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Full Paper

Estimation of natural frequencies of symmetrically laminated plates with all edges clamped using neural networks

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Abstract: Neural networks were applied to the estimation of the natural frequencies of symmetrically laminated rectangular plates with all edges clamped. The networks were employed to determine the relationships between the natural frequency data obtained from the extended Kantorovich method by training. After a successful training, the neutral networks generated a new algorithm based on the root mean square error. To verify the accuracy of the new algorithm, it was tested with the new data. The results show the new algorithm's ability to estimate the natural frequencies within an acceptable accuracy range.

Keywords: natural frequency, laminated plates, clamped edges, neural network

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INTRODUCTION

Composite materials are increasingly being used in engineering fields due to their properties such as high strength-to-weight ratios, good fatigue and corrosion resistance. Composite materials have a plate-type structure because they are constructed in the form of a thin layer with small thickness. These materials are called laminated plates. In engineering fields, laminated plates are subjected to vibration in many applications. Laminated plates will be damaged if their natural frequencies coincide with the frequency of the external excitation. Therefore, the natural frequencies of vibration are significant in the design of laminated plates.

An analytical method such as the Navier method or the Levy method [1] can be applied to the estimation of the natural frequency of laminated plates. However, the method is only possible for laminated plates with two opposite edges being simply supported. For laminated plates with other boundary conditions, an approximate method such as the Galerkin method [2-3], the Rayleigh-Ritz method [4-5] or the extended Kantorovich method [6-7] is usually employed. The natural frequencies obtained from an approximate method are not in the formula form. Thus, an approximate method is inappropriate in a practical laminated plate design in which the material properties, boundary conditions or aspect ratios vary.

Many researchers [9-12] have investigated the complicated relationships between the natural frequencies obtained from an approximate method and the fibre orientations, number of layers, material properties, length-to-thickness ratios, and aspect ratios of laminated rectangular plates by using a neural network. All the researches were about neural network optimisation which gives the best natural frequency under a given state of the fibre orientations, number of layers, material properties, length-to-thickness ratios, and aspect ratios to certain boundary conditions of laminated rectangular plates.

In any practical situation, however, laminated rectangular plates are used with various aspect ratios. Therefore, the study of the relationship between natural frequencies and aspect ratios of laminated rectangular plates still has the major role in the design of laminated rectangular plates. A study on the relationship between natural frequencies and aspect ratios of laminated rectangular plates using regression analysis has also been reported [13]. The drawback of regression analysis, however, is the large number of terms in the polynomial equation. The purpose of this research is to apply neural networks to finding the relationship between the natural frequencies and aspect ratios of laminated rectangular plates. The neural networks have so far been considered to be limited to finding the relationship between the natural frequencies and the aspect ratios of isotropic plates [14-15].

METHODS

To find the relationship between the natural frequencies and aspect ratios of symmetrically laminated rectangular plates with all edges clamped, the process consisted of an extended Kantorovich method for calculating natural frequencies that were consistent with aspect ratios until the natural frequencies converged. The results from this process generated the relationship between natural frequencies and aspect ratios by a neural network.

Extended Kantorovich Method

The extended Kantorovich method [8] is an approximate method for reducing partial differential equations to ordinary differential equations. Each ordinary differential equation has a solution that does not satisfy the boundary conditions. By using the iterative calculation procedure, the final solution is induced to satisfy the boundary conditions, and natural frequencies are evaluated.

The natural frequencies of symmetrically laminated rectangular plates with all edges clamped, as shown in Figure 1, are governed by partial differential equation (1):

$$\frac{1}{2}\int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}}\left[D_{11}\left(\frac{\partial^2 w}{\partial x^2}\right)^2+2D_{12}\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right)+4D_{16}\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial x\partial y}\right)+D_{22}\left(\frac{\partial^2 w}{\partial y^2}\right)^2\right.$$

$$\left.+4D_{26}\left(\frac{\partial^2 w}{\partial y^2}\right)\left(\frac{\partial^2 w}{\partial x\partial y}\right)+4D_{66}\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right]dxdy-\frac{1}{2}\int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}}m(\omega w)^2\,dx\,dy=0\,, \qquad (1)$$

where a is the width of the plate in coordinate x, b is the width of the plate in coordinate y, D_{ij} is the bending stiffness of the laminated plate, w is the lateral deflection, m is the mass per unit area of the plate, and ω is the natural circular frequency.

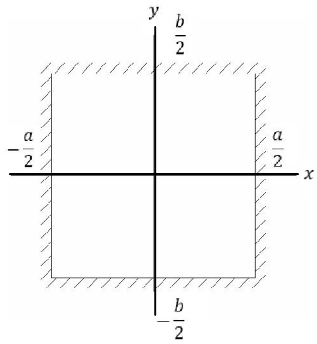

Figure 1. A plate with all edges clamped

By using the extended Kantorovich method, the partial differential equation (1) is reduced to an ordinary differential equation with respect to coordinate x, as equation (2):

$$\frac{d^4X(x)}{dx^4} + A_1\frac{d^2X(x)}{dx^2} + A_2X(x) = 0, \tag{2}$$

where $X(x)$ is the lateral deflection in coordinate x,

$$A_1 = \left(\frac{2S_{2y}D_{12} - 4S_{4y}D_{66}}{S_{3y}D_{11}}\right), \qquad A_2 = \left(\frac{S_{1y}D_{22} - S_{3y}m\omega^2}{S_{3y}D_{11}}\right),$$

$$S_{1y} = \int_{-\frac{b}{2}}^{\frac{b}{2}}\left(\frac{\partial^2 Y}{\partial y^2}\right)^2 dy, \qquad S_{2y} = \int_{-\frac{b}{2}}^{\frac{b}{2}}\left(Y\frac{\partial^2 Y}{\partial y^2}\right)dy,$$

$$S_{3y} = \int_{-\frac{b}{2}}^{\frac{b}{2}} Y^2 dy, \text{ and } \qquad S_{4y} = \int_{-\frac{b}{2}}^{\frac{b}{2}}\left(\frac{\partial Y}{\partial y}\right)^2 dy.$$

The solutions of equation (2) for symmetric and antisymmetric bending-mode vibrations are written as equations (3.1) and (3.2) respectively:

$$X(x) = C_{1x}\cos(p_1x) + C_{2x}\cosh(p_2x), \tag{3.1}$$

$$X(x) = C_{1x}\sin(p_{1x}) + C_{2x}\sinh(p_2x), \tag{3.2}$$

where C_{1x} and C_{2x} are constants.

Substituting equations (3.1) and (3.2) into the boundary conditions $dX(x)/dx = 0$ and $X(x) = 0$ [16] yields a non-trivial solution for symmetric and antisymmetric bending-mode vibrations as equations (4.1) and (4.2) respectively:

$$p_2\tanh\left(\frac{p_2a}{2}\right) + p_1\tan\left(\frac{p_1a}{2}\right) = 0, \tag{4.1}$$

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$$p_2 \coth\left(\frac{p_2 b}{2}\right) - p_1 \cot\left(\frac{p_1 b}{2}\right) = 0, \tag{4.2}$$

where p_1 and p_2 are modal parameters in coordinate x, and

$$p_1^2 - p_2^2 = \frac{2S_{2y}D_{12} - 4S_{4y}D_{66}}{S_{3y}D_{11}}, \tag{5}$$

$$p_1^2 p_2^2 = \frac{S_{3y}m\omega^2 - S_{1y}D_{22}}{S_{3y}D_{11}}. \tag{6}$$

Similarly, ordinary differential equation with respect to coordinate y can be written as equation (7):

$$\frac{d^4Y(y)}{dy^4} + B_1\frac{d^2Y(y)}{dy^2} + B_2Y(y) = 0, \tag{7}$$

where $Y(y)$ is the lateral deflection in the coordinate y,

$$B_1 = \left(\frac{2S_{2x}D_{12} - 4S_{4x}D_{66}}{S_{3x}D_{22}}\right), \qquad B_2 = \left(\frac{S_{1x}D_{11} - S_{3x}m\omega^2}{S_{3x}D_{22}}\right),$$

$$S_{1x} = \int_{-\frac{a}{2}}^{\frac{a}{2}} \left(\frac{\partial^2 X}{\partial x^2}\right)^2 dx, \qquad S_{2x} = \int_{-\frac{a}{2}}^{\frac{a}{2}} \left(X\frac{\partial^2 X}{\partial x^2}\right) dx,$$

$$S_{3x} = \int_{-\frac{a}{2}}^{\frac{a}{2}} X^2 dx, \text{ and} \qquad S_{4x} = \int_{-\frac{a}{2}}^{\frac{a}{2}} \left(\frac{\partial X}{\partial x}\right)^2 dx.$$

The solutions of equation (7) for symmetric and antisymmetric bending-mode vibrations are written as equations (8.1) and (8.2) respectively:

$$Y(y) = C_{1y}\cos(q_1 y) + C_{2y}\cosh(q_2 y), \tag{8.1}$$

$$Y(y) = C_{1y}\sin(q_1 y) + C_{2y}\sinh(q_2 y), \tag{8.2}$$

where C_{1y} and C_{2y} are constants.

Substituting equations (8.1) and (8.2) into the boundary conditions $dY(y)/dy = 0$ and $Y(y) = 0$ [16] yields a non-trivial solution for symmetric and antisymmetric bending-mode vibrations as equations (9.1) and (9.2) respectively:

$$q_2 \tanh\left(\frac{q_2 b}{2}\right) + q_1 \tan\left(\frac{q_1 b}{2}\right) = 0, \tag{9.1}$$

$$q_2 \coth\left(\frac{q_2 b}{2}\right) - q_1 \cot\left(\frac{q_1 b}{2}\right) = 0, \tag{9.2}$$

where q_1 and q_2 are modal parameters in coordinate y, and

$$q_1^2 - q_2^2 = \frac{2S_{2x}D_{12} - 4S_{4x}D_{66}}{S_{3x}D_{22}}, \tag{10}$$

$$q_1^2 q_2^2 = \frac{S_{3x} m\omega^2 - S_{1x} D_{11}}{S_{3x} D_{22}}. \tag{11}$$

Iterative calculation procedure

By using the following steps, the solution equations (3) and (8) are forced to satisfy the boundary conditions, and the natural frequencies are evaluated.

(1) Choose a trial function. If the trial function is a function of x, S_{1x} through S_{4x} are calculated to determine the eigenvalue in equation (9.1) or (9.2).
(2) Calculate the circular natural frequency by substituting the eigenvalue into equation (11).
(3) Substitute the eigenvalue into the solution equations (8.1) and (8.2). This yields the eigenvector.
(4) Calculate S_{1y} through S_{4y} from the eigenvector and determine the eigenvalue in equation (4.1) or (4.2).
(5) Calculate the circular natural frequency by substituting the eigenvalue into equation (6).
(6) Substitute the eigenvalue into the solution equations (3.1) and (3.2). This yields the eigenvector.
(7) Compare the circular natural frequencies obtained from step (2) and step (5). If the difference is acceptable, the final solution is the eigenvector obtained from step (3) and step (6). Otherwise, continue the iterative calculation by repeating steps (1) to (6).

Neural Networks

A neural network is a mathematical model that imitates the human brain [17]. It is used to find the patterns and relationships within a complicated given data set. Neural networks are composed of processing units called neurones. Each neurone performs some simple computations and connections between the neurones with weights to constitute a neural network model. The model is trained by using a given data set as input, typically referred to as the training set. After a successful training, the neural network model can estimate new data from the same or similar sources.

To find the relationship between natural frequencies and aspect ratios of symmetrically laminated rectangular plates with all edges clamped, a back propagation neural network was designed using Weka 3.7.4 software [18]. The network consists of one input layer, one hidden layer and one output layer (Figure 2). The neurones in the input layer correspond to the aspect ratios. The aspect ratio r_i will generate the weight w_i to form the product $w_i r_i$ to the hidden layer as equation (12):

$$s = \sum_{i=0}^{n} w_i r_i, \tag{12}$$

where s is the sum, i is the positive integer number, n is the number of neurones in the input layer, w_i is the weight, and r_i is the aspect ratio of the plate.

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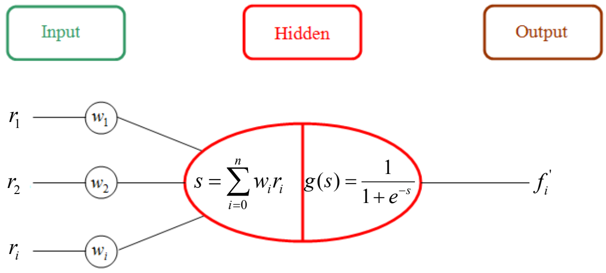
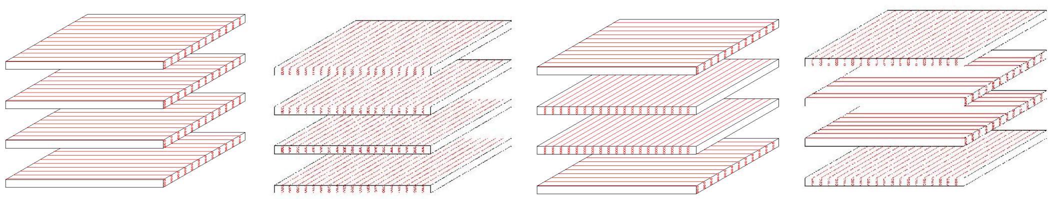


Figure 2. The neural network diagram

The number of neurones in the hidden layer depends on the fibre orientation adjusted for a suitable relationship between the natural frequencies and the aspect ratios. The fibre orientations of symmetrically laminated plates for this study are $[0]_4$, $[0/90]_s$, $[90/0]_s$ and $[90]_4$ (Figure 3). The hidden layer transfers the sum s from equation (12) by means of the sigmoid function to produce the output layer as equation (13):

$$g(s)=\frac{1}{1+e^{-s}}, \tag{13}$$

where $g(s)$ is the sigmoid function.



(a) $[0]_4$ (b) $[90]_4$ (c) $[0/90]_s$ (d) $[90/0]_s$

Figure 3. Fibre orientations of symmetrically laminated rectangular plates

The neurones in the output layer correspond to the natural frequencies. These frequencies were compared to the natural frequencies obtained from the extended Kantorovich method. The root mean square error (RMSE) as equation (14) is the index of comparison:

$$RMSE=\sqrt{\frac{1}{m}\sum_{i=1}^{m}(f_i^{'}-f_i)^2}, \tag{14}$$

where $f_i^{'}$ is the natural frequencies obtained from the output layer, f_i is the natural frequencies obtained from the extended Kantorovich method, and m is the number of data set.

Neural network model procedure

To model the neural network for symmetrically laminated rectangular plates that are composed of $[0]_4$, $[0/90]_s$, $[90/0]_s$ and $[90]_4$ fibre orientations, the procedure can be divided into six steps:

(1) Prepare the natural frequencies of $[0]_4$ laminated rectangular plates that were calculated using the extended Kantorovich method from the aspect ratios 1:1 to 1:5 by increments of 0.1, for a total of 41 data sets, to train the neural network.

(2) Assign the aspect ratios as the input data sets. The natural frequencies are the output data sets, and the sigmoid function is the transfer function from the hidden layer to the output layer.
(3) Specify parameters in Weka 3.7.4 software as follows:
   Hidden layer = 1 – 10
   Learning rate = 0.1 – 0.5
   Momentum = 0.5 – 1.0
   Epoch = 1,000 – 20,000
(4) Model the neural network in Weka 3.7.4 software by adjusting the parameters in step (3); a suitable neural network must have the smallest RMSE.
(5) Take random data sets of the natural frequencies that are not the training data sets from the aspect ratios 1:1 to 1:5, for a total 12 data sets, to test the neural network.
(6) Repeat steps (1)-(5) by changing the training and testing data sets from the natural frequencies and aspect ratios of $[0]_4$ to those of $[0/90]_s$, $[90/0]_s$ and $[90]_4$.

RESULTS AND DISCUSSION

By adjusting parameters in the Weka 3.7.4 software to model the neural network, it was found that increasing the number of neurones in the hidden layer and the number of epochs for the training process decreased the RMSE. Subsequently, in the training process the suitable neural networks for the $[0]_4$, $[0/90]_s$, $[90/0]_s$ and $[90]_4$ rectangular plates with all edges clamped were 1:3:1, 1:3:1, 1:5:1 and 1:3:1 respectively as shown in Table 1. The output neurones corresponded to the natural frequencies obtained from the neural network model which had the smallest RMSE as shown in Table 2.

Table 1. Parameters of neural networks

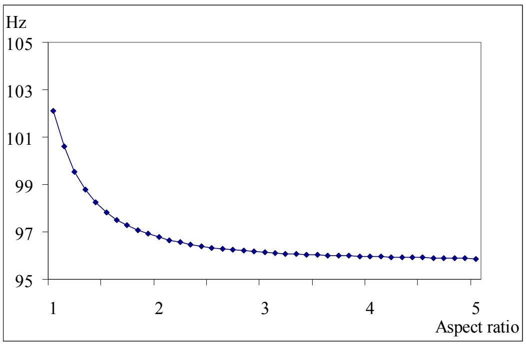
Parameter	Laminated plate			
	$[0]_4$	$[0/90]_s$	$[90/0]_s$	$[90]_4$
Input layer	1	1	1	1
Hidden layer	3	3	5	3
Output layer	1	1	1	1
Learning rate	0.27	0.22	0.23	0.19
Momentum	0.60	0.70	0.60	0.80
Epoch	20,000	20,000	20,000	15,000

Table 2. RMSE of neural network model

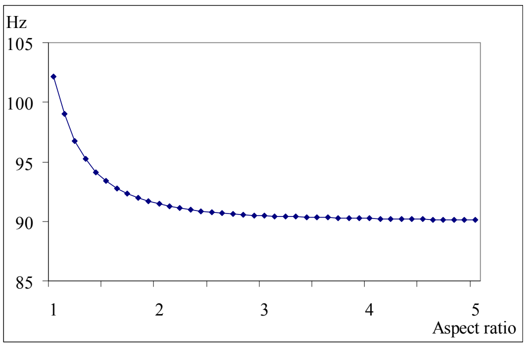
Laminated plate	Neural network model	RMSE	
		Training	Testing
$[0]_4$	1:3:1	0.0096	0.0064
$[0/90]_s$	1:3:1	0.0177	0.0114
$[90/0]_s$	1:5:1	0.0743	0.0626
$[90]_4$	1:3:1	0.0895	0.0757

The neural network model from Table 2 was used to evaluate the natural frequencies with various aspect ratios of the $[0]_4$, $[0/90]_s$, $[90/0]_s$ and $[90]_4$ rectangular plates with all edges clamped. It can be observed that increasing the aspect ratios decreases the natural frequencies of laminated rectangular plates, which eventually stabilise (Figure 4). Furthermore, by comparing the natural frequencies obtained from the neural network model and the extended Kantorovich method, it was found that the correlation coefficient is 1.000 as shown in Figure 5, which indicates that the natural frequencies obtained from the neural networks are in good agreement with those obtained from the extended Kantorovich method.

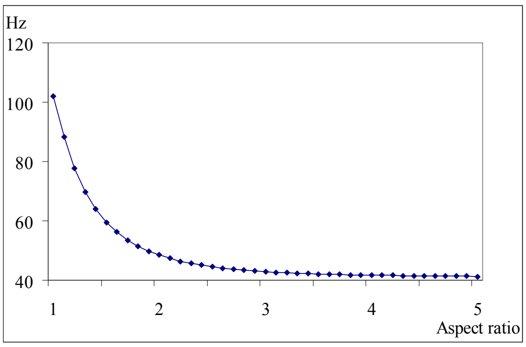
To verify the accuracy, the neural network model was tested by new data of natural frequencies and aspect ratios which had not been employed to establish a neural network model. The comparison of natural frequencies obtained from the neural networks and the extended Kantorovich method results in a correlation coefficient of 1.000 (Figure 6), which implies that the neural networks can accurately estimate the natural frequencies of symmetrically laminated rectangular plates with all edges clamped.



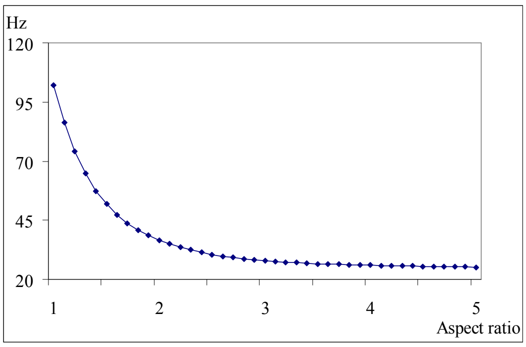
(a) $[0]_4$



(b) $[0/90]_s$



(c) $[90/0]_s$



(d) $[90]_4$

Figure 4. Natural frequencies of laminated rectangular plates obtained from neural network model

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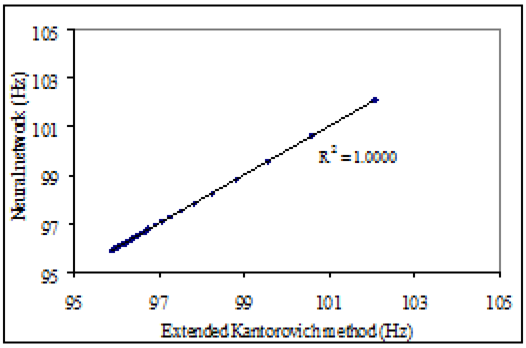
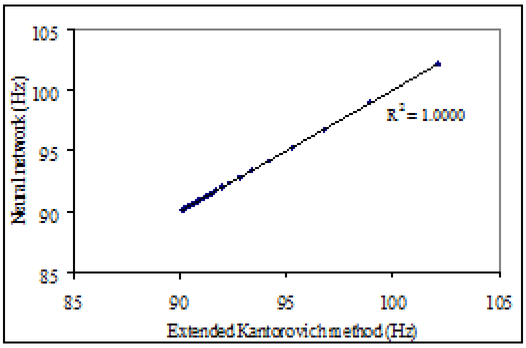
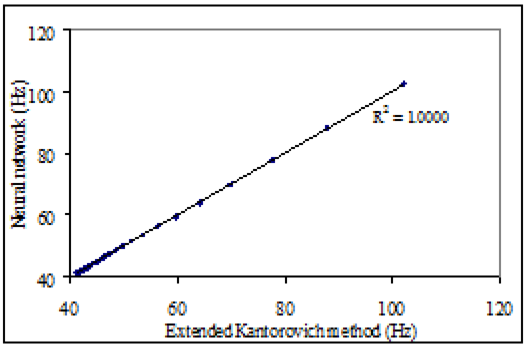


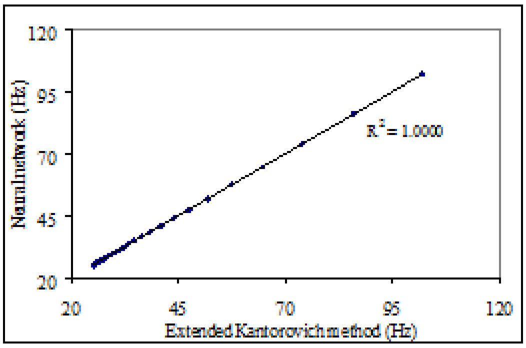
(a) $[0]_4$



(b) $[0/90]_s$

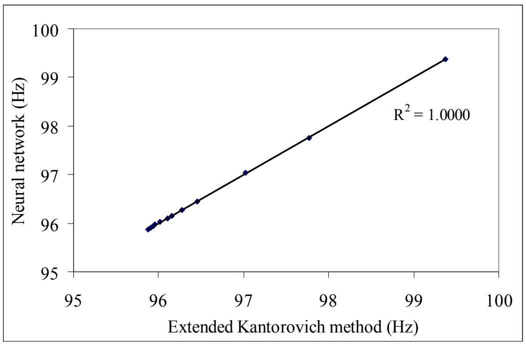


(c) $[90/0]_s$

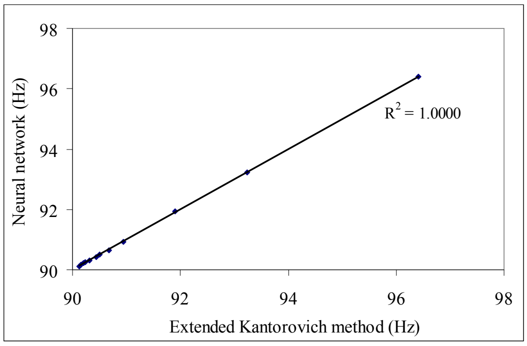


(b) $[90]_4$

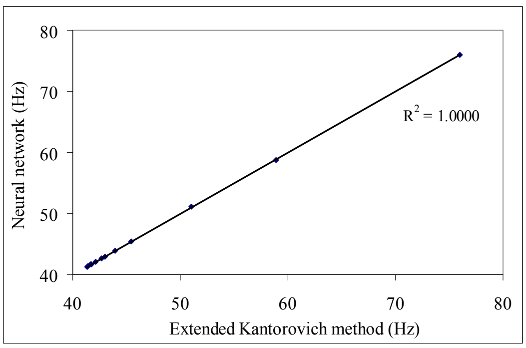
Figure 5. Comparison of natural frequencies of laminated rectangular plates from neural network model and extended Kantorovich method for training data set



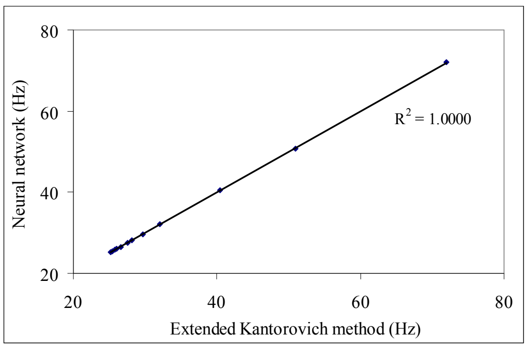
(a) $[0]_4$



(b) $[0/90]_s$



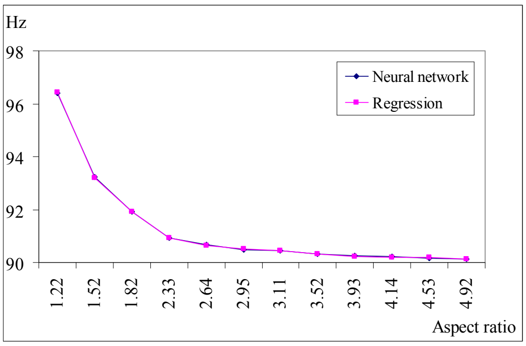
(c) $[90/0]_s$



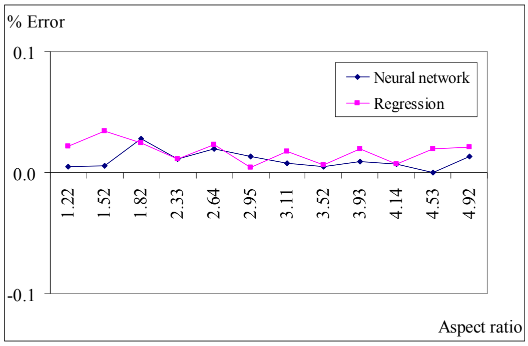
(d) $[90]_4$

Figure 6. Verification the natural frequencies of laminated rectangular plates between the neural network model and the extended Kantorovich method for test data set

In order to illustrate the worthiness of the neural network model, the natural frequencies and percentage errors of the $[90/0]_s$ rectangular plates derived from the model are compared with those derived from the regression analysis [13] as shown in Figure 7. It is clear that the results on the natural frequencies by the neural network model show excellent agreement with those by the regression analysis. The errors of the neural network model are even slightly lower than those of the regression analysis (Figure 7b).



(a)



(b)

Figure 7. Comparison of simulation results on natural frequencies (a) and percentage errors (b) of $[90/0]_s$ rectangular plates with all edges clamped obtained from neural network model vs regression analysis

CONCLUSIONS

The neural network can estimate the natural frequencies of symmetrically laminated rectangular plates with all edges clamped within an acceptable accuracy range. Using the neural network, the relationship between natural frequencies and aspect ratios can be established without solving the partial differential equations. Only the aspect ratios as the input data sets and the natural frequencies as the output data sets are required for adequately training the neural network to have the smallest RMSE in order to establish the relationship between the natural frequencies and the aspect ratios by the training process. Subsequently in the training process the neural network can rapidly estimate the natural frequencies from the same aspect ratio source. Thus, in a practical laminated rectangular plate design in which the aspect ratios vary, the neural network is simpler to use than approximate methods such as Galerkin method, Rayleigh-Ritz method and extended Kantorovich method.

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