left(-\frac{\pi\ mA}{v_{\pm}\tau_c h}\right) \tag{32}$$

so that m/h will be measured from the oscillations. Introducing the flux quantisation in the argument of Sinh factor, we obtain,

$$\hbar v_{\pm}\omega_c = \frac{\hbar v_{\pm} n_2 h}{mA} \tag{33}$$

which does not have the charge but measures m/h. In the experiments the factor measured is m*g*/n so it is clear that the mass and g get mixed [14].

Spin-Charge Locking

The charge of the electron may be described by matrices just as the angular momentum is. When the spin is aligned along the charge, such as s_x parallel to e_x, the arrangement is called the spin-charge locking. Taking our effective charge expression for $e_{eff}/e=(1/2)g_{\pm}$ we find the dot product of spin and charge to find,

$$e^*.s' = (2l+1)^{-1}\left(l+\frac{1}{2}\pm s\right).s' = (2l+1)^{-1}\left(1.s'+\frac{1}{2}(1).s'+s.s\right) \tag{34}$$

which produces spin-orbit and spin-spin interactions and there is spin divided by 2*l*+1. That is what makes it difficult to detect this type of effect.

Conclusions

We have found the correct explanation of the experimentally observed quantum Hall effect. We find that angular momentum gives rise to fractional charge. Therefore, there is a spin-charge effect, i.e. under high magnetic fields the spin determines the charge.

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