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Technical Note

Some results on λ- statistical convergence on time scales

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Abstract: Some sets of sequences on a time scale are introduced by considering the various sequences $\mu_{\Delta_\lambda(t)}$ and $\mu_{\Delta_\beta(t)}$ in the class Λ. Furthermore, some inclusion results on these sets are obtained.

Keywords: statistical convergence, sequence spaces, time scales

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INTRODUCTION

The development of sequence spaces is nowadays effected by the introduction of various new convergence methods such as statistical convergence. The idea of statistical convergence goes back to a crucial study of Zygmund [1]. This notion was first defined for real sequences by Steinhaus [2] and Fast [3]. Schoenberg [4] called it *D*-convergence. Later on it was studied and linked with summability by Fridy [5], Çolak [6], Connor [7], Maddox [8], Rath and Tripathy [9], Šalát [10], Tripathy [11], Moricz [12] and many others [13-18]. Recently, the statistical convergence has many applications in different fields, e.g. measure theory, locally convex spaces, approximation theory, probability, and Banach spaces. The concept is described briefly.

Statistical convergence is related to the density of subsets of $\mathbb{N}$. The natural density of a subset A of $\mathbb{N}$ is given by

$$\delta(A)=\lim_{n\to\infty}\frac{1}{n}\left|\{k\le n\ :\ k\in A\}\right|,$$

if the limit exists, where $|.|$ indicates the cardinality of any set [10].

A complex sequence $(x_k)_{k=1}^{\infty}$ is statistically convergent to some number L, if for $\forall\varepsilon>0$, $\delta(\{k\in\mathbb{N}:|x_k-L|\ge\varepsilon\})$ is zero. L is necessarily unique and it is called the statistical limit of (x_k), and written as $Stat-\lim x_k=L$. The space of all statistically convergent sequences is denoted by S [5,10]. Leindler [19] defined the generalised de la Vallée-Poussin mean as follows.

Let $\lambda=(\lambda_n)$ be a non-decreasing sequence of positive real numbers which approach ∞ with $\lambda_{n+1}\le\lambda_n+1$, $\lambda_1=1$. Then

$$t_n(x) = \frac{1}{\lambda_n} \sum_{k \in I_n} x_k,$$

where $I_n = [n - \lambda_n + 1, n]$. Throughout this study Λ denotes the set of all such sequences.

Borwein [20] and Maddox [21] introduced and studied strongly summable sequences of functions. Then Mursaleen [22] introduced λ-density and λ-statistical convergence by letting $K \subset \mathbb{N}$ and defining the $\lambda -$density of K by

$$\delta_\lambda(K) = \lim_{n\to\infty} \frac{1}{\lambda_n} \left| \{ n - \lambda_n + 1 \le k \le n \ : \ k \in K \} \right|,$$

where $\delta_\lambda(K)$ transforms to the natural density $\delta(K)$ for $\lambda_n = n$ and $\forall n \in \mathbb{N}$. The sequence (x_k) is λ-statistically convergent to L if, for $\forall \varepsilon > 0$, $\lim_{n\to\infty} \frac{1}{\lambda_n} \left| \{ k \in I_n \ : \ |x_k - L| \ge \varepsilon \} \right|$ has zero natural density [22]. After that Nuray [23] studied λ-strong summable and λ- statistically convergent functions by using the above notions. Now we need to give some information about the historical improvement of time scale calculus and its structure.

A time scale $\mathbb{T}$ is an arbitrary, non-empty and closed subset of $\mathbb{R}$. Time scale calculus, introduced by Hilger [24], allows to the unification of the usual differential and integral calculus with one variable. One can replace the range of definition $(\mathbb{R})$ of the functions under consideration by $\mathbb{T}$. For $t \in \mathbb{T}$, the forward jump operator $\sigma \ : \ \mathbb{T} \to \mathbb{T}$ can be defined by

$$\sigma(t) = \inf\{ s \in \mathbb{T} : s > t \}.$$

The graininess function $\mu \ : \ \mathbb{T} \to [0,\infty)$ is defined by $\mu(t) = \sigma(t) - t.$ Here, we put $\inf \phi = \sup \mathbb{T}$ where ϕ is an empty set. A closed interval of $\mathbb{T}$ is given by $[a,b]_{\mathbb{T}} = \{ t \in \mathbb{T} : a \le t \le b \}$. Open intervals or half-open intervals can be defined similarly [25]. There are many studies on time scales for different areas [e.g. 26-27].

Let A denote the family of all left-closed and right-open intervals of $\mathbb{T}$ of the form $[a,b)_{\mathbb{T}}$. Let $m : A \to [0,\infty)$ be a set function on A such that $m([a,b)_{\mathbb{T}}) = b - a$. Here, it is known that m is a countably additive measure on A. Now the Caratheodory extension of the set function m associated with family A is said to be the Lebesque $\Delta -$measure on $\mathbb{T}$ and is denoted by μ_Δ. In this case it is known that if $a \in \mathbb{T} - \{\max \mathbb{T}\}$, then the single-point set $\{a\}$ is $\Delta -$ measurable and $\mu_\Delta(a) = \sigma(a) - a$. If $a,b \in \mathbb{T}$ and $a \le b$, then $\mu_\Delta((a,b)_{\mathbb{T}}) = b - \sigma(a)$. If $a,b \in \mathbb{T} - \{\max \mathbb{T}\}$, $a \le b$, then $\mu_\Delta((a,b]_{\mathbb{T}}) = \sigma(b) - \sigma(a)$ and $\mu_\Delta([a,b])_{\mathbb{T}}) = \sigma(b) - a$ [28].

Let $\mathbb{T} \subset [0,\infty)$ and there exists a subset $\{ t_k \ : \ k \in \mathbb{N} \} \subset \mathbb{T}$ with $0 = t_0 < t_1 < t_2 ...$ and $\lim_{k\to\infty} t_k = \infty$. The spaces of continuous functions are defined on a time scale by Batit [29]:

$$\ell_\infty(\mathbb{T}) = \left\{ f | f \ : \ \mathbb{T} \to \mathbb{R}, \sup_{t\in\mathbb{T}} |f(t)| < \infty \right\},$$

$$c(\mathbb{T}) = \left\{ f | f \ : \ \mathbb{T} \to \mathbb{R}, \lim_{t\to\infty} f(t) = \ell, \text{for some } \ell \right\},$$

$$c_0(\mathbb{T}) = \left\{ f | f \ : \ \mathbb{T} \to \mathbb{R}, \lim_{t\to\infty} f(t) = 0 \right\}.$$

Additionally, there are some studies about statistical convergence on time scales in the literature. For instance, Seyyidoglu and Tan [30] gave some new notations such as $\Delta -$convergence and $\Delta -$Cauchy by using $\Delta -$density and investigated their relations. Recently,

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Turan and Duman [28] studied statistical convergence of $\Delta-$measurable real-valued functions defined on time scales. Furthermore, λ-density and λ- statistical convergence have been explained by Yilmaz et al [31].

Definition 1. Let Ω be a $\Delta_\lambda-$measurable subset of T. Then $\Omega(t,\lambda)$ is defined by

$$\Omega(t,\lambda)=\{s\in[t-\lambda_t+t_0,t]_{\mathsf{T}}\ :\ s\in\Omega\},$$

for $t\in\mathsf{T}$. In this case the $\lambda-$density of Ω on T is defined as follows:

$$\delta_{\mathsf{T}}^{\lambda}(\Omega)=\lim_{t\to\infty}\frac{\mu_{\Delta_\lambda}(\Omega(t,\lambda))}{\mu_{\Delta_\lambda}([t-\lambda_t+t_0,t]_{\mathsf{T}})}\quad, \tag{1}$$

if the above limit exists [31]. If one takes $\lambda_t=t$ in (1), the classical density of Ω on T is obtained as

$$\delta_{\mathsf{T}}(\Omega)=\lim_{t\to\infty}\frac{\mu_{\Delta}(\Omega(t))}{\mu_{\Delta}([t_0,t]_{\mathsf{T}})},$$

provided that the above limit exists [28, 30].

Definition 2. Let $f\ :\ \mathsf{T}\to\mathsf{R}$ be a $\Delta_\lambda-$measurable function. Then f is λ-statistically convergent to L on T if

$$\lim_{t\to\infty}\frac{\mu_{\Delta_\lambda}(\{s\in[t-\lambda_t+t_0,t]_{\mathsf{T}}\ :\ |f(s)-L|\geq\varepsilon\})}{\mu_{\Delta_\lambda}([t-\lambda_t+t_0,t]_{\mathsf{T}})}=0,$$

for $\forall\varepsilon>0$. In this case $\boldsymbol{s}_{\mathsf{T}}^{\lambda}-\lim_{t\to\infty}(f(t))=L$. The set of all λ-statistically convergent functions on T is denoted by $\boldsymbol{s}_{\mathsf{T}}^{\lambda}$ [31].

MAIN RESULTS

In this section the connections between $\boldsymbol{s}_{\mathsf{T}}^{\lambda}$ and $\boldsymbol{s}_{\mathsf{T}}^{\beta}$; $[w,\lambda]_{\mathsf{T}}$ and $[w,\beta]_{\mathsf{T}}$; and S_{T}^{λ} and $[w,\beta]_{\mathsf{T}}$ for various sequences $\mu_{\Delta_\lambda(t)}$ and $\mu_{\Delta_\beta(t)}$ are determined in the class Λ. Also, the results of Çolak [6] are generalised to a time scale.

Theorem 1. Let $\mu_{\Delta_{\lambda(t)}},\mu_{\Delta_{\beta(t)}}\in\Lambda$ such that $\mu_{\Delta_{\lambda(t)}}\leq\mu_{\Delta_{\beta(t)}}$ for all $t\in\mathsf{T}$.

i) If

$$\liminf_{t\to\infty}\frac{\mu_{\Delta_{\lambda(t)}}([t-\lambda_t+t_0,t]_{\mathsf{T}})}{\mu_{\Delta_{\beta(t)}}([t-\beta_t+t_0,t]_{\mathsf{T}})}>0\quad, \tag{2}$$

then $\boldsymbol{s}_{\mathsf{T}}^{\beta}\subseteq\boldsymbol{s}_{\mathsf{T}}^{\lambda}$.

ii) If

$$\lim_{t\to\infty}\frac{\mu_{\Delta_{\lambda(t)}}([t-\lambda_t+t_0,t]_{\mathsf{T}})}{\mu_{\Delta_{\beta(t)}}([t-\beta_t+t_0,t]_{\mathsf{T}})}=1\quad, \tag{3}$$

then $\boldsymbol{s}_{\mathsf{T}}^{\lambda}\subseteq\boldsymbol{s}_{\mathsf{T}}^{\beta}$.

Proof. i) Suppose that $\mu_{\Delta_{\lambda(t)}}\leq\mu_{\Delta_{\beta(t)}}$ for all $t\in\mathsf{T}$ and (2) is satisfied. Then $I_t\subset J_t$, and so for $\varepsilon>0$, we have

$$\mu_{\Delta_{\beta(t)}}(\{s\in[t-\beta_t+t_0,t]_{\mathsf{T}}\ :\ |f(s)-L|\geq\varepsilon\})\geq\mu_{\Delta_{\lambda(t)}}(\{s\in[t-\lambda_t+t_0,t]_{\mathsf{T}}\ :\ |f(s)-L|\geq\varepsilon\}).$$

Therefore,

$$\frac{\mu_{\Delta_{\beta(t)}}\left(\left\{s\in[t-j_t+t_0,t]_{\mathbb{T}}\ :\ |f(s)-L|\geq\varepsilon\right\}\right)}{\mu_{\Delta_{\beta(t)}}\left([t-\beta_t+t_0,t]_{\mathbb{T}}\right)}$$
$$\geq\frac{\mu_{\Delta_{\lambda(t)}}\left([t-\lambda_t+t_0,t]_{\mathbb{T}}\right)}{\mu_{\Delta_{\beta(t)}}\left([t-\beta_t+t_0,t]_{\mathbb{T}}\right)}\frac{1}{\mu_{\Delta_{\lambda(t)}}\left([t-\lambda_t+t_0,t]_{\mathbb{T}}\right)}\times\mu_{\Delta_{\lambda(t)}}\left(\left\{s\in[t-\lambda_t+t_0,t]_{\mathbb{T}}\ :\ |f(s)-L|\geq\varepsilon\right\}\right)$$

for $\forall t\in\mathbb{T}$, where $J_t=[t-\beta_t+1,t]$. Hence by using (2) and taking the limit $t\to\infty$, we get $s_{\mathbb{T}}^{\beta}\subseteq s_{\mathbb{T}}^{\lambda}$.

ii) Let f be a $\Delta_\lambda-$measurable function and $s_{\mathbb{T}}^{\lambda}-\lim f(s)=L$. Since $I_t\subset J_t$ for all $t\in\mathbb{T}$, we can write

$$\frac{\mu_{\Delta_{\beta(t)}}\left(\left\{s\in[t-\beta_t+t_0,t]_{\mathbb{T}}\ :\ |f(s)-L|\geq\varepsilon\right\}\right)}{\mu_{\Delta_{\beta(t)}}\left([t-\beta_t+t_0,t]_{\mathbb{T}}\right)}=\frac{\mu_{\Delta_{\beta(t)}}\left(\left\{t-\beta_t+t_0\leq s\leq t\ :\ |f(s)-L|\geq\varepsilon\right\}\right)}{\mu_{\Delta_{\beta(t)}}\left([t-\beta_t+t_0,t]_{\mathbb{T}}\right)}+\frac{\mu_{\Delta_{\lambda(t)}}\left(\left\{s\in[t-\lambda_t+t_0,t]_{\mathbb{T}}\ :\ |f(s)-L|\geq\varepsilon\right\}\right)}{\mu_{\Delta_{\beta(t)}}\left([t-\beta_t+t_0,t]_{\mathbb{T}}\right)}$$

$$\leq\frac{\mu_{\Delta_{\beta(t)}}\left(s\in[t-\beta_t+t_0,t]_{\mathbb{T}}\right)-\mu_{\Delta_{\lambda(t)}}\left(s\in[t-\lambda_t+t_0,t]_{\mathbb{T}}\right)}{\mu_{\Delta_{\beta(t)}}\left([t-\beta_t+t_0,t]_{\mathbb{T}}\right)}+\frac{\mu_{\Delta_{\lambda(t)}}\left(\left\{s\in[t-\lambda_t+t_0,t]_{\mathbb{T}}\ :\ |f(s)-L|\geq\varepsilon\right\}\right)}{\mu_{\Delta_{\lambda(t)}}\left([t-\lambda_t+t_0,t]_{\mathbb{T}}\right)}$$

$$\leq\left(1-\frac{\mu_{\Delta_{\lambda(t)}}\left([t-\lambda_t+t_0,t]_{\mathbb{T}}\right)}{\mu_{\Delta_{\beta(t)}}\left([t-\beta_t+t_0,t]_{\mathbb{T}}\right)}\right)+\frac{\mu_{\Delta_{\lambda(t)}}\left(\left\{s\in[t-\lambda_t+t_0,t]_{\mathbb{T}}\ :\ |f(s)-L|\geq\varepsilon\right\}\right)}{\mu_{\Delta_{\lambda(t)}}\left([t-\lambda_t+t_0,t]_{\mathbb{T}}\right)},$$

for $\forall t\in\mathbb{T}$. Since $\lim_{t\to\infty}\frac{\mu_{\Delta_{\lambda(t)}}\left([t-\lambda_t+t_0,t]_{\mathbb{T}}\right)}{\mu_{\Delta_{\beta(t)}}\left([t-\beta_t+t_0,t]_{\mathbb{T}}\right)}=1$ by (3), the term in the above inequality tends to 0. Furthermore, since $s_{\mathbb{T}}^{\lambda}-\lim f(s)=L$, the second term of the right hand side of the above inequality goes to 0 as $t\to\infty$. Therefore, $s_{\mathbb{T}}^{\lambda}\subseteq s_{\mathbb{T}}^{\beta}$.

From Theorem 1, we can give the following corollaries.

Corollary 1. Let $\mu_{\Delta_{\lambda(t)}},\mu_{\Delta_{\beta(t)}}\in\Lambda$ such that $\mu_{\Delta_{\lambda(t)}}\leq\mu_{\Delta_{\beta(t)}}$ for all $t\in\mathbb{T}$. If (3) holds, then $s_{\mathbb{T}}^{\lambda}=s_{\mathbb{T}}^{\beta}$.

If we take $\mu_{\Delta_{\lambda(t)}}=\lambda(t)$, $t\in\mathbb{T}$ in the above corollary, we get the following corollary.

Corollary 2. Let $\mu_{\Delta_{\lambda(t)}}\in\Lambda$. If $\lim_{t\to\infty}\frac{\mu_{\Delta_{\lambda(t)}}\left([t-\lambda_t+t_0,t]_{\mathbb{T}}\right)}{\lambda(t)}=1$, then we have $s_{\mathbb{T}}^{\lambda}=s_{\mathbb{T}}$.

Now we give the concept of strong $\lambda-$Cesàro summability on $\mathbb{T}$.

Definition 3. $f\ :\ \mathbb{T}\to\mathbb{R}$ is strongly $\lambda-$Cesàro summable on $\mathbb{T}$ if there exists some $L\in\mathbb{R}$ such that

$$\lim_{t\to\infty}\frac{1}{\mu_{\Delta_{\lambda(t)}}\left([t-\lambda_t+t_0,t]_{\mathbb{T}}\right)}\int_{[t-\lambda_t+t_0,t]_{\mathbb{T}}}|f(s)-L|\Delta s=0,$$

where f is a $\Delta_\lambda-$measurable function.

In this case $[W,\lambda]_{\mathbb{T}}-\lim f(s)=L$ [31]. The set of all strongly λ-Cesàro summable functions on $\mathbb{T}$ is denoted by $[W,\lambda]_{\mathbb{T}}$. The Lebesque $\Delta-$integral on time scales was introduced by Cabada and Vivero [32].

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Theorem 2. Let $\mu_{\Delta_{\lambda(t)}}, \mu_{\Delta_{\beta(t)}} \in \Lambda$ such that $\mu_{\Delta_{\lambda(t)}} \leq \mu_{\Delta_{\beta(t)}}$ for all $t \in \mathrm{T}$. Then we get:
i) If (2) holds, then $[W,\lambda]_{\mathrm{T}} \subseteq [W,\beta]_{\mathrm{T}}$;
ii) If (3) holds, then $\ell_{\infty}(\mathrm{T}) \cap [W,\lambda]_{\mathrm{T}} \subseteq [W,\beta]_{\mathrm{T}}$.

Proof. i) Suppose that $\mu_{\Delta_{\lambda(t)}} \leq \mu_{\Delta_{\beta(t)}}$ for all $t \in \mathrm{T}$. Then $I_t \subset J_t$ for all $t \in \mathrm{T}$ so that we may write

$$\frac{1}{\mu_{\Delta_{\beta(t)}}\left([t-\beta_t+t_0,t]_{\mathrm{T}}\right)} \int_{[t-\beta_t+t_0,t]_{\mathrm{T}}} |f(s)-L|\Delta s \geq \frac{1}{\mu_{\Delta_{\lambda(t)}}\left([t-\lambda_t+t_0,t]_{\mathrm{T}}\right)} \int_{[t-\lambda_t+t_0,t]_{\mathrm{T}}} |f(s)-L|\Delta s,$$

for all $t \in \mathrm{T}$. This implies that

$$\begin{aligned}
&\frac{1}{\mu_{\Delta_{\beta(t)}}\left([t-\beta_t+t_0,t]_{\mathrm{T}}\right)} \int_{[t-\beta_t+t_0,t]_{\mathrm{T}}} |f(s)-L|\Delta s \\
&\geq \frac{\mu_{\Delta_{\lambda(t)}}\left([t-\lambda_t+t_0,t]_{\mathrm{T}}\right)}{\mu_{\Delta_{\beta(t)}}\left([t-\beta_t+t_0,t]_{\mathrm{T}}\right)} \frac{1}{\mu_{\Delta_{\lambda(t)}}\left([t-\lambda_t+t_0,t]_{\mathrm{T}}\right)} \int_{[t-\lambda_t+t_0,t]_{\mathrm{T}}} |f(s)-L|\Delta s.
\end{aligned}$$

Then by taking the limit $t \to \infty$ in the last inequality, we obtain $[W,\lambda]_{\mathrm{T}} \subseteq [W,\beta]_{\mathrm{T}}$.

ii) Let $f \in \ell_{\infty}(\mathrm{T}) \cap [W,\lambda]_{\mathrm{T}}$ and (3) hold. Since $f \in \ell_{\infty}(\mathrm{T})$, then there exists $M > 0$ such that $|f(s)| \leq M$ for $\forall s \in \mathrm{T}$. Also, now since $\mu_{\Delta_{\lambda(t)}} \leq \mu_{\Delta_{\beta(t)}}$ and $\frac{1}{\mu_{\Delta_{\beta(t)}}} \leq \frac{1}{\mu_{\Delta_{\lambda(t)}}}$, $I_t \subset J_t$ for $\forall t \in \mathrm{T}$, and we may write

$$\begin{aligned}
&\frac{1}{\mu_{\Delta_{\beta(t)}}\left([t-\beta_t+t_0,t]_{\mathrm{T}}\right)} \int_{[t-\beta_t+t_0,t]_{\mathrm{T}}} |f(s)-L|\Delta s \\
&\leq \frac{1}{\mu_{\Delta_{\beta(t)}}\left([t-\beta_t+t_0,t]_{\mathrm{T}}\right)} \int_{J_t/I_t} |f(s)-L|\Delta s + \frac{1}{\mu_{\Delta_{\beta(t)}}\left([t-\beta_t+t_0,t]_{\mathrm{T}}\right)} \int_{I_t} |f(s)-L|\Delta s \\
&\leq \left(\frac{\mu_{\Delta_{\beta(t)}}\left([t-\beta_t+t_0,t]_{\mathrm{T}}\right) - \mu_{\Delta_{\lambda(t)}}\left([t-\lambda_t+t_0,t]_{\mathrm{T}}\right)}{\mu_{\Delta_{\beta(t)}}\left([t-\beta_t+t_0,t]_{\mathrm{T}}\right)} \right) M + \frac{1}{\mu_{\Delta_{\lambda(t)}}\left([t-\lambda_t+t_0,t]_{\mathrm{T}}\right)} \int_{I_t} |f(s)-L|\Delta s
\end{aligned}$$

for $\forall t \in \mathrm{T}$. Since $\lim_{t\to\infty} \frac{\mu_{\Delta_{\lambda(t)}}\left([t-\lambda_t+t_0,t]_{\mathrm{T}}\right)}{\mu_{\Delta_{\beta(t)}}\left([t-\beta_t+t_0,t]_{\mathrm{T}}\right)} = 1$ and $f \in [W,\lambda]_{\mathrm{T}}$, the first and second terms of the right hand side of the above inequality tends to 0 as $t \to \infty$, where $1 - \frac{\mu_{\Delta_{\lambda(t)}}\left([t-\lambda_t+t_0,t]_{\mathrm{T}}\right)}{\mu_{\Delta_{\beta(t)}}\left([t-\beta_t+t_0,t]_{\mathrm{T}}\right)} \geq 0$ for all $t \in \mathrm{T}$.
This implies that $\ell_{\infty}(\mathrm{T}) \cap [W,\lambda]_{\mathrm{T}} \subseteq [W,\beta]_{\mathrm{T}}$.

From Theorem 2, we can give the following corollary.

Corollary 3. Let $\mu_{\Delta_{\lambda(t)}}, \mu_{\Delta_{\beta(t)}} \in \Lambda$ such that $\mu_{\Delta_{\lambda(t)}} \leq \mu_{\Delta_{\beta(t)}}$ for all $t \in \mathrm{T}$. If (3) holds, then $\ell_{\infty}(\mathrm{T}) \cap [W,\lambda]_{\mathrm{T}} = \ell_{\infty}(\mathrm{T}) \cap [W,\beta]_{\mathrm{T}}$.

CONCLUSIONS

For the summability theory, generalisation of some concepts, notations and theorems is an important issue. So we extend the study of Çolak [6] to an arbitrary time scale.

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