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Full Paper

Soft Wijsman convergence

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Abstract: This study presents results of the Wijsman convergence of sequences of sets, a topic with numerous applications, within the framework of soft set theory. We introduce the concept of Wijsman convergence for sequences of soft closed sets in soft metric spaces and provide several foundational results. Additionally, we define the notions of soft Wijsman boundedness and soft Wijsman Cauchy sequences and explore their interrelationships and connections to convergence. In parallel to Hausdorff convergence, we first define the concept of soft Hausdorff distance and then introduce soft Hausdorff convergence with relevant results. Finally, we make a comparison between soft Wijsman convergence and soft Hausdorff convergence.

Keywords: soft set, soft Wijsman convergence, soft Hausdorff convergence, soft Wijsman bounded

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INTRODUCTION

One of the tools for constructing or understanding a mathematical structure is the concept of convergence. A review of the literature reveals a variety of different definitions of convergence. These distinct concepts of convergence can be compared with one another to provide alternative approaches to various problems. Many applied areas of mathematics, such as optimisation and variational problems, require the analysis of sets or approximations involving sets. This approach facilitates a better understanding of other concepts and different methodologies. The first work on the convergence of sets was conducted by Painlevé in 1902 in his lectures on analysis [1, 2], and it was subsequently popularised and developed by Hausdorff [3] and Kuratowski [4]. Given that any set can be viewed as an element of the defined mathematical structure, the convergence of sets has been addressed from a topological perspective over time [1].

One notable type of convergence for sets is Wijsman convergence, which is characterised by the distance from a point to a set as determined by a metric function. Let (X, d) be a metric space, and let $\{A_k\}$ be a sequence of closed sets, with A being a non-empty closed set in X. The sequence $\{A_k\}$ is said to converge to A in the Wijsman sense if, for every $x \in X$, the sequence $(d(x, A_k))$ is $d-$convergent to $d(x, A)$. This notion was originally introduced by Wijsman and has been applied to sequences of convex subsets in Euclidean spaces [5, 6].

In metric spaces, Hausdorff convergence [3] is defined in terms of uniform convergence of the distance function, whereas Wijsman convergence is characterised by pointwise convergence of the distance function. Subsequently, various generalisations of this type of convergence, its application to different mathematical structures, and comparisons with other types of convergence have been explored [1, 2, 7, 8].

Other fundamental concept in our study is the notion of a soft set, which is a generalisation of classical set. A soft set over X is denoted as F^E, where $F: E \to P(X)$ is a mapping. The collection of all soft sets over X is represented by $SS(X)$ [9]. In this definition the set X is termed the universe set while the set E is referred to as the parameter set.

Uncertainty is a prevalent issue in fields such as decision-making, economics, engineering, medicine, social sciences and environmental science, which renders classical set theory insufficient. While various theories exist in the literature to address uncertainty, each has its own limitations. One promising approach is soft set theory, initially introduced by Molodtsov [9]. Soft sets provide a parametric method for dealing with uncertainty and have since been applied to a wide range of mathematical structures.

The fundamental operations and properties associated with soft sets were first established by Maji et al. [10] and later enhanced by Ali et al. [11]. Since then the theory of soft sets has rapidly evolved, with contributions from various mathematicians. Researchers have investigated several algebraic structures derived from soft sets, including soft groups, soft semirings and soft rings. Furthermore, the concept of soft topology, along with several related topological ideas, was introduced independently by Çağman et al. [12] and Shabir and Naz [13].

Soft points represent a specific category of soft sets. However, their definition has appeared in various forms in the literature, sometimes referred to as soft elements [14-18]. For a comparative overview of these definitions, we recommend consulting Alcantud et al. [19]. The convergence of sequences of soft points within the framework of soft topology was addressed by Demir and Özbakır [20].

The extension of the concept of metric to soft theory was initially proposed by Das and Samanta [16, 21] through the definition of a metric on the set of soft points. In these works various topological concepts—including soft open sets, soft closed sets, soft limit points, soft closure and soft completeness—were defined in the context of soft metrics, in addition to the properties typically recognised in classical metric theory, leading to notable results. Alcantud et al. [19] have also recently offered a comprehensive and in-depth review of soft set theory.

The main aim of this study is to define the concepts of Wijsman and Hausdorff convergence for sequences of soft closed sets in soft metric spaces and to explore the fundamental properties of these concepts. To the best of our knowledge, a Wijsman-type convergence has not been addressed in the literature for soft metric spaces and thus our motivation is to partially fill this gap while also establishing a connection between soft set theory and broader mathematical structures. Since soft sets provide a powerful tool for modelling uncertainty, it is believed that this study will make significant contributions to both mathematical and applied fields.

In this work classical theories examining the convergence processes of sets in topological spaces have been utilised. Although there are several types of convergence for sequences of sets, only the concepts of Wijsman and Hausdorff convergences are applied to closed soft set sequences in soft metric spaces in this study. In parallel with classical theory, the metric used for Wijsman convergence is defined through the distance between each set and a point, whereas a different distance concept is adopted for Hausdorff convergence. To better understand the differences and similarities between these two types of convergence, these two concepts are compared. Additionally, in order to gain a deeper understanding of the nature of soft Wijsman convergence, the concepts of Wijsman boundedness and Wijsman Cauchy sequences are defined, and the relationships between these concepts are discussed.

KEY CONCEPTS AND RESULTS ASSOCIATED WITH SOFT SET THEORY

As a preliminary remark for the reader's convenience, this section presents the essential definitions and results that will be utilised in soft set theory. Throughout this work we denote the universe set by X and the set of all parameters by E.

Definition 1 [9]**.** A soft set over X is denoted as F^E, where $F: E \to P(X)$ is a mapping. The collection of all soft sets over X is represented by $SS(X)$.

Based on Molodtsov's definition, several key operations in soft theory that are relevant to this study are outlined below.

Definition 2. Let $F^E, G^E \in SS(X)$.
(i) If $F(g) = \emptyset_s$ for every $g \in E$, then F^E is referred to null soft set $\emptyset_s$. Similarly, if for every $g \in E$, $F(g) = X$, then F^E is referred to absolute soft set X_s [10].
(ii) $F^E \subset G^E$ holds if $F(g) \subset G(g)$ for $g \in E$. Hence F^E is equal to G^E (denoted by $F^E =_s G^E$) if $F(g) = G(g)$ for every $g \in E$ [10].
(iii) The soft complement of F^E is the soft set $(F^E)^c =_s (F^c)^E$ such that $F^c: E \to P(X)$, $F^c(g) = X \backslash F(g)$ for every $g \in E$ [13].
(iv) The union $F^E \cup G^E$ is a soft set $H^E \in SS(X)$ such that $H: E \to P(X)$, $H(g) = F(g) \cup G(g)$ for every $g \in E$ [10].
(v) The intersection $F^E \cap G^E$ is a soft set $I^E \in SS(X)$ such that $I: E \to P(X)$, $I(g) = F(g) \cap G(g)$ for every $g \in E$ [10].

By selecting a specific function F that characterises the soft set, we can derive various types of soft sets commonly discussed in the literature:

(i) A soft set F^E is referred to as a singleton soft set if $F(g)$ is a singleton for every $g \in E$.
(ii) More specifically, any singleton soft set F^E is called a soft element. If, for each $g \in E$, $F(g) = \{x_g\}$, this soft element is denoted as $\tilde{x}$. The collection of all soft elements over X is denoted by $SE(X)$.
(iii) A soft set F^E is termed a soft point, represented as P_e^x, if there exists a $g \in E$ such that $F(g) = \{x\}$ for some $x \in X$ and $F(e') = \emptyset_s$ for all $e' \in E - \{g\}$. The set of all soft points over X is denoted by $SP(X)$. Notably, every soft set can be expressed as a union of soft points.
(iv) A soft set F^E is defined as a soft real set if F is a mapping given by $F: E \to B(\mathbb{R})$, where $B(\mathbb{R})$ is the collection of all non-empty bounded subsets of $\mathbb{R}$.
(v) If a soft real set F^E is a soft element, it is termed a soft real number. If, for each $g \in E$, there exists $r_g \in \mathbb{R}$ such that $F(g) = \{r_g\}$, this soft real number is denoted by $\tilde{r}$. The set of all soft real numbers is represented by $\mathbb{R}^E$.

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(vi) If $F(g) = \{r\}$ for every $g \in E$, then F^E is referred to as a constant soft real number, denoted by $\bar{r}$.
(vii) For $\tilde{r}, \tilde{s} \in \mathbb{R}^E$, we express $\tilde{r_g} \leq_s \tilde{s_g}$ if and only if $r_g \leq s_g$ for every $g \in E$. Thus, $\leq$ establishes a partial order relation on $\mathbb{R}^E$. If $\tilde{r_g} \geq_s \bar{0}$, meaning $r_g \geq 0$ for all $g \in E$, then $\tilde{r}$ is classified as a non-negative soft real number. The set of all non-negative soft real numbers is denoted by $\mathbb{R}_+^E$. Similarly, we denote $\tilde{r} <_s \tilde{s}$ if $r_g < s_g$ for every $g \in E$ [16]. Consequently, we can define the supremum and infimum of any set in $\mathbb{R}_+^E$, analogous to classical definitions.

Remark 1. The concepts of soft elements and soft points are distinct from each other. For instance, let $X = \mathbb{R}$ and $E = \{e_i = [i, i+1]: i \in \mathbb{N}^+\}$, where the function $F: E \to P(X)$ is defined such that $F(e_i)$ represents the count of prime numbers within the interval e_i. In this context the soft set

$$F^E = \{e_1 = \{1\}, e_2 = \{2\}, e_3 = \{1\}, \dots, e_8 = \{0\}, \dots\}$$

is a soft element and even a soft real number. However, it is not a soft point. Additionally, it is clear that the set

$$F^E =_s \cup \{P_{e_i}^{x_i}: \{x_i\} = F(e_i)\}$$

can be expressed accordingly.

Definition 3 [16]. A function $d: SP(X) \times SP(X) \to \mathbb{R}_+^E$ is defined as a soft metric if it satisfies the following criteria. For all $P_{e_1}^x, P_{e_2}^y, P_{e_3}^z \in SP(X)$:
(sM1) Non-negativity: $d(P_{e_1}^x, P_{e_2}^y) \geq_s \bar{0}$ (i.e. $d(P_{e_1}^x, P_{e_2}^y)(e) \geq 0$ for every $e \in E$).
(sM2) Identity of Indiscernibles: $d(P_{e_1}^x, P_{e_2}^y) =_s \bar{0}$ if and only if $P_{e_1}^x = P_{e_2}^y$ (i.e. $d(P_{e_1}^x, P_{e_2}^y)(e) = 0$ for every $e \in E$ if and only if $e_1 = e_2$ and $x = y$).
(sM3) Symmetry: $d(P_{e_1}^x, P_{e_2}^y) =_s d(P_{e_2}^y, P_{e_1}^x)$ (i.e. $d(P_{e_1}^x, P_{e_2}^y)(e) = d(P_{e_2}^y, P_{e_1}^x)(e)$ for every $e \in E$).
(sM4) Triangle Inequality: $d(P_{e_1}^x, P_{e_2}^y) \leq_s d(P_{e_1}^x, P_{e_3}^z) + d(P_{e_2}^y, P_{e_3}^z)$ (i.e.$d(P_{e_1}^x, P_{e_2}^y)(e) \leq d(P_{e_1}^x, P_{e_3}^z)(e) + d(P_{e_2}^y, P_{e_3}^z)(e)$ for every $e \in E$).

In this framework the triplet (X, d, E) is referred to as a soft metric space.

Definition 4 [16]. Let (X, d) be a soft metric, $P_{e_1}^a \in SP(X)$ and $\tilde{r} \in \mathbb{R}_+^E$.
(i) The soft open ball centred at $P_{e_1}^a$ of radius $\tilde{r}$ is the set

$$B(P_{e_1}^a, \tilde{r}) =_s \{P_e^x \in SP(X): d(P_{e_1}^a, P_e^x) <_s \tilde{r}\}.$$

(ii) The soft closed ball centred at $P_{e_1}^a$ of radius $\tilde{r}$ is the set

$$B[P_{e_1}^a, \tilde{r}] =_s \{P_e^x \in SP(X): d(P_{e_1}^a, P_e^x) \leq_s \tilde{r}\}.$$

Remark 2. As stated by Das and Samanta [16], the collection of soft open balls forms the basis for a soft topology induced by the soft metric d, denoted as τ_d. Consequently, (X, τ_d) is a soft topological space. Conventionally, if the complement of the set $(Y^E)^c$ is a soft open set, then the soft set Y^E is termed a soft closed set.

Definition 5 [16]. Let (X, d) be a soft metric space and $F^E, G^E \in SS(X)$.
(i) The distance of any soft point P_e^x to the soft set F^E is defined as the non-negative soft real number $D(P_e^x, F^E)$, which is expressed as follows:

$$D(P_e^x, F^E)(g) = \inf\{d(P_e^x, P_h^y)(g): P_h^y \in SP(F^E)\} \quad \text{for every} \quad g \in E.$$

In particular, if P_e^x belongs to F^E, then $D(P_e^x, F^E) =_s \bar{0}$.

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(ii) The distance between two soft sets F^E and G^E is defined as the non-negative soft real number $D(F^E, G^E)$, expressed as

$$D(F^E, G^E)(g) = \inf\{d(P_e^x, P_h^y)(g): P_e^x \in SP(F^E), P_h^y \in SP(G^E)\} \text{ for every } g \in E.$$

Remark 3. It is well established that all soft elements are defined using soft points, such as

$$\tilde{x} =_s \bigcup \{P_e^x : P_e^x \in \tilde{x}\}.$$

Thus, considering a soft element as a soft set, it follows that

$$D(\tilde{x}, F^E) \leq_s D(P_e^x, F^E) \text{ for every } P_e^x \in \tilde{x}.$$

Definition 6 [20]. Let $(P_{e_k}^{x_k})$ be a sequence of soft points in the soft metric space (X, d). The sequence $(P_{e_k}^{x_k})$ is said to be convergent to a soft point P_e^x if the assumption $d(P_{e_k}^{x_k}, P_e^x) \overset{\tilde{d}}{\rightarrow} \overline{0}$ holds as $n \to \infty$ and we denote this by $P_{e_k}^{x_k} \overset{\tilde{d}}{\rightarrow} P_e^x$. This means for every choosen arbitrary $\tilde{\varepsilon} >_s \overline{0}$ there exists a natural number k_0 such that $\overline{0} \leq_s d(P_{e_k}^{x_k}, P_e^x) <_s \tilde{\varepsilon}$ whenever $k > k_0$.

Definition 7 [16]. Let (X, d) be a soft metric space and $F^E \in SS(X)$.

(i) A soft point P_e^x is defined as a soft limit point of F^E if every soft open ball centered at P_e^x contains at least one soft point of F^E that is distinct from P_e^x.

(ii) The soft set formed by the collection of all soft points and soft limit points of F^E in the soft metric space (X, d) is referred to as the soft closure of F^E in (X, d). This set is denoted by $\overline{F^E}$.

SOFT WIJSMAN CONVERGENCE

In this section we define Wijsman convergence in soft metric spaces and provide its general properties.

Definition 8. Let (X, d) be a soft metric space and a sequence of soft closed sets $\{A_k^E\}$ and A^E be non-empty soft closed sets in X_s. We say that the sequence $\{A_k^E\}$ is soft Wijsman convergent to A^E if, for every $g \in E$ and for all $P_e^x \in SP(X)$,

$$\lim_{k\to\infty} D(P_e^x, A_k^E)(g) = D(P_e^x, A^E)(g).$$

This convergence is denoted by $A_k^E \overset{sW}{\longrightarrow} A^E$. In other word,

$$A_k^E \overset{sW}{\longrightarrow} A^E \Leftrightarrow \quad D(P_e^x, A_k^E)(g) \overset{\mathbb{R}}{\rightarrow} D(P_e^x, A^E)(g)$$

$$\Leftrightarrow \text{ for each } \tilde{\varepsilon} > \overline{0}, \text{ there exists } k_0 \in \mathbb{N} \text{ such that}$$

$$d\left(D(P_e^x, A_k^E), D(P_e^x, A^E)\right)(g) < \varepsilon_e \text{ for every } k > k_0.$$

Example 1. Let $E = \{g\}$ and $X = \mathbb{R} \times \mathbb{R}$. We define a sequence of soft closed sets $\{A_k^E\}$ as follows:

$$\{A_k^E\} =_s \{g = \{(x, y): x^2 + y^2 - 2ky = 0\}\}.$$

Considering the soft metric given by

$$d(P_g^{(x_1,y_1)}, P_g^{(x_2,y_2)}) =_s \{g = \{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\}\},$$

it is observed that $\{A_k^E\}$ converges in the soft Wijsman sense to the soft closed set $A^E =_s \{g = \{(x, y): y = 0\}\}$.

Example 2. Let $E = \{g\}$ and $X = \mathbb{R}$. We define a sequence of soft closed sets $\{A_k^E\}$ as follows:

$$A_k^E =_s \begin{cases} g = [2, k], & \text{if } k \geq 2 \text{ and } k \text{ is a perfect square,} \\ g = \{1\}, & \text{otherwise.} \end{cases}$$

The soft metric is considered as

$$d(P_g^x, P_g^y) =_s \{g = \{|x - y|\}\}.$$

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For $k \geq 2$ when k is a perfect square, it is found that for $x \geq 2$, $D(P_g^x, A_k^E) =_s \overline{0}$, while for $x < 2$, $D(P_g^x, A_k^E) =_s \{g = \{2 - x\}\}$. Conversely, when k is not a perfect square, it is noted that $D(P_g^x, A_k^E) =_s \{g = \{|1 - x|\}\}$ for all $x \in X$. Consequently, it is concluded that the sequence of soft closed sets does not converge to a single value; thus, it is not soft Wijsman convergent.

Lemma 1. Let (X, d) be a soft metric space, $A^E \in SS(X)$, and $P_e^x \in SP(X)$. If $D(P_e^x, A^E) =_s \overline{0}$ and $P_e^x \notin A^E$, then it follows that $P_e^x \in (A^E)^d$.

Proof. Given that $D(P_e^x, A^E) =_s \overline{0}$ and $P_e^x \notin A^E$, it can be concluded that for every $g \in E$,

$$\inf\{d(P_e^x, P_{e'}^a)(g): P_{e'}^a \in A^E\} =_s 0.$$

As a result, there exists a $P_{e'}^a \in A^E$ such that for every $\tilde{\varepsilon} >_s \overline{0}$, the condition $0 < d(P_e^x, P_{e'}^a)(g) < \varepsilon_g$ holds. Thus, for any $\tilde{\varepsilon} >_s \overline{0}$, we find that $B(P_e^x, \tilde{\varepsilon}) \cap A^E \neq_s \emptyset_s$. Given the hypothesis $P_e^x \notin A^E$, we can express $B(P_e^x, \tilde{\varepsilon}) \backslash \{P_e^x\} \cap A^E \neq_s \emptyset_s$. This leads to the conclusion that $P_e^x \in (A^E)^d$.

The following theorem describes soft points for which their soft distance to a soft set equals zero.

Theorem 1. Let (X, d) be a soft metric space, and let $A^E \in SS(X)$. Consequently, the closure of A^E can be expressed as follows:

$$\overline{A^E} =_s \{P_e^x \in SP(X): D(P_e^x, A^E) =_s \overline{0}\}.$$

Proof. Let us consider $P_e^x \in SP(X)$ such that $D(P_e^x, A^E) =_s \overline{0}$. According to Lemma 1, P_e^x must either belong to A^E or $(A^E)^d$. Since both $A^E \subseteq \overline{A^E}$ and $(A^E)^d \subseteq \overline{A^E}$ hold true in either scenario, it follows that $P_e^x \in \overline{A^E}$. Therefore, we conclude that

$$\{P_e^x \in SP(X): D(P_e^x, A^E) =_s \overline{0}\} \subseteq \overline{A^E}. \quad (1)$$

Conversely, assume $P_e^x \in \overline{A^E}$. Let us consider the case where $D(P_e^x, A^E) =_s \tilde{r} >_s \overline{0}$. It is clear that $P_e^x \notin A^E$. Moreover, there exists a $g_0 \in E$ such that $D(P_e^x, A^E)(g_0) = r_{g_0} > 0$. This indicates that $d(P_e^x, P_{e'}^a)(g_0) \geq r_{g_0}$ for every $P_{e'}^a \in A^E$. Consequently, we have

$$B\left(P_e^x, \frac{1}{2}\tilde{r}\right) \cap A^E =_s \emptyset_s.$$

This shows that $P_e^x \notin (A^E)^d$, which leads to the conclusion that $P_e^x \notin \overline{A^E}$, resulting in a contradiction. Therefore, it must be the case that $D(P_e^x, A^E) =_s \overline{0}$. Hence we can assert that

$$\overline{A^E} \subseteq \{P_e^x \in SP(X): D(P_e^x, A^E) =_s \overline{0}\}. \quad (2)$$

The proof is thus completed by combining (1) and (2).

Theorem 2. The soft Wijsman limit of the sequence of soft closed sets $\{A_k^E\}$ is unique.

Proof. Let $\{A_k^E\}$ represent a sequence of soft closed sets that converges in the Wijsman sense to two distinct soft closed sets, A^E and B^E. For every $P_e^x \in SP(X)$, the sequence $D(P_e^x, A_k^E)$ converges to both $D(P_e^x, A^E)$ and $D(P_e^x, B^E)$. Since both $D(P_e^x, A^E)$ and $D(P_e^x, B^E)$ are soft elements, Theorem 3.3 [21] implies that

$$D(P_e^x, A^E) =_s D(P_e^x, B^E).$$

Consequently, for any $P_e^x \in A^E$, it follows that $D(P_e^x, A^E) =_s \overline{0}$, which in turn implies $D(P_e^x, B^E) =_s \overline{0}$. Therefore, by Theorem 1 and the closedness of B^E, we deduce that $P_e^x \in B^E$, thereby confirming that $A^E \subseteq B^E$.

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By applying a symmetrical argument, we can reverse the reasoning to conclude that $B^E \subseteq A^E$. Thus, we can ultimately establish that $A^E =_s B^E$.

Remark 4. In a soft topological space every soft point is also a soft set and is closed [22]. Since Wijsman and metric convergences provide different perspectives on convergence, it is pertinent to explore the relationship between Wijsman convergence and metric convergence within the context of soft points in soft metric spaces. Analysing the compatibility of these two convergence concepts or examining whether one convergence type implies or contrasts the other is a natural inquiry.

As stated in Remark 2, the collection of soft open balls forms the basis for a soft topology induced by the soft metric d, denoted by τ_d. Then the sequence of soft points $\left(P_{e_k}^{x_k}\right)$ is said to be convergent (in short $sd-convergent$ and denoted by $P_{e_k}^{x_k} \overset{sd}{\to} P_e^x$) to $P_e^x \in SP(X)$ provided that $d\left(P_{e_k}^{x_k}, P_e^x\right)(f) \to \overline{0}$ for every $f \in E$ [16].

Theorem 3. Let $\left(P_{e_k}^{x_k}\right)$ be a sequence of soft points in the soft metric space (X, d). The sequence $\left(P_{e_k}^{x_k}\right)$ converges to a soft point P_f^y in the sense of soft Wijsman convergence if and only if $\left(P_{e_k}^{x_k}\right)$ is sd −convergent to P_f^y. That is,

$$P_{e_k}^{x_k} \overset{sW}{\longrightarrow} P_f^y \iff P_{e_k}^{x_k} \overset{sd}{\to} P_f^y.$$

Proof. As can be easily inferred from the definitions, for any $P_e^x, P_f^y \in SP(X)$ and $g \in E$, the following relation is established:

$$D\left(P_e^x, P_f^y\right)(g) = \inf\left\{d\left(P_e^x, P_f^y\right)(g): P_e^x \in SP(X)\right\} = d\left(P_e^x, P_f^y\right)(g).$$

Assuming that $P_{e_k}^{x_k} \overset{sW}{\longrightarrow} P_f^y$, it follows that for every $P_e^x \in SP(X)$ and $g \in E$,

$$D\left(P_e^x, P_{e_k}^{x_k}\right)(g) \overset{\mathbb{R}}{\to} D\left(P_e^x, P_f^y\right)(g).$$

Consequently, it follows that

$$d\left(P_e^x, P_{e_k}^{x_k}\right)(g) \overset{\mathbb{R}}{\to} d\left(P_e^x, P_f^y\right)(g).$$

In particular, when $P_e^x = P_f^y$ is chosen, it can be concluded that $d\left(P_e^x, P_{e_k}^{x_k}\right)(g) \overset{\mathbb{R}}{\to} 0$. This implies that $d\left(P_e^x, P_{e_k}^{x_k}\right) \overset{\mathbb{R}}{\to} \overline{0}$, indicating that $P_{e_k}^{x_k} \overset{sd}{\to} P_f^y$.

On the other hand, for any arbitrarily chosen $P_e^x \in SP(X)$ and for every $g \in E$, the following inequality is satisfied:

$$\left|D\left(P_e^x, P_{e_k}^{x_k}\right)(g) - D\left(P_e^x, P_f^y\right)(g)\right| = \left|d\left(P_e^x, P_{e_k}^{x_k}\right)(g) - d\left(P_e^x, P_f^y\right)(g)\right| \leq d\left(P_{e_k}^{x_k}, P_f^y\right)(g).$$

Thus, it is straightforward to see that the convergence $P_{e_k}^{x_k} \overset{sd}{\to} P_f^y$ implies that $P_{e_k}^{x_k} \overset{sW}{\longrightarrow} P_f^y$.

SOFT WIJSMAN BOUNDEDNESS

In this section the concept of boundedness, which is an important notion in mathematics, is addressed in the context of soft set theory from the perspective of Wijsman convergence.

Definition 9. Let (X, d) be a soft metric space, $\{A_k^E\}$ a sequence of soft closed sets, and A^E a soft closed set. If the condition $\sup\{D(P_e^x, A_k^E)(g): k \in \mathbb{N}\} < \infty$ holds for at least one $P_e^x \in SP(X)$ and for each $g \in E$, then the sequence $\{A_k^E\}$ is said to be soft Wijsman bounded.

Theorem 4. Let (X, d) be a soft metric space, $\{A_k^E\}$ a sequence of soft closed sets, and A^E a soft closed set. If the sequence $\{A_k^E\}$ is soft Wijsman bounded, then for every $P_e^x \in SP(X)$, the condition $\sup\{D(P_e^x, A_k^E)(g): k \in \mathbb{N}\} < \infty$ holds for each $g \in E$.

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Proof. Assume that the sequence $\{A_k^E\}$ is soft Wijsman bounded. Then there exists a soft point P_f^y such that $\sup\{D(P_f^y, A_k^E)(g): k \in \mathbb{N}\} < \infty$ for each $g \in E$. Let P_e^x be any chosen element from $SP(X)$ such that $P_e^x \neq P_f^y$. Consequently, the following inequality holds for every $k \in \mathbb{N}$ and for each $g \in E$:

$$D(P_e^x, A_k^E)(g) \leq d(P_e^x, P_f^y)(g) + D(P_f^y, A_k^E)(g).$$

Since the sequence $\{A_k^E\}$ is soft Wijsman bounded, it follows that

$$D(P_e^x, A_k^E)(g) \leq d(P_e^x, P_f^y)(g) + \sup_{k\in\mathbb{N}}\{D(P_f^y, A_k^E)(g)\} < \infty.$$

This indicates that $D(P_e^x, A_k^E)$ is bounded from above for each $k \in \mathbb{N}$. Consequently, we arrive at the conclusion that $\sup\{D(P_e^x, A_k^E)(g): k \in \mathbb{N}\} < \infty$.

Corollary 1. Let (X, d) be a soft metric space and let $\{A_k^E\}$ be a sequence of soft closed sets. If there exists at least one $k_0 \in \mathbb{N}$ such that $\bigcap_{k>k_0} A_k^E \neq_s \emptyset_s$, then the sequence $\{A_k^E\}$ is soft Wijsman bounded.

Theorem 5. Let (X, d) be a soft metric space and let $\{A_k^E\}$ denote a sequence of soft closed sets. If this sequence $\{A_k^E\}$ converges to $A^E \in SS(X)$ in the sense of soft Wijsman convergence, then it is also soft Wijsman bounded.

Proof. Assume $A_k^E \xrightarrow{sW} A^E$. Then for any $P_e^x \in SP(X)$ and for each $g \in E$, it follows that $D(P_e^x, A_k^E)(g) \to D(P_e^x, A^E)(g)$. In particular, if $P_e^x \in A^E$, we have $D(P_e^x, A_k^E)(g) \to D(P_e^x, A^E)(g) = 0$.

From this, for every $\tilde{\varepsilon} >_s \overline{0}$, there exists at least one $k_0 \in \mathbb{N}$ such that $D(P_e^x, A_k^E)(g) < \varepsilon_g$ for all $k > k_0$. Define $M_g = \{D(P_e^x, A_1^E), \dots, D(P_e^x, A_{k_0}^E)\}$. Then for every $k \in \mathbb{N}$ and for each $g \in E$, the following inequality holds:

$$D(P_e^x, A_k^E)(g) < \varepsilon_g + M_g.$$

Thus, there exists at least one $P_e^x \in SP(X)$ such that $\sup\{D(P_e^x, A_k^E)(g): k \in \mathbb{N}\} < \infty$, indicating that the sequence $\{A_k^E\}$ is soft Wijsman bounded.

Definition 10. Let $\{A_k^E\}$ be a sequence of soft closed sets in the soft metric space (X, d). We define $\{A_k^E\}$ as a soft Wijsman Cauchy sequence (abbreviated as $sW-$Cauchy) if, for any $P_e^x \in SP(X)$ and every $g \in E$, the sequence $\left(D(P_e^x, A_k^E)(g)\right)$ forms a Cauchy sequence in the metric space $(\mathbb{R}, d_{\mathbb{R}})$.

Alternatively, $\{A_k^E\}$ is an $sW-$Cauchy sequence if, for every $\tilde{\varepsilon} >_s \overline{0}$ and for each $P_e^x \in SP(X)$, there exists a natural number k_0 such that for all $m, k > k_0$ and for every $g \in E$, the following condition holds:

$$d_{\mathbb{R}}\left(D(P_e^x, A_k^E)(g), D(P_e^x, A_m^E)(g)\right) < \varepsilon_g.$$

Theorem 6. Consider $\{A_k^E\}$ as a $sW-$Cauchy sequence in the soft metric space (X, d). If there exists a subsequence of $\{A_{k_n}^E\}$ that converges to a soft set A^E, then the sequence $\{A_k^E\}$ also converges to A^E.

Proof. Let $\tilde{\varepsilon} >_s \overline{0}$ and $P_e^x \in SP(X)$ be chosen arbitrarily. Because the sequence is $sW-$Cauchy, there exists an $N_1 \in \mathbb{N}$ such that for all $k, k_n > N_1$, we have

$$d_{\mathbb{R}}\left(D(P_e^x, A_k^E)(g), D(P_e^x, A_{k_n}^E)(g)\right) < \frac{\varepsilon_g}{2}.$$

Additionally, since the subsequence converges, there exists an $N_2 \in \mathbb{N}$ such that for all $n_k > N_2$,

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$$d_{\mathbb{R}}\big(D\big(P_e^x, A_{k_n}^E\big)(g), D(P_e^x, A^E)(g)\big) < \frac{\varepsilon_g}{2}.$$

By taking $N = \max\{N_1, N_2\}$, for every $k, k_n > N$, we derive

$$\begin{aligned} &d_{\mathbb{R}}\big(D\big(P_e^x, A_k^E\big)(g), D(P_e^x, A^E)(g)\big) \\ &\qquad \le d_{\mathbb{R}}\big(D\big(P_e^x, A_k^E\big)(g), D\big(P_e^x, A_{k_n}^E\big)(g)\big) + d_{\mathbb{R}}\big(D\big(P_e^x, A_{k_n}^E\big)(g), D(P_e^x, A^E)(g)\big) \\ &\qquad < \varepsilon_g. \end{aligned}$$

This establishes that the sequence $\{A_k^E\}$ converges to A^E in the soft Wijsman sense.

Theorem 7. In the soft metric space (X, d), every $sW-$convergent sequence is also an $sW-$Cauchy sequence.

Proof. Assume that $\{A_k^E\} \subset SS(X)$ and that $A_k^E \xrightarrow{sW} A^E \in SS(X)$. Then for every $P_e^x \in SP(X)$, it follows that $D\big(P_e^x, A_k^E\big) \xrightarrow{d} D(P_e^x, A^E)$. Consequently, for any $\tilde{\varepsilon} >_s \overline{0}$ there exists an $N \in \mathbb{N}$ such that the following inequality holds for every $P_e^x \in SP(X)$, every $g \in E$ and all $k > N$:

$$d_{\mathbb{R}}\big(D\big(P_e^x, A_k^E\big)(g), D(P_e^x, A^E)(g)\big) < \frac{\varepsilon_g}{2}.$$

Thus, for every $k, m > N$, we have

$$\begin{aligned} &d_{\mathbb{R}}\big(D\big(P_e^x, A_k^E\big)(g), D(P_e^x, A_m^E)(g)\big) \le d_{\mathbb{R}}\big(D\big(P_e^x, A_k^E\big)(g), D(P_e^x, A^E)(g)\big) \\ &+ d_{\mathbb{R}}(D(P_e^x, A_m^E)(g), D(P_e^x, A^E)(g)) \\ &< \frac{\varepsilon_g}{2} + \frac{\varepsilon_g}{2} = \varepsilon_g. \end{aligned}$$

This demonstrates that $\{A_k^E\}$ is an $sW-$Cauchy sequence.

SOFT HAUSDORFF DISTANCE

Wijsman convergence utilises the metric distance from a point to a set. In contrast, Hausdorff convergence employs a different notion of distance defined in relation to the metric, rather than using the metric itself.

Definition 11. Let (X, d) be a soft metric space and let $A^E, B^E \in SS(X)$. The soft semi-Hausdorff distance between the sets A^E and B^E is a soft element defined as follows:

$$h_1(A^E, B^E)(g) = \sup\{D(P_e^b, A^E)(g) : P_e^b \in B^E\}$$

and

$$h_2(A^E, B^E)(g) = \sup\{D(P_e^a, B^E)(g) : P_e^a \in A^E\}$$

for every $g \in E$.

The soft Hausdorff distance $h(A^E, B^E)$ between A^E and B^E is a soft element defined as follows:

$$h(A^E, B^E)(g) = \max\big\{\sup\{D(P_e^b, A^E)(g) : P_e^b \in B^E\}, \sup\{D(P_e^a, B^E)(g) : P_e^a \in A^E\}\big\}$$

for every $g \in E$. It is evident that

$$h(A^E, B^E) =_s \max\{h_1(A^E, B^E), h_2(A^E, B^E)\}.$$

Example 3. Let $X = \{1,2,3,4,5,6\}$ and $E = \{1,2,3,4\}$. Consider the sets

$$A^E =_s \{1 = \{2,3\}, 2 = \{5,6\}, 3 = \{1\}, 4 = \emptyset\}$$

and

$$B^E =_s \{1 = \{2,6\}, 2 = \{4\}, 3 = \varnothing, 4 = \{1,6\}\}.$$

According to the soft metric defined as

$$d : SP(X) \times SP(X) \to \mathbb{R}^E, \quad d(P_{e_1}^x, P_{e_2}^y) =_s |\overline{x} - \overline{y}| + |\overline{e_1} - \overline{e_2}|,$$

we find that $h_1(A^E, B^E) =_s \overline{1}$ and $h_2(A^E, B^E) =_s \overline{2}$. Consequently, it follows that $h(A^E, B^E) =_s \overline{2}$.

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Remark 5. From Example 3 it is evident that $h_1(A^E, B^E)$ and $h_2(A^E, B^E)$ may not be equal. Additionally, we can deduce the following straightforward observations:
(i) If $h_1(A^E, B^E) =_s h_2(A^E, B^E)$, then it follows that $h(A^E, B^E) =_s h_1(A^E, B^E) =_s h_2(A^E, B^E)$.
(ii) If $h_1(A^E, B^E) =_s h(A^E, B^E)$, then it implies that $h_1(A^E, B^E) \geq_s h_2(A^E, B^E)$.

Remark 6. Let us consider the universe and parameter sets in Example 2, along with the soft metric. Accordingly, for the sets $A^E =_s \{e = [0,1]\}$ and $B^E = \{e = (0,1)\}$, we observe that $h(A^E, B^E) =_s \overline{0}$ holds while $A^E \neq_s B^E$.

Upon reviewing the aforementioned Example 3, it is evident that the soft Hausdorff distance does not generally define a soft metric. However, we can present the following proposition, the proof of which is omitted as it will be readily apparent.

Proposition 1. The soft Hausdorff distance is an soft pseudo metric.

Lemma 2. Let (X, d) be a soft metric space and let $A^E, B^E \in SS(X)$. In this context the following statements are valid:
(i) If $h_1(A^E, B^E) =_s \overline{0}$, then $A^E \subseteq \overline{B^E}$.
(ii) If $h_2(A^E, B^E) =_s \overline{0}$, then $B^E \subseteq \overline{A^E}$.
(iii) If $h(A^E, B^E) =_s \overline{0}$, then $\overline{A^E} = \overline{B^E}$.

Proof. To prove assertion (i), assume that $h_1(A^E, B^E) =_s \overline{0}$. This implies that $\sup\{D(P_e^a, B^E): P_e^a \in A^E\} =_s \overline{0}$. Consequently, for every $P_e^a \in A^E$, we have $D(P_e^a, B^E) =_s \overline{0}$. According to Theorem 1, this leads to the conclusion that $P_e^a \in \overline{B^E}$. Thus, we establish that $A^E \sqsubseteq \overline{B^E}$. The proof of assertion (ii) follows a similar argument by interchanging the roles of A^E and B^E. Assertion (iii) is a direct consequence of assertions (i) and (ii).

In conclusion, as indicated in Proposition 1, the soft Hausdorff distance is not a soft metric on the family of soft sets, but rather a pseudometric. However, when considering a more restricted class instead of all soft sets, we can derive the following result with the assistance of Proposition 1 and Lemma 2(iii).

Corollary 2. Let (X, d) be soft metric space and $clSS(X)$ be set of all soft closed sets. Then the Hausdorff distance function on the set of all soft closed sets

$$h: clSS(X) \times clSS(X) \to \mathbb{R}^E$$

is a soft metric.

SOFT HAUSDORFF CONVERGENCE

In this section we define Hausdorff convergence within the context of soft set theory using soft Hausdorff distance. We also examine its relationship with soft Wijsman convergence.

Definition 12. Let (X, d) be soft metric space, $\{A_k^E\}$ be sequence of non-empty soft closed sets, and $A^E \in clSS(X)$. We say that the sequence $\{A_k^E\}$ is soft Hausdorff convergent to A^E if

$$\lim_{n\to\infty} h\left(A_k^E, A^E\right) =_s \overline{0}.$$

This convergence is denoted by $A_k^E \xrightarrow{sH} A^E$. In other word,

$$A_k^E \xrightarrow{sH} A^E \iff h\left(A_k^E, A^E\right)(g) \xrightarrow{\mathbb{R}^E} 0 \quad \text{for every } g \in E$$
$$\iff \text{For every } \tilde{\varepsilon} >_s \overline{0}, \quad \text{there exists } k_0 \in \mathbb{N} \text{ such that}$$

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$$h(A_k^E, A^E)(g) < \varepsilon_g \quad \text{for every } k \geq k_0.$$

Definition 13. Let (X, d) be soft metric space. The expansion of a soft set A^E with radius $\tilde{\varepsilon}$ is defined as the set

$$\mathbb{B}(A^E, \tilde{\varepsilon}) =_s \{P_e^x \in SP(X): D(P_e^x, A^E) <_s \tilde{\varepsilon}\}.$$

Lemma 3. Let (X, d) be soft metric space. For every $A^E, B^E \in SS(X)$, the following assertions hold:
(i) $h_1(A^E, B^E) =_s \inf\{\tilde{\varepsilon} > \overline{0}: A^E \subseteq \mathbb{B}(B^E, \tilde{\varepsilon})\}$.
(ii) $h_2(A^E, B^E) =_s \inf\{\tilde{\varepsilon} > \overline{0}: B^E \subseteq \mathbb{B}(A^E, \tilde{\varepsilon})\}$.

Proof. Let us define $h_1(A^E, B^E) =_s \tilde{\delta} \geq_s \overline{0}$. For every $P_e^b \in B^E$ and for any $g \in E$, it follows that

$$D(P_e^b, A^E)(g) = \inf\{d(P_e^b, P_f^a)(g): P_f^a \in A^E\} \leq \delta_g.$$

This indicates that $A^E \subseteq \mathbb{B}(B^E, \tilde{\delta})$. Therefore, we can assert that

$$h_1(A^E, B^E) \in \{\tilde{\varepsilon} >_s \overline{0}: A^E \subseteq \mathbb{B}(B^E, \tilde{\varepsilon})\}.$$

Now let us pick an arbitrary $\tilde{\varepsilon} \in \{\tilde{\varepsilon} >_s \overline{0}: A^E \subseteq \mathbb{B}(B^E, \tilde{\varepsilon})\}$. For each $P_e^a \in A^E$, it holds that $D(P_e^a, B^E) <_s \tilde{\varepsilon}$, which leads to

$$h_1(A^E, B^E) =_s \sup\{D(P_e^a, B^E): P_e^a \in A^E\} <_s \tilde{\varepsilon}.$$

Thus, we can conclude that

$$h_1(A^E, B^E) <_s \inf\{\tilde{\varepsilon} >_s \overline{0}: A^E \subseteq \mathbb{B}(B^E, \tilde{\varepsilon})\}.$$

As a result, the equality in statement (i) is verified. The proof of the equality in statement (ii) follows a similar line of reasoning.

Theorem 8. Let (X, d) be a soft metric space and let $\{A_k^E\}$ denote a sequence of non-empty soft closed sets, with $A^E \in clSS(X)$. The convergence $A_k^E \xrightarrow{sH} A^E$ is characterised as follows:

$$A_k^E \xrightarrow{sH} A^E \Leftrightarrow \text{For every } \tilde{\varepsilon} >_s \overline{0}, \text{ there exists } n_0 \in \mathbb{N} \text{ such that, for every } n \geq n_0,$$
$$A^E \subseteq \mathbb{B}(A_k^E, \tilde{\varepsilon}) \text{ and } A_k^E \subseteq \mathbb{B}(A^E, \tilde{\varepsilon}).$$

Proof. Assume that $A_k^E \xrightarrow{sH} A^E$ holds, i.e. $\lim_{n\to\infty} h(A_k^E, A^E) =_s \overline{0}$. This is equivalent to the assertion that for every $\tilde{\varepsilon} >_s \overline{0}$ there exists an $n_0 \in \mathbb{N}$ such that for all $n > n_0$ and for every $g \in E$, the inequality $h(A_k^E, A^E)(g) < \varepsilon_g$ holds. It follows that

$$h_1(A_k^E, A^E)(g) < \varepsilon_g \quad \text{and} \quad h_2(A_k^E, A^E)(g) < \varepsilon_g.$$

As a result, from Lemma 3 we observe that the convergence $A_k^E \xrightarrow{sH} A^E$ is equivalent to the conditions that for every $n > n_0$, $A^E \subseteq \mathbb{B}(A_k^E, \tilde{\varepsilon})$ and $A_k^E \subseteq \mathbb{B}(A^E, \tilde{\varepsilon})$ are satisfied.

Theorem 9. In a soft metric space (X, d), for a sequence of soft closed sets $\{A_k^E\}$, soft Hausdorff convergence implies soft Wijsman convergence. That is,

$$A_k^E \xrightarrow{sH} A^E \Rightarrow A_k^E \xrightarrow{sW} A^E.$$

Proof. Assume that $A_k^E \xrightarrow{sH} A^E$. Hence for every $\tilde{\varepsilon} >_s \overline{0}$ there exists $n_0 \in \mathbb{N}$ such that $h(A^E, A_k^E) <_s \tilde{\varepsilon}$ holds for every $n \geq n_0$. By Theorem 8 we observe that $A^E \subseteq \mathbb{B}(A_k^E, \tilde{\varepsilon})$ and $A_k^E \subseteq \mathbb{B}(A^E, \tilde{\varepsilon})$ for every $n \geq n_0$. Let us choose arbitrary $P_e^x \in SP(X)$. Thus, we obtain that, for every $n \geq n_0$ and for every $g \in E$,

$$D(P_e^x, \mathbb{B}(A_k^E, \tilde{\varepsilon}))(g) \leq D(P_e^x, A^E)(g) \quad \text{and} \quad D(P_e^x, \mathbb{B}(A^E, \tilde{\varepsilon}))(g) \leq D(P_e^x, A_k^E)(g).$$

On the other hand, using the triangle inequality, for every $n \geq n_0$ and for every $g \in E$, the following inequalities hold:

$$D(P_e^x, A_k^E)(g) - \varepsilon_g \leq D(P_e^x, \mathbb{B}(A_k^E, \tilde{\varepsilon}))(g) \quad \text{and} \quad D(P_e^x, A^E)(g) - \varepsilon_g \leq D(P_e^x, \mathbb{B}(A^E, \tilde{\varepsilon}))(g).$$

From these, it follows that:

$$D(P_e^x, A_k^E)(g) - \varepsilon_g \leq D(P_e^x, A^E)(g) \quad \text{and} \quad D(P_e^x, A^E)(g) - \varepsilon_g \leq D(P_e^x, A_k^E)(g).$$

Thus, for each $n \geq n_0$ and for every $g \in E$, we obtain:

$$|D(P_e^x, A_k^E)(g) - D(P_e^x, A^E)(g)| \leq \varepsilon_g.$$

This implies that for every $P_e^x \in SP(X)$ and for every $g \in E$, the convergence

$$D(P_e^x, A_k^E)(g) \overset{\mathbb{R}}{\rightarrow} D(P_e^x, A^E)(g)$$

holds, indicating that $A_k^E \overset{sW}{\longrightarrow} A^E$.

Example 4. Let $E = \{e\}$ and $X = \mathbb{R}$. We define a sequence of soft closed sets $\{A_k^E\}$ as follows:

$$A_k^E =_s \begin{cases} e = \{0, \frac{1}{k}\}, & \text{if } k \text{ is even}, \\ e = \{0, k\}, & \text{if } k \text{ is odd}. \end{cases}$$

We consider the soft metric defined by

$$d(P_e^x, P_e^y) =_s \{e = \{|x - y|\}\}.$$

When k is even, it is observed that for every $P_e^x \in SP(X)$,

$$D(A_k^E, P_e^x) =_s \{e = \{\inf\{|x - 0|, |x - \tfrac{1}{k}|\}\}\}.$$

Similarly, when k is odd, it is found that for every $P_e^x \in SP(X)$,

$$D(A_k^E, P_e^x) =_s \{e = \{\inf\{|x - 0|, |x - k|\}\}\}.$$

As $k \to \infty$, it can be seen that in both cases, $D(A_k^E, P_e^x) =_s \{e = \{|x|\}$. On the other hand, it is noted that $D(A^E, P_e^x) =_s \{e = \{|x|\}\}$ holds true only for $A^E = P_e^0$. Consequently, it is concluded that $A_k^E \overset{sW}{\longrightarrow} A^E$. However, it is determined that $h_1(A_k^E, A^E) =_s \overline{0}$ and $h_2(A_k^E, A^E) \neq_s \overline{0}$. Thus, it follows that $\lim_{k\to\infty} h(A_k^E, A^E) \neq_s \overline{0}$. As a result, it is established that the sequence A_k^E of soft closed sets is not soft Hausdorff convergent.

CONCLUSIONS

By utilising a soft metric defined on a specific type of soft set known as soft points, we have defined the Wijsman convergence of the sequences of closed soft sets and presented several fundamental results. Additionally, concepts such as soft Wijsman boundedness and soft Wijsman Cauchy sequences have been defined, along with a discussion of their interrelationships. Furthermore, we have defined the Hausdorff distance and Hausdorff convergence as alternative forms of convergence for sequences of sets in the context of soft sets, deriving several results and exploring their relationship with Wijsman convergence.

Future research may delve deeper into the concept of Hausdorff convergence for sequences of soft sets. While it has been demonstrated that Wijsman convergence necessitates Hausdorff convergence, the converse of this proposition has yet to be established. Understanding the conditions under which Wijsman and Hausdorff convergences coincide may serve as a motivation for researchers working in this area.

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