

Full Paper

A multi-criteria model for maintenance job scheduling

Oladimeji F. Ogunwolu¹, Sunday A. Oke^{2*}, and Oluwatoyin P. Popoola¹

¹ Department of Systems Engineering, Faculty of Engineering, University of Lagos, Nigeria

² Department of Mechanical Engineering, Faculty of Engineering, University of Lagos, Nigeria

* Author to whom correspondence should be addressed, email: sa_ok@yahoo.com

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Abstract: This paper presents a multi-criteria maintenance job scheduling model, which is formulated using a weighted multi-criteria integer linear programming maintenance scheduling framework. Three criteria, which have direct relationship with the primary objectives of a typical production setting, were used. These criteria are namely minimization of equipment idle time, manpower idle time and lateness of job with unit parity. The mathematical model constrained by available equipment, manpower and job available time within planning horizon was tested with a 10-job, 8-hour time horizon problem with declared equipment and manpower available as against the required. The results, analysis and illustrations justify multi-criteria consideration. Thus, maintenance managers are equipped with a tool for adequate decision making that guides against error in the accumulated data which may lead to wrong decision making. The idea presented is new since it provides an approach that has not been documented previously in the literature.

Keywords: multi-criteria analysis, decision-making, maintenance-scheduling, optimisation, truck plant

Introduction

Contemporary industrial systems strive for both quality and cost-effectiveness as a result of the challenges of optimally managing their processes and facilities through effective and efficient maintenance schemes [1-7]. Thus, a properly planned and implemented maintenance scheduling programme would guarantee efficient utilisation of machines, rapid response to demands and the

achievement of specific deadlines [8]. However, maintenance could be linked with the multi-facet objective of industrial systems. Hence, modelling, simulating, and analysing industrial system problems with multi-objective formulations are more realistic than formulating models and solving them through a single objective of system. Consequently, when such multi-objective frameworks have unit-parity, a single multi-criteria objective formulation may be used in place of the inherently multi-objective formulation with unit disparity. These arguments inform the methods used in this work.

In the maintenance scheduling literature, a number of approaches have been used in tackling the problem [9-12]. In addition, Jarvis [13] developed a heuristic computerised maintenance system for planning and scheduling. This model includes a scheme that allocates manpower to jobs based on First In First Out (FIFO) queue discipline. Roberts and Escudero [14] proposed a deterministic integer linear programming model in which personnel availability is given. The model is not multi-criteria. Roberts and Escudero [15] further revised the earlier work to obtain a minimum size model. Worrall and Mert [16] proposed dynamic heuristic and deterministic rules for maintenance planning and scheduling which does not address a multi-criteria concern. Duffuaa and Al-Sultan [17,18] proposed a stochastic formulation of the maintenance scheduling problem as an extension of Roberts and Escudero's [14] work. This again did not address a multi-criteria or multi-objective proposition. Based on the obvious gaps identifiable in the literature, this work addresses the multi-criteria concern. The approach is similar to the framework presented by Ogunwolu and Popoola [19].

The paper consists of four sections. The introduction provides the motivation for the study, the definition of the problem, the research objective, a literature review and the expected contributions. The methodology employed for the maintenance job scheduling is presented in section two. This shows the framework of the multi-criteria integer linear programming (MILP) problem formulated and solved. Section three presents the results obtained in running the model and its variants. It also shows a number of deductions made in line with the method of analysis. Discussions are also made on the optimal values and other values of importance. In section four, conclusions are made to justify the current approach and future directions given.

Methodology

Preamble

The maintenance job scheduling problem was modeled as a multi-criteria integer linear programming problem with three criteria. The three criteria used run parallel to the three major goals of typical manufacturing system. They are based on minimisation of equipment idle times, personnel idle times and delay in scheduling and presented in the next sub-section. These criteria have time-parity and are hence integrated as a single objective problem with integrated criteria. The problem space is constrained by specified jobs arrival (due routine maintenance) times, job-specific and unique time commencement of maintenance, as well as the need for the required personnel and equipment to be met by the available personnel and equipment at any time point within the schedule time-horizon. The schedule time-horizon is discretised at one-hour intervals of time. Job arrivals and maintenance, durations also have integral values.

The main model

The maintenance problem on hand is modeled as

$$\text{Minimize } \left\{ \sum_{\ell} \sum_k \left(TE_{\ell k(\ell)} - \sum_i \sum_j x_{ij} t_i n_{i\ell k(\ell)}^e \right) + \sum_p \left(TM_p - \sum_i \sum_j x_{ij} t_i n_{ip}^m \right) + \sum_i \sum_j x_{ij} (j - a_i) \right\}$$

subject to:

$$\sum_i x_{ij} (j - a_i - 1) \geq 0 \text{ for each } j \tag{1}$$

$$\sum_j x_{ij} \leq 1 \text{ for each job } i \tag{2}$$

$$\sum_i x_{ij} \geq 0 \text{ for each } j \tag{3}$$

$$\sum_r x_{ir} n_{i\ell k(\ell)}^e \leq E_{\ell k(\ell)} \text{ for each } i, \ell, k \tag{4}$$

$$\sum_r x_{ir} n_{ip}^m \leq M_p \text{ for each } i, p \tag{5}$$

$$x_{ij} = \begin{cases} 1 & \text{if job } i \text{ is scheduled at time } j \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

$$j \leq T \tag{7}$$

$$a_i + 2 \leq r \leq j - 1 \tag{8}$$

T = scheduling time-horizon

$E_{\ell k(\ell)}$ = the maximum available number of Equipment of group ℓ type $k(\ell)$

M_p = the maximum available number of manpower of category p available

t_i = duration of job i

$n_{i\ell k(\ell)}^e$ = number of equipment group ℓ type $k(\ell)$ needed for job i

n_{ip}^m = number manpower category p needed for job i

a_i = due time for routine maintenance of job i within schedule time-horizon

Model Criteria

Criterion 1 measures the idle time of equipment. It constitutes the first term in the objective function and is expressed as a difference of the total time of availability of all equipment (i.e. $TE_{\ell k(\ell)}$) and the time the equipment are in use (i.e. $\sum_i \sum_j x_{ij} t_i n_{i\ell k(\ell)}^e$) over the schedule horizon. *Criterion 2*

measures personnel idle time. It constitutes the second term in the objective function and is expressed as a difference of the total time of availability of all personnel of different trades, (i.e. TM_p) and the time they are engaged, (i.e. $\sum_i \sum_j x_{ij} t_i n_{ip}^m$) within the schedule horizon. *Criterion 3* measures the job

delay. It constitutes the third term in the objective function and is expressed as the difference between the actual schedule time, j and the job arrival time, a_i over all jobs and the schedule horizon.

The three criteria that result in downtime for the system considered here are idle time of equipment, personnel idle time, and job delays. This downtime can be classified as avoidable and unavoidable. Avoidable idle time relates to those created artificially by the system due to non-provision of resources needed for the maintenance job when needed. Unavoidable idle time relates to those due to naturally uncontrollable situations such as unplanned absence of key maintenance staff at work, thereby creating a job implementation vacuum. In general, other causes of idle time/downtime include (i) inefficient material ordering system, which leads to delay in the receipt of materials; (ii) untimely release of funds to execute projects by the accounts department; and (iii) unexpected damage to funding equipment or its non-functionality.

Model Constraints

Constraint 1 stipulates the earliest time a job can be scheduled. *Constraint 2* constrains the model to schedule a job once or not schedule at all. *Constraint 3* allows none or more than one job to be scheduled at any discrete time point, j. *Constraint 4* stipulates that the maximum number of individual equipment group tools in use at any time j cannot exceed the maximum available. *Constraint 5* stipulates that the maximum number engaged of an individual personnel group at any time j cannot exceed the maximum available. *Constraint 6* is the binary integer decision variable for determining schedule time points that optimises the objective function. *Constraint 7* constrains all schedule time points to be within the stipulated time-horizon T. *Constraint 8* specifies a range of value of discrete time point based on maintenance duration of individual jobs during which relevant equipment and personnel are kept in use.

Model Variants

From the above, variants of the main model for different combinations of criteria are obtainable (Table 1). For all variants, constraints (1), (2), (3), (6) and (7) govern. Here, for each variant the governing criteria are marked (√) and additional constraints (apart from the common ones) marked as (†).

Table 1. Variants of the model with criteria combinations and variable constraints

Variant	Criteria Combinations			Constraints		
	1	2	3	4	5	8
Main	√	√	√	†	†	†
1	√			†		†
2		√			†	†
3			√			
4	√	√		†	†	†
5	√		√	†		†
6		√	√		†	†

Case study

In order to verify the model in a case study, hypothetical data is used to illustrate the working of the model through the generation of a test problem. The data mimics that of a truck manufacturing company (Dynamics) located in Lagos, Nigeria. This company mostly engages skilled electricians, mechanical technicians, and labourers in the maintenance of its equipment. It is assumed that as soon as an equipment breaks down, it is ready for maintenance. Maintenance scheduling in Dynamics is typically an iterative process, which involves various steps with the aim of achieving cost effective maintenance. In Dynamics, a planned maintenance schedule is a properly organised report that would effectively handle the maintenance activities performed on the various equipment in the plant. It takes into consideration the frequency of activities that would be performed on equipment and facilities. It also optimally allots man-hours to activities based on the availability of manpower. Other concerns include safety precautions to be observed, the tools, materials and test equipment that should be made available. Each division in the organisation will submit request for a planned maintenance event. A planning group will be tasked with the responsibility of coordinating the entire maintenance request to produce a “least-cost” maintenance schedule observing reliability constraints. This schedule will be passed to the generation division, which will review the schedule in terms of resource constraints (e.g. availability of personnel and equipment) and suggest revision. The planning group incorporates these decisions in order to produce a revised maintenance schedule. Typically, there are multiple iterations between the planning group and the generation division before a maintenance schedule is approved.

The multi-criteria model was tested with a short term 10-job scheduling problem with time horizon of 8 hours (one day) with deterministic arrival times (due time for routine maintenance). The job parameter specifications giving the available time, ideally the routine maintenance due times that fall within the schedule time-horizon, and the estimated duration of time required for maintenance of individual jobs are given in Table 2.

Table 2. Job parameter specifications

Job i	Arrival time, a_i	Maintenance Duration t_i
1	0	2
2	4	3
3	3	2
4	2	2
5	5	2
6	4	3
7	3	1
8	7	2
9	6	1
10	5	3

It should be noted that constraint 1 of the Mathematical model above constrains the earliest schedule time of a job as 1 hour after the due time for its routine maintenance. This is ideal for practical considerations. Table 3 gives the schedule of needed and available (maximum number of) equipment. The equipment is classified as tool groups for simplicity with three mechanical tool groups and two for electrical.

Table 3. Job equipment needs and availability

Job	Equipment					Max.
	Mechanical			Electrical		
	1	2	3	1	2	
i	4	5	4	4	6	Required
1	1	2	2	1	2	
2	2	2	1	1	2	
3	2	2	2	1	2	
4	1	1	2	1	2	
5	2	1	2	2	1	
6	2	2	1	1	2	
7	2	1	2	1	2	
8	1	2	2	2	2	
9	2	2	1	1	2	
10	1	2	2	1	1	

Personnel available (maximum number) and those required for the individual jobs are specified in Table 4. Three classes of manpower: mechanical, electrical and supporting labour force, are specified.

Table 4. Job manpower needs and availability

Job	Manpower			Max
	Mechanical	Electrical	Labour	
i	7	6	7	Required
1	2	3	2	
2	3	2	3	
3	3	2	2	
4	1	2	1	
5	2	1	1	
6	3	2	3	
7	2	2	3	
8	2	1	1	
9	2	2	2	
10	1	1	2	

Method of solution

The test problem was run with the Integer Linear Programming module of the Quantitative Systems for Business Plus (QSB+) software for the solution of the main model formulated and its variants.

Methods of analysis

Apart from the optimal objective values realised, five other approaches are used in this paper to compare the results obtained and make necessary deductions:

1. *The number of jobs scheduled.* The model by Constraint (2) allows a job to be scheduled or unscheduled within the given time-horizon. The measure of the number of jobs scheduled or unscheduled is a measure of satisfactoriness of the model variant.
2. *The number of uncompleted jobs.* There are possibilities of jobs scheduled which cannot be completed within the time-horizon. This is a backlog which the next horizon schedule has to start with. The number of jobs completed together with the time duration left for completion can also serve as a basis of comparison of variants of the model.
3. *Equipment utilisation indices.* Hourly equipment utilisation index (HEUI) for each variant h of the model can be defined as

$$\lambda_{hj}^e = \sum_k \sum_\ell \left[\frac{\sum_i (x_{ij} + x_{ir}) n_{\ell k(\ell)}^e}{E_{\ell k(\ell)}} \right] \text{ for each } j \text{ and } x_{ij} = 1.$$

Thus, the total equipment utilisation index for each variant h within the time horizon T is

$$\eta_h^e = \frac{1}{T} \sum_{j=1}^T \lambda_{hj}^e.$$

4. *Manpower utilisation indices.* Hourly manpower utilisation index (HMUI) for each variant λ_{hj}^m is defined similar to HEUI as

$$\lambda_{hj}^m = \sum_p \left[\frac{\sum_i (x_{ij} + x_{ir}) n_p^m}{M_p} \right] \text{ for each } j \text{ and } x_{ij} = 1.$$

The total manpower utilisation index for a variant h is also defined as

$$\eta_h^m = \frac{1}{T} \sum_{j=1}^T \lambda_{hj}^m.$$

5. *Number of branch and bound iterations required.* The QSB+ solution procedure used zero integer tolerance and newest branching schemes. The number of iterations for the model and its variant is thus a level ground for their comparison.

Results and Discussion

The schedules obtained from running the model and its variants are as in Table 5. A number of deductions can be made in line with the methods of analysis enumerated in the previous section. These are summarised in Tables 6, 7 and 8. Other analyses emerged as in Tables 9 and 10. Statistical tests using t-Test were administered on the results obtained in Tables 7 and 8. These analyses are shown in Tables 9 and 10 respectively. The aim of this analysis is to find out whether or not there are significant

differences between the main model and one of its variants. In using statistical test for the problem, the first step taken is to formulate the null hypothesis and alternative hypothesis also. If the mean of the main model is represented by μ_1 , and that of the first variant (and for any of the variants in subsequent analysis) is μ_2 , then the null hypothesis is stated as: $\mu_1 - \mu_2 = 0$. The alternative hypothesis is stated as: $\mu_1 - \mu_2 \neq 0$. The level of significance, α , equals 0.05.

Table 5. Job schedules for model and its variants

Model/ Variant	Time Point, j (hourly)							
	1	2	3	4	5	6	7	8
Main	1		4	3	7	5, 6	9	2, 8
1	1		4	3	7	5,6		2, 10
2	1		4	3,7	6	5	9	2,8,10
3*	1		4	3,7	2,6	5,10	9	8
4	1		4	3	7	5,6		2,8
5	1		4	3	7	5,6		2,10
6	1		4	3,7	2	5	9	6,8,10

* Constraint 2 relaxed as an equality constraint

The criterion is that the null hypothesis should be rejected if $t < -1.89$ or $t > 1.89$, where 1.89 is the value of $t_{0.05}$ for 7 degrees of freedom. Since the means and variances of the two samples are 0.59 and 0.09 for sample 1, and 0.61 and 0.06 for sample 2, t equals -0.23. Then, since $t = -0.23$ does not exceed 1.89, the null hypothesis must be accepted. We conclude that the values from the two methods are the same, hence, significant difference does not exist between them. This analysis is only for data in Table 7, between the main method and variant 1. Similar analysis is then performed for the main method and other variants. This is also extended to the data in Table 8. The decisions are stated in Tables 9 and 10.

Table 6. Deductions from optimal schedules

Model / variant	Optimal value (hrs)	No. of iterations	No. of jobs un- scheduled	Jobs un- completed	
				No.	Hours
Main	37	11	1	1	3
1	43	17	2	2	4
2	43	16	0	3	5
3	11	1	0	1	1
4	99	25	2	2	3
5	58	13	2	2	4
6	58	27	0	3	5

Optimal values

Apart from variant 3, a single criterion variant with trivial solution, the other non-single criteria variants have comparably better optima. For virtually all the measures of analysis identified, the non-single criteria variants and the main model have equally good or better comparative measures.

Table 7. Variants’ equipment utilisation indices

Model/ Variant	Hourly Equipment Utilisation Index								Total
	1	2	3	4	5	6	7	8	
Main	0.35	0.35	0.20	0.55	0.70	0.60	1.00	1.00	0.60
1	0.35	0.35	0.30	0.70	0.74	0.70	0.70	1.00	0.60
2†	0.35	0.35	0.30	1.00	0.40	0.70	1.04	1.00	0.68
3†	0.35	0.35	0.30	1.00	1.09	1.04	1.04	1.43	0.83
4	0.35	0.35	0.30	0.70	0.74	0.70	0.70	1.00	0.60
5	0.35	0.35	0.30	0.70	0.74	0.70	0.70	1.00	0.60
6	0.35	0.35	0.30	1.00	0.74	0.74	1.04	1.04	0.68

†Some indices are greater than 1 since the measure is not reckoned within the variants.

Table 8. Variants’ manpower utilisation indices

Model/ Variant	Hourly Manpower Utilisation Index								Total
	1	2	3	4	5	6	7	8	
Main	0.35	0.35	0.20	0.70	0.74	0.70	0.90	1.00	0.63
1	0.35	0.35	0.20	0.55	0.70	0.60	0.60	1.00	0.54
2	0.35	0.35	0.20	0.90	0.75	0.60	0.90	0.90	0.62
3†	0.35	0.35	0.20	0.90	1.15	0.80	1.20	1.20	0.77
4	0.35	0.35	0.20	0.55	0.70	0.60	0.60	1.00	0.54
5	0.35	0.35	0.20	0.55	0.70	0.60	0.60	1.00	0.54
6	0.35	0.35	0.20	0.90	0.75	0.60	0.90	0.80	0.62

†Some indices are greater than 1 since the measure is not reckoned within the variants.

Table 9. t-Test paired two samples for means (analysis of data from Table 7)

Description	Main	Variant 1	Main	Variant 2	Main	Variant 3	Main	Variant 4	Main	Variant 5	Main	Variant 6
Mean	0.59	0.61	0.59	0.64	0.59	0.83	0.59	0.61	0.59	0.61	0.59	0.70
Variance	0.09	0.06	0.09	0.11	0.09	0.18	0.09	0.06	0.09	0.06	0.09	0.10
Correlation	0.89		0.79		0.89		0.89		0.89		0.89	
Hypothesised mean	0		0		0		0		0		0	
Df	7		7		7		7		7		7	
t Stat	-0.23		-0.67		-3.07		-0.23		-0.23		-1.93	
P(T≤ t) one tail	0.41		0.26		0.01		0.41		0.41		0.05	
t Critical one tail	1.89		1.89		1.89		1.89		1.89		1.89	
P(T≤ t) two tail	0.82		0.52		0.02		0.82		0.82		0.10	
t Critical two tail	2.36		2.36		2.36		2.36		2.36		2.36	
Decision	Not significant		Not significant		Significant		Not significant		Not significant		Significant	

Table 10. t-Test paired two samples for means (analysis of data from Table 8)

Description	Main	Variant t 1	Main	Variant t 2	Main	Variant t 3	Main	Variant t 4	Main	Variant t 5	Main	Variant t 6
Mean	0.62	0.54	0.62	0.62	0.62	0.77	0.62	0.54	0.62	0.54	0.62	0.61
Variance	0.08	0.06	0.08	0.08	0.08	0.17	0.08	0.06	0.08	0.06	0.08	0.08
Correlation	0.93		0.95		0.97		0.93		0.93		0.92	
Hypothesise d mean	0		0		0		0		0		0	
Df	7		7		7		7		7		7	
t Stat	1.94		-0.04		-2.79		1.94		1.94		0.28	
P(T≤ t) one tail	0.05		0.49		0.01		0.05		0.05		0.39	
t Critical one tail	1.89		1.89		1.89		1.89		1.89		1.89	
P(T≤ t) two tail	0.09		0.97		0.03		0.09		0.09		0.79	
t Critical two tail	2.36		2.36		2.36		2.36		2.36		2.39	
Decision	Significant		Not significant		Significant		Significant		Significant		Not significant	

Conclusion

Professionals in the industry, particularly maintenance managers are facing many challenges in the rapidly changing modern world. The stiff competition in the global international business is forcing decision makers and researchers to review maintenance practices towards a more efficient maintenance organisation. In this paper an attempt has been made to demonstrate the development and application of the multi-criteria approach to maintenance job scheduling in order to generate more robust maintenance job schedules than with single criterion. From the literature, no prior research seems to have been documented in this regard. Thus, this appears to be an important contribution to knowledge. An important benefit of the present work to the maintenance scheduling community is the ability to capture holistic information of the system being studied. From the case study considered we have shown that it is feasible to implement the model in a particular instance. Further applications in different environments are possible with slight modifications in model parameters. Future studies may consider extension to the model by incorporating fuzziness, stochastic elements, and genetic optimisation techniques in order to have a wide range of data capture and analysis.

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