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Full Paper

Optimal approach to enhancing population mean estimation using auxiliary parameters

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Abstract: This study presents an efficient method for estimating the population mean of a study variable using a Searls exponential ratio estimator. The approach is based on simple random sampling and leverages known population parameters of an auxiliary variable for enhanced estimation of population mean. Using large sample approximations up to the first order, the expressions for the bias and mean squared error of the proposed estimator are derived. To evaluate its performance, the estimator is compared conceptually and mathematically with several existing estimators. Theoretical results are further validated through both simulated and real-world data sets. Findings from these evaluations demonstrate that the proposed estimator consistently outperforms many competing estimators, particularly at higher population mean values. Therefore, it holds promise for improving population mean estimation across a range of practical applications.

Keywords: auxiliary variable, population mean estimation, Searls estimator, mean squared error

INTRODUCTION

Sampling becomes essential when dealing with large populations to save time, cost and resources including manpower. In such scenarios direct computation of population parameters is often impractical, making estimation a more feasible alternative. The sample mean is typically used as an unbiased estimator of the population mean of the study variable. However, despite its unbiased nature, the sample mean often exhibits substantial variability. Hence improving estimation efficiency particularly by minimising the mean squared error (MSE) becomes a key objective. To this end, auxiliary information from a secondary variable that is strongly correlated (positively or negatively) with the study variable can be highly valuable. When such a correlation is strong and positive, ratio estimators are commonly employed to enhance the precision of the population mean estimates.

Over the years, many researchers have proposed refinements and extensions of ratio-type estimators using auxiliary variables. For instance, Chatterjee et al. [1] introduced a regression estimator for the population mean based on a standard regression model. Noor-ul-Amin et al. [2] applied a two-parameter ratio estimator to exponentially weighted moving average control charts. Jerajuddin and Kishun [3] suggested a classic ratio estimator for highly positively correlated variables while Zaman and Bulut [4] proposed a modified ratio estimator via regression methods. Kadilar and Cingi [5] also developed efficient ratio estimators using known auxiliary information. Several other advancements have followed. Zaman [6] and Pal et al. [7] explored efficient estimator families using known auxiliary parameters. Singh and Solanki [8] enhanced estimation using auxiliary attribute information while Lui [9] introduced composite ratio estimators. Irfan et al. [10] introduced ratio-type estimators for estimating the finite population mean. Subramani [11] used known medians of auxiliary variables for efficient estimation of $\bar{Y}$. Javed et al. [12] presented the estimators combining traditional and non-traditional auxiliary variables. Sinha and Bharti [13] presented a ratio estimator of $\bar{Y}$. Ahmad et al. [14] incorporated dual auxiliary information. Singh and Chaudhary [16] introduced a predictive ratio estimator, which Subzar et al. [17] expanded under ranked set sampling. Subzar et al. [18] introduced an effective method for computing $\bar{Y}$ using auxiliary attibutes.

Haq et al. [19] developed a ratio-cum-exponential estimator of the population cumulative distribution function. Ahmad et al. [20] and Singh et al. [21] introduced improved Searls-type predictive estimators, and Singh and Vishwakaram [22] proposed a biased estimator with reduced MSE using known parameters. Singh et al. [23] evaluated estimator performance using the sine inverse Rayleigh model. Khalid et al. [24] proposed generalised exponential estimators using linear transformations. Bhushan and Kumar [25] suggested an enhanced regression-type estimator using two known auxiliary attributes. Yadav [26] used the coefficient of skewness for improved estimation. John and Inyang [27] proposed two exponential ratio estimators for estimating the population mean under simple random sampling. Searls [28] laid foundational work with exponential-type estimators using auxiliary attributes and demonstrated that an estimator constructed as a constant multiple of the corresponding parameter estimator achieves greater efficiency than the original form. Grover and Kaur [29–30] contributed extended ratio-type exponential estimators and robust regression-based enhancements. Yadav et al. [31] addressed missing data via optimal imputation strategies while Singh et al. [32] and Yadav et al. [33] introduced advanced families of exponential-type estimators. There were some very recent good articles on efficient estimation of population mean using auxiliary parameters [34-39].

The estimation of the finite population mean has long been a central focus in survey sampling, leading to the development of numerous ratio, regression and product-type estimators that leverage auxiliary information [40-42]. More recently, research has advanced into highly specific and sophisticated techniques including the integration of novel frameworks such as the neutrosophic approach to enhancing estimation [43, 44], the use of dual or multiple auxiliary variables and attributes, often incorporating known population parameters such as the median or quartiles to boost efficiency [45-48]. While these efforts have led to significant improvements in precision [48, 49], there remains a critical need for efficient and robust estimators that maintain simplicity while maximising the benefit from basic auxiliary knowledge. Specifically, the potential of the Searls exponential ratio estimator, particularly when tailored to utilise the known population mean of a single auxiliary variable under simple random sampling, has not been fully explored or optimised in the context of recent advancements [50].

This study aims to fill this gap by proposing a novel and efficient modified Searls exponential ratio estimator. We derive its large sample properties and rigorously compare its performance against several established and contemporary estimators using extensive empirical and simulation analysis. This development highlights the continuous pursuit in sampling theory for more efficient estimators with sampling distributions closely aligned with the true population parameters. Motivated by this goal, the current study introduces an exponential ratio estimator for the population mean when auxiliary information is available. The bias and MSE of the proposed estimator are derived using a first-order approximation.

REVIEW OF EXISTING ESTIMATORS

The real-valued functions y and x are defined on the finite population $U = (U_1, U_2, \ldots, U_N)$. A random sample size n is selected from the population U. To estimate the population mean $\overline{Y}$ of the study variable Y, the classical ratio estimator is commonly used, especially when the population mean $\overline{X}$ of an auxiliary variable is known. This estimator takes advantage of the positive correlation between Y and X to improve estimation efficiency. Yadav [26] contributed to enhancing the precision of such estimators by incorporating known auxiliary parameters such as the coefficients of variation, skewness, kurtosis and the correlation coefficient between Y and X. Expanding on this approach, Dansawad and Lurdjariyaporn [27] proposed a generalised family of estimators that combines multiple forms of auxiliary information. The generalised family of estimators is given by

$$\mu = \bar{y}\left(\frac{\overline{X}+\delta_i}{\bar{x}+\delta_i}\right)^g \exp\left(\frac{\overline{X}-\bar{x}}{\overline{X}+\bar{x}}\right), \quad i = 1,2,\ldots,6 \tag{1}$$

where δ_i is either a real constant or a function of the known X and g takes the value zero or one. For $g = 0$ the suggested class reduces to exponential ratio estimator and for $g = 1$ it becomes ratio-cum-exponential ratio estimator. Real-world data were employed to compare the performance of various estimators within the proposed family and to identify the optimal estimator under the given distributional setting. Table 1 represents the members of Dansawad and Lurdjariyaporn's family of estimators μ for $g = 1$ [27].

The MSE of μ is given by Dansawad and Lurdjariyaporn [27] as

$$\text{MSE}(\mu) = \begin{cases} \lambda\overline{Y}^2 C_y^2, & i = 0, \\ \lambda\overline{Y}^2\left[C_y^2 + \frac{3}{4}\left(3C_x^2 - 4C_{yx}\right)\right], & i = 1, \\ \lambda\overline{Y}^2\left[C_y^2 + \frac{(1+2\varphi_i)}{4}\left((1+2\varphi_i)C_x^2 - 4C_{yx}\right)\right], & i = 2,3,\ldots,6 \end{cases} \tag{2}$$

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Table 1. Members of μ family of estimators

No.	Estimator	δ_i
1	$\mu_1=\bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$	0
2	$\mu_2=\bar{y}\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$	C_x
3	$\mu_3=\bar{y}\left(\frac{\bar{X}+\beta_1}{\bar{x}+\beta_1}\right)\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$	β_1
4	$\mu_4=\bar{y}\left(\frac{\bar{X}+\rho}{\bar{x}+\rho}\right)\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$	P
5	$\mu_5=\bar{y}\left(\frac{\bar{X}+S_x}{\bar{x}+S_x}\right)\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$	S_x
6	$\mu_6=\bar{y}\left(\frac{\bar{X}+\beta_2}{\bar{x}+\beta_2}\right)\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$	β_2

PROPOSED CLASS OF ESTIMATOR

Motivated by the above-mentioned studies, we introduce a new family of Searls-type exponential estimators, combining the approach of Dansawad and Lurdjariyaporn [27] with Searls' methodology to achieve improved estimation performance. The set of estimators under simple random sampling for $g=1$ is given as

$$t=k\,\bar{y}\left(\frac{\bar{X}+\delta_i}{\bar{x}+\delta_i}\right)^g\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right),\qquad i=1,2,\ldots,6 \tag{3}$$

where k is taken as a constant, which is to be obtained such that the estimator t has the lowest MSE. Table 2 lists some of the members of the introduced family of estimators.

Table 2. Estimators from proposed t family of estimators

No.	Estimator	δ_i
1	$t_1=k\bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$	0
2	$t_2=\bar{y}\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$	C_x
3	$t_3=\bar{y}\left(\frac{\bar{X}+\beta_1}{\bar{x}+\beta_1}\right)\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$	β_1
4	$t_4=\bar{y}\left(\frac{\bar{X}+\rho}{\bar{x}+\rho}\right)\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$	ρ
5	$t_5=\bar{y}\left(\frac{\bar{X}+S_x}{\bar{x}+S_x}\right)\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$	S_x
6	$t_6=\bar{y}\left(\frac{\bar{X}+\beta_2}{\bar{x}+\beta_2}\right)\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$	β_2

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To evaluate the bias and the MSE of the proposed estimator t, we use the following first-order approximation:

$$\bar{x}=\bar{X}(1+\xi_1) \text{ and } \bar{y}=\bar{Y}(1+\xi_0) \tag{4}$$

such that $E(\xi_0)=E(\xi_1)=0$ and $E(\xi_0^2)=\lambda C_y^2$, $E(\xi_1^2)=\lambda C_x^2$, $E(\xi_0\xi_1)=\lambda C_{yx}=\rho C_y C_x$, where $\lambda=\frac{1}{n}-\frac{1}{N}$.

The bias and MSE of the proposed estimator t are derived using a first-order Taylor series expansion. On expressing t_i in terms of errors ξ_i $(i=0,1)$, then on multiplying its terms and using first-order approximating, we get

$$t=k\bar{Y}(1+\xi_0)(1+\varphi_i\xi_1)^{-g}\exp\left(\frac{-\xi_1}{2+\xi_1}\right), \text{ where, } \varphi_i=\frac{\bar{X}}{\bar{X}+\delta_i}$$

$$t=k\bar{Y}(1+\xi_0)(1+\varphi_i\xi_1)^{-g}\exp\left(\left(\frac{-\xi_1}{2}\right)\left(1+\frac{\xi_1}{2}\right)^{-1}\right),$$

$$t=k\bar{Y}(1+\xi_0)\left(1-g\varphi_i\xi_1+\frac{g(g+1)}{2}\varphi_i^2\xi_1^2\right)\left(1-\frac{\xi_1}{2}+\frac{3\xi_1^2}{8}\right),$$

$$t=k\bar{Y}(1+\xi_0)\left(1-\frac{\xi_1}{2}+\frac{3\xi_1^2}{8}-g\varphi_i\xi_1+\frac{g\varphi_i\xi_1^2}{2}+\frac{g(g+1)}{2}\varphi_i^2\xi_i^2\right),$$

$$t=k\bar{Y}\left(1+\xi_0-\frac{\xi_1}{2}+\frac{3\xi_1^2}{8}-g\varphi_i\xi_1+\frac{g\varphi_i\xi_1^2}{2}+\frac{g(g+1)}{2}\varphi_i^2\xi_1^2+\xi_0-\frac{\xi_0\xi_1}{2}-g\varphi_i\xi_0\xi_1\right) \tag{5}$$

Subtracting the population mean $\bar{Y}$ from both sides of equation (5), we get

$$t-\bar{Y}=k\bar{Y}\left(1-\frac{\xi_1}{2}+\frac{3\xi_1^2}{8}-g\varphi_i\xi_1+\frac{g\varphi_i\xi_1^2}{2}+\frac{g(g+1)}{2}\varphi_i^2\xi_1^2+\xi_0-\frac{\xi_0\xi_1}{2}-g\varphi_i\xi_0\xi_1\right)-\bar{Y}. \tag{6}$$

By squaring both sides of equation (6) and retaining terms up to the first order of approximation, we obtain

$$(t-\bar{Y})^2=k^2\bar{Y}^2\left(1-\frac{\xi_1}{2}+\frac{3\xi_1^2}{8}-g\varphi_i\xi_1+\frac{g\varphi_i\xi_1^2}{2}+\frac{g(g+1)}{2}\varphi_i^2\xi_1^2+\xi_0-\frac{\xi_0\xi_1}{2}-g\varphi_i\xi_0\xi_1\right)^2$$
$$+\bar{Y}^2-2k\bar{Y}\left(1-\frac{\xi_1}{2}+\frac{3\xi_1^2}{8}-g\varphi_i\xi_1+\frac{g\varphi_i\xi_1^2}{2}+\frac{g(g+1)}{2}\varphi_i^2\xi_1^2+\xi_0-\frac{\xi_0\xi_1}{2}-g\varphi_i\xi_0\xi_1\right) \tag{7}$$

$$(t-\bar{Y})^2=k^2\bar{Y}^2\begin{pmatrix}1+\xi_0^2+\frac{\xi_1^2}{4}+g^2\varphi_i^2\xi_1^2-\xi_1+\frac{3}{4}\xi_1^2-2g\varphi_i\xi_1+g\varphi_i\xi_1^2+g(g+1)\varphi_i^2\xi_1^2\\+2\xi_0-\xi_0\xi_1-2g\varphi_i\xi_0\xi_1+g\varphi_i\xi_1^2-\xi_0\xi_1-2g\varphi_i\xi_0\xi_1\end{pmatrix}$$
$$+\bar{Y}^2-2k\bar{Y}\left(1-\frac{\xi_1}{2}+\frac{3\xi_1^2}{8}-g\varphi_i\xi_1+\frac{g\varphi_i\xi_1^2}{2}+\frac{g(g+1)}{2}\varphi_i^2\xi_1^2+\xi_0-\frac{\xi_0\xi_1}{2}-g\varphi_i\xi_0\xi_1\right) \tag{8}$$

The MSE (t) is obtained by taking the expectation of both sides of equation (8), yielding

$$MSE(t)=k^2\bar{Y}^2\left(1+\lambda C_x^2+g^2\varphi_i^2\lambda C_x^2+\lambda C_y^2+g\varphi_i\lambda C_x^2+g(g+1)\varphi_i^2\lambda C_x^2-\rho\lambda C_y C_x-\right.$$
$$\left.2g\varphi_i\lambda\rho C_y C_x+g\varphi_i\lambda C_x^2-\lambda\rho C_y C_x-2g\varphi_i\lambda\rho C_y C_x\right)+\bar{Y}^2-2k\bar{Y}^2\left(1+\frac{3}{8}\lambda C_x^2+\frac{g\varphi_i\lambda C_x^2}{2}+\frac{g(g+1)}{2}\varphi_i^2\lambda C_x^2-\right.$$
$$\left.\frac{\rho\lambda C_y C_x}{2}-g\varphi_i\lambda\rho C_y C_x\right)$$

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$$\mathrm{MSE(t)} = \overline{Y}^2\left[(k-1)^2+k^2\lambda C_y^2+\lambda C_x^2\left\{k^2\left(1+g^2\varphi_i^2+g(g+1)\varphi_i^2+2g\varphi_i\right)-2k\left(\frac{3}{8}+\frac{g\varphi_i}{2}+\frac{g(g+1)}{2}\varphi_i^2\right)\right\} - 2\lambda\rho C_y C_x\left\{k^2\left(1+2g\varphi_i\right)-\frac{k}{2}\left(1+2g\varphi_i\right)\right\}\right]$$

$$\mathrm{MSE(t)} = \overline{Y}^2\left[1 + k^2\left\{1+\lambda\left[C_y^2 + C_x^2\left(1+g^2\varphi_i^2+g(g+1)\varphi_i^2+2g\varphi_i\right) - 2\lambda\rho C_y C_x\left(1+2g\varphi_i\right)\right]\right.\right.$$
$$\left.\left. - 2k\left[1+\lambda C_x^2\left(\frac{3}{8}+\frac{g\varphi_i}{2}+\frac{g(g+1)}{2}\varphi_i^2\right) - \frac{1}{2}\lambda\rho C_y C_x\left(1+2g\varphi_i\right)\right]\right\}\right]$$

$$\mathrm{MSE(t)} = \overline{Y}^2[1 + k^2P - 2kQ] \tag{9}$$

Taking the partial derivative of equation (9) with respect to k and setting it to zero to minimise the MSE of t, we obtain the optimum value of k as

$$k=\frac{P}{Q}, \tag{10}$$

where $P = 1 + \lambda C_x^2\left(\frac{3}{8}+\frac{g\varphi_i}{2}+\frac{g(g+1)}{2}\varphi_i^2\right) - \frac{1}{2}\lambda\rho C_y C_x(1+2g\varphi_i)$

$Q = 1 + \lambda\left[C_y^2 + C_x^2(1+g^2\varphi_i^2+g(g+1)\varphi_i^2+2g\varphi_i) - 2\rho C_y C_x(1+2g\varphi_i)\right].$

By substituting the optimum value of k in equation (9), the minimum MSE of the proposed estimator is given by

$$\mathrm{MSE_{min}}(t)= \overline{Y}^2\left(1-\frac{P^2}{Q}\right). \tag{11}$$

Furthermore, equation (11) can be rewritten as

$$\mathrm{MSE_{min}}(t)=\begin{cases}\overline{Y}^2\left(1-\frac{1}{(1+\lambda C_y^2)}\right), & i=0,\\ \overline{Y}^2\left(k^2\lambda C_y^2+3\lambda C_x^2(2k^2-\frac{5}{4}k)-3\lambda C_{yx}(2k^2-k)+(k-1)^2\right), & i=1,\\ \overline{Y}^2[k^2\lambda C_y^2+\lambda C_x^2\{k^2\left(3\phi_i^2+2\varphi_i+1\right)-k\left(8\phi_i^2+4\varphi_i+3\right)\} & i=2,3,4,5,6\\ -2\lambda C_{yx}\left\{k^2\left(1+2\varphi_i\right)-\frac{k}{2}\left(1+2\varphi_i\right)\right\}+(k-1)^2]. & \end{cases} \tag{12}$$

Equation (12) can be used to express the MSE of the ratio estimators presented in Table 2. The optimal values of k for which the MSE of t are minimised are given by

$$k_i=\begin{cases}\frac{1}{1+\lambda C_y^2}, & i=0,\\ \frac{1+\frac{15}{8}\lambda C_x^2-\frac{3}{2}\lambda C_{yx}}{1+\lambda C_y^2+6\lambda\{C_x^2-\rho C_y C_x\}}, & i=1,\\ \frac{1+\frac{\lambda C_x^2}{8}\{8\varphi_i^2+4\varphi_i+3\}-\frac{\lambda C_{yx}}{2}\{1+2\varphi_i\}}{1+\lambda C_y^2+\lambda C_x^2\{3\varphi_i^2+2\varphi_i+1\}-2\lambda\rho C_y C_x\{1+2\varphi_i\}}. & i=2,3,4,5,6\end{cases} \tag{13}$$

By substituting the optimal value of k from equation (13) into equation (12), the minimum MSE of t is obtained as

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$$\mathrm{MSE}(t)=\begin{cases}\overline{Y}^2\left(1-\frac{1}{(1+\lambda C_y^2)}\right), & i=0,\\ \overline{Y}^2\left(1-\frac{A^2}{B}\right), & i=1,\\ \overline{Y}^2\left(1-\frac{C^2}{D}\right), & i=2,3,4,5,6.\end{cases} \tag{14}$$

where

$$A=1+\frac{15}{8}\lambda C_x^2-\frac{3}{2}\lambda\rho C_y C_x,$$
$$B=1+\lambda C_y^2+6\lambda\{C_x^2-\rho C_y C_x\},$$
$$C=1+\frac{\lambda C_x^2}{8}\{8\varphi_i^2+4\varphi_i+3\}-\frac{\lambda\rho C_y C_x}{2}\{1+2\varphi_i\},$$
$$D=1+\lambda C_y^2+\lambda C_x^2\{3\varphi_i^2+2\varphi_i+1\}+2\lambda\rho C_y C_x\{1+2\varphi_i\}.$$

The proposed class of estimators t is compared with existing estimators μ of $\bar{Y}$. The criteria demonstrating its superiority over these estimators are given in detail below. Specifically, we compare the minimum MSE of the proposed estimator with the MSEs of various competing estimators under simple random sampling. The comparison is summarised as follows:

(i) $\mathrm{MSE}_{\min}(t)< MSE\,(\mu_1)$ if,

$$\overline{Y}^2\left(1-\frac{A^2}{B}\right)-\lambda\overline{Y}^2\left[C_y^2+\frac{3}{4}\left(3C_x^2-4C_{yx}\right)\right]<0. \tag{15}$$

(ii) $\mathrm{MSE}_{\min}(t)< MSE\,(\mu_i)$ if,

$$\overline{Y}^2\left(1-\frac{C^2}{D}\right)-\lambda\overline{Y}^2\left[C_y^2+\frac{(1+2\varphi_i)}{4}\left((1+2\varphi_i)C_x^2-4C_{yx}\right)\right]<0. \tag{16}$$

(iii) $\mathrm{MSE}_{\min}(t)< MSE\,(t_i)$ if $\overline{Y}^2\left(1-\frac{C^2}{D}\right)-\overline{Y}^2\left(1-\frac{A^2}{B}\right)<0. \quad (17)$

PRACTICAL STUDY

To empirically validate the theoretical results under simple random sampling, two real populations were considered. The first data set from Searls [28] includes X as the number of cities in 1920 and Y as the number of cities in 1930. The second data set from Singh and Horn [51] has X representing the number of bearing lime trees and Y denoting the area under lime cultivation (in acres). Table 3 presents the parameter values for both populations.

Table 3. Parametric information of populations

No.	Parameter	First population	Second population
1	N	49	22
2	n	20	5
3	$\overline{Y}$	103.1429	22.6209
4	$\overline{X}$	127.7959	1467.5455
5	S_x	123.1212	2562.1449
6	S_y	104.4051	33.0469
7	ρ_{yx}	0.981742	0.9022

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To evaluate the effectiveness of the proposed family of estimators, the % relative efficiencies (PREs) of competing estimators are compared for both populations. The formula used to calculate the PRE for each estimator is given by

$$\mathrm{PRE}(., \mu_0) = \frac{\mathrm{MSE}(\mu_0)}{\mathrm{MSE}(.)} \times 100. \tag{18}$$

Table 4 presents the MSEs and PREs of various estimators for both real populations, calculated using the formula above. The results indicate that the proposed class of estimators t attains the lowest MSE in both cases (11.71418 for first population and 30.97018 for second population), thus outperforming all competing estimators and demonstrating superior efficiency.

Table 4. MSEs and PREs of various estimators for both populations with respect to $\mu_0 = \bar{y}$

Estimator	First population		Second population	
	MSE	PRE	MSE	PRE
μ_0	322.5634	100.0000	168.7795	100.0000
μ_1	75.81063	425.4856	165.2394	102.1424
μ_2	73.77833	437.2061	164.8129	102.4067
μ_3	65.03474	495.9863	163.1403	103.4567
μ_4	73.74029	437.4316	165.0188	102.2789
μ_5	11.81487	2730.147	34.29408	492.1533
μ_6	60.09701	536.7378	162.3229	103.9776
t_0	313.0709	103.0321	126.9174	132.9837
t_1	73.11102	441.1966	50.73327	332.6801
t_2	71.23871	452.7922	50.76488	332.4729
t_3	63.13499	510.9106	50.8855	331.6848
t_4	71.20363	453.0153	50.74966	332.5726
t_5	11.71418	2753.614	30.97018	544.9742
t_6	58.52301	551.1736	50.94249	331.3137

Figure 1 compares the PREs of the proposed t family of estimators with those of the mentioned competing estimators for both populations. The introduced estimators consistently exhibit lowest MSEs and highest PREs, indicating a better fit and improved efficiency over the mentioned existing estimators.

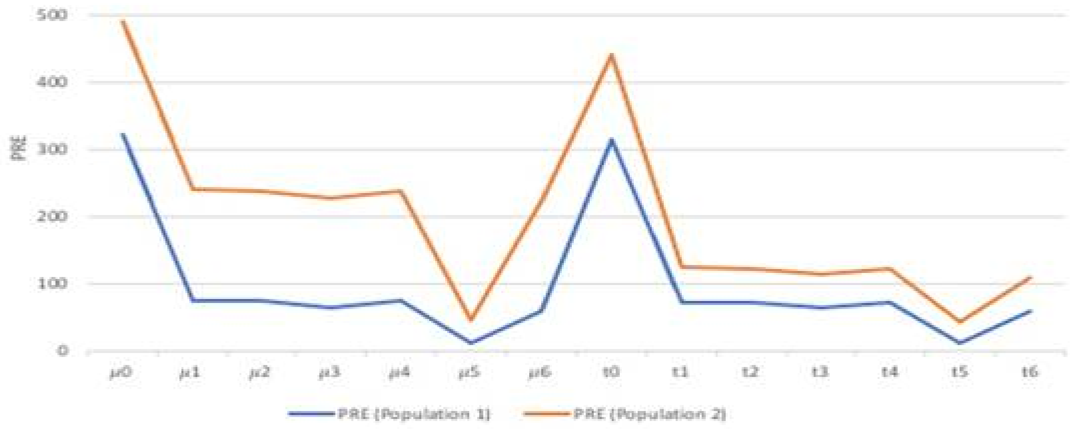

Figure 1. PREs of different estimators with respect to μ_0 for both populations

CONCLUSIONS

The findings of the study indicate that although the ratio estimator is biased, it consistently outperforms the traditional sample mean estimator. Moreover, the real data analyses demonstrate that the proposed estimator achieves lower MSE and higher PRE, confirming its superiority over previously suggested estimators. Consequently, this family of estimators offers a valuable tool for enhancing population mean estimation across various applications by incorporating known auxiliary information in terms of auxiliary parameters. Thus, the proposed estimator may be recommended for improved estimation of $\bar{Y}$ in different areas of applications.

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