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Interval-valued spherical fuzzy matrix and its applications in multi-attribute decision-making process

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Abstract: Despite significant advancement in matrix problem research over the past few decades, there remains a considerable gap in the literature when it comes to addressing real-life problems. The application of matrix is widespread in various real-life scenarios. In the literature researchers have always been more interested in studying matrices in fuzzy settings including intuitionistic fuzzy, picture fuzzy and spherical fuzzy environments. Inspired by the notion of interval-valued spherical fuzzy sets, we extend the theory of spherical fuzzy matrix into interval-valued spherical fuzzy matrix (IVSFM) to represent more flexibly uncertain and vague information. In this context we establish significant definitions and theorems about the given matrices. Further, we introduce the methodology for determining the determinant and adjoint of IVSFM. Finally, we propose a new score function for the interval-valued spherical fuzzy sets and prove its validity with the help of basic properties. Further, the application of the proposed concepts is shown by real-life decision-making for a career placement assessment.

Keywords: decision-making, interval-valued spherical fuzzy matrix, determinant of interval-valued spherical fuzzy matrix, adjoint of interval-valued spherical fuzzy matrix, score function

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INTRODUCTION

Decision-making is the cognitive process of selecting a choice or action among multiple alternatives, a fundamental aspect of human life essential in various contexts, from personal to professional. It involves assessing information, considering consequences and aligning choices with goals and values. Decisions can range from simple daily choices to complex strategic plans. Effective decision-making necessitates critical thinking, problem-solving skills and emotional intelligence. It plays a crucial role in shaping our lives, determining success and mitigating risks.

Understanding the decision-making process helps individuals and organisations make well-informed, rational choices that lead to desired outcomes and progress. The application of matrices in decision-making problems plays a pivotal role in various fields. Matrices provide a structured framework that aids in evaluating and comparing different alternatives. They enable decision-makers to quantify and analyse multiple factors or criteria simultaneously, facilitating a systematic and comprehensive approach. Matrices allow for the organisation of information and the identification of relationships between variables, providing a visual representation that enhances understanding and aids in making informed decisions [1-3]. By assigning weights and scores to various elements, matrices help prioritise options and determine the most favourable course of action. Overall, matrices are used in a variety of fields of science and technology to represent data in a meaningful way. However, various sorts of uncertain data are involved in decision-making, making it challenging to solve the issue in a traditional matrix. These problems can stem from data unpredictability, inadequate information and other factors.

To deal with the situation of vague data, fuzzy matrix (FM) plays a fundamental role in dealing with such a situation. Zadeh [4] developed the fuzzy set to deal with uncertainty in practical situations. The FM is defined by Thomason [5] after the fuzzy set is introduced. Kim and Roush [6] worked on the generalisation of the FM over boolean algebra. Pal [7] defined the FM with fuzzy rows and columns and presented some properties with the binary operator. Ragab and Eman [8] gave some results on the max-min composition and worked on the construction of an idempotent FM. Ragab and Emam [9] solved the determinant and adjoint of a square FM and discussed some properties defined on it. Pal [10] extended the FM to an interval-valued fuzzy matrix (IVFM) with an interval-valued fuzzy row and column. Meenakshi and Kaliraja [11] used the interval-valued FM for solving medical diagnosis problems. Mandal and Pal [12] described some methods of finding the ranks of IVFM. A number of researchers have worked on FM but they considered only the membership of the element.

Atanassov [13] introduced the concept of intuitionistic fuzzy sets with this kind of situation in mind. Khan et al. [14] defined the concept of an intuitionistic fuzzy matrix (IFM). Padder and Murugadas [15] worked on max-min operations on an IFM and discussed the convergence of transitive IFM. Pal and Khan [16] proposed some operations on the IFM. Moreover, Khan and Pal [17] have presented the concept of an interval-valued intuitionistic fuzzy matrix (IVIFM). Silambarasan [18] defined the Hamacher operations of IVIFM and proved some important properties.

In the above studies the concept of FM and IFM has been strongly enforced in various areas, yet the concept of neutrality is not considered in FM and IFM. The FM and IFM fail to attain any satisfactory result when the neutral membership degree is calculated independently in real-life problems. After that, Dogra and Pal [19] proposed the picture fuzzy matrix (PFM) using the concept of Cuong and Kreinovich [20] and introduced the method of determinant and adjoint of a PFM. Silambarasan [21] also defined some algebraic operations and properties of the PFM. Khalil et al. [22] worked on new operations on interval-valued picture fuzzy sets. Liu et al. [23] introduced the similarity measures for interval-valued picture fuzzy sets and their applications in decision-making problems. Further, Silambarasan [24] defined a spherical fuzzy matrix (SFM) using the theory of Gundogdu and Kahraman [25] and proposed some important properties and algebraic operations for the SFM. Muthukumaran et al. [26] defined the n-hyperspherical neutrosophic matrices and compared them with the SFM. Gundogdu and Kahraman [25, 27] extended the spherical fuzzy set into the interval-valued spherical fuzzy set. Menekse and Akdag [28] worked on risk analysis of hospital buildings using an interval-valued picture fuzzy set. Otay [29] worked on tech-centre location selection by interval-valued spherical fuzzy AHP-based MULTIMOORA methodology. The present work aims to present the notion of interval-valued spherical fuzzy matrix (IVSFM) and its important features. The key operations are as follows:

(i) We explore the SFM into an IVSFM.
(ii) Based on PFM and SFM, we introduce some important definitions and theorems.
(iii) The method of finding the determinant and adjoint is proposed.
(iv) We propose the score function for the interval-valued spherical fuzzy set with some basic properties.
(v) The real-life application of decision-making is performed by using the proposed score function.
(vi) Some comparative studies are illustrated.

PRELIMINARIES

In this section we discuss some basic definitions of the related work.

Definition 1 [5]**.** The fuzzy matrix A of order $m \times n$ is defined as

$$A = (< a_{ij}, a_{ij\mu} >),$$

where $a_{ij\mu}$ is called the membership degree of a_{ij}, satisfying the condition $0 \le a_{ij\mu} \le 1$.

Definition 2 [14]**.** The intuitionistic fuzzy matrix A of order $m \times n$ is defined as

$$A = (< a_{ij}, (a_{ij\mu}, a_{ij\nu}) >),$$

where $a_{ij\mu}$, $a_{ij\nu}$ are the degree of membership and degree of non-membership value of a_{ij} respectively with condition $0 \le a_{ij\mu} + a_{ij\nu} \le 1$.

Definition 3 [17]**.** An IVIFM A of order $m \times n$ is defined as

$$A = (< a_{ij}, (a_{ij\mu}, a_{ij\nu}) >),$$

where $a_{ij\mu}$ membership degree and $a_{ij\nu}$ non-membership degree are both the subsets of [0,1], which are denoted respectively by $a_{ij\mu}$=$[a_{ij\mu L}, a_{ij\mu U}]$ and $a_{ij\nu}$=$[a_{ij\nu L}, a_{ij\nu U}]$ under the condition $a_{ij\mu U}$+$a_{ij\nu U} \le 1$ for $i = 1,2, \dots\dots m$ and $j = 1,2 \dots\dots n$.

Definition 4 [24]**.** A spherical fuzzy matrix A of order $m \times n$ is defined as

$$A = (< a_{ij}, (a_{ij\mu}, a_{ij\eta}, a_{ij\nu}) >),$$

where $a_{ij\mu} \in [0,1]$, $a_{ij\eta} \in [0,1]$, $a_{ij\nu} \in [0,1]$ are the measure of membership, neutrality and non-membership degree of a_{ij} respectively, and $i = 1,2 \dots\dots m$; $j = 1,2 \dots\dots n$, satisfying $0 \le a_{ij\mu}^2 + a_{ij\eta}^2 + a_{ij\nu}^2 \le 1$.

Definition 5 [27]**.** An interval-valued spherical fuzzy set A on X is defined as

$$A = \{< x, [\mu_A^L(x), \mu_A^U(x)], [\eta_A^L(x), \eta_A^U(x)], [\nu_A^L(x), \nu_A^U(x)] > / [\mu_A^L(x), \mu_A^U(x)], [\eta_A^L(x), \eta_A^U(x)], [\nu_A^L(x), \nu_A^U(x)] \in D[0,1], (\mu_A^U(x))^2 + (\eta_A^U(x))^2 + (\nu_A^U(x))^2 \le 1, x \in X\},$$

where $[\mu_A^L(x), \mu_A^U(x)], [\eta_A^L(x), \eta_A^U(x)], [\nu_A^L(x), \nu_A^U(x)]$ are the membership, neutrality and non - membership degree of A at x respectively.

The interval of the indeterminacy degree relative to A for each $x \in X$ is defined as

$$\pi_A(x) = [\pi^L{}_A(x), \pi^U{}_A(x)] = [\sqrt{1 - \mu_A{}^L(x))^2 - \eta_A{}^L(x))^2 - (\nu_A{}^L(x))^2}, \sqrt{1 - \mu_A{}^U(x))^2 - \eta_A{}^U(x))^2 - (\nu_A{}^U(x))^2}].$$

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INTERVAL-VALUED SPHERICAL FUZZY MATRIX (IVSFM)

In this section we introduce the IVSFM, along with the important definition and theorem for the proposed matrix.

Definition 6. An interval-valued spherical fuzzy matrix A of order $m \times n$ is defined as
$A = (< a_{ij}, (a_{ij\mu}, a_{ij\eta}, a_{ijv}) >)$,
where $a_{ij\mu} = [a_{ij\mu L}, a_{ij\mu U}] \in [0,1]$, $a_{ij\eta} = [a_{ij\eta L}, a_{ij\eta U}] \in [0,1]$,
$a_{ijv} = [a_{ijvL}, a_{ijvU}] \in [0,1]$ with the condition $(a_{ij\mu U})^2 + (a_{ij\eta U})^2 + (a_{ijvU})^2 \le 1$, and $a_{ij\mu}$, $a_{ij\eta}$ and a_{ijv} are the membership, neutrality and non-membership degree of the element a_{ij} respectively.

Definition 7. Let A and B be two IVSFM such that
$A = ([a_{ij\mu L}, a_{ij\mu U}], [a_{ij\eta L}, a_{ij\eta U}], [a_{ijvL}, a_{ijvU}])$,
$B = ([b_{ij\mu L}, b_{ij\mu U}], [b_{ij\eta L}, b_{ij\eta U}], [b_{ijvL}, b_{ijvU}])$.
Then we write $A \le B$ if following case is true:
$a_{ij\mu L} \le b_{ij\mu L}, a_{ij\mu U} \le b_{ij\mu U}; a_{ij\eta L} \le b_{ij\eta L}, a_{ij\eta U} \le b_{ij\eta U}; a_{ijvL} \ge b_{ijvL}, a_{ijvU} \ge b_{ijvU}$.

Definition 8. Let $A = ([a_{ij\mu L}, a_{ij\mu U}], [a_{ij\eta L}, a_{ij\eta U}], [a_{ijvL}, a_{ijvU}])$ and
$B = ([b_{ij\mu L}, b_{ij\mu U}], [b_{ij\eta L}, b_{ij\eta U}], [b_{ijvL}, b_{ijvU}])$ be two IVSFM of order $m \times n$. Then we define the following operators:

(i) $A^c = ([a_{ijvL}, a_{ijvU}], [a_{ij\eta L}, a_{ij\eta U}], [a_{ij\mu L}, a_{ij\mu U}])$
(ii) $A \vee B = ([max(a_{ij\mu L}, b_{ij\mu L}), max(a_{ij\mu U}, b_{ij\mu u})][min(a_{ij\eta L}, b_{ij\eta L}), min(a_{ij\eta u}, b_{ij\eta U})]$ $[min(a_{ijvL}, b_{ijvL}), min(a_{ijvU}, b_{ijvU})])$
(iii) $A \wedge B = ([min(a_{ij\mu L}, b_{ij\mu L}), min(a_{ij\mu U}, b_{ij\mu u})][min(a_{ij\eta L}, b_{ij\eta L}), min(a_{ij\eta u}, b_{ij\eta U})]$ $[max(a_{ijvL}, b_{ijvL}), max(a_{ijvU}, b_{ijvU})])$
(iv) $A^T = ([a_{ji\mu L}, a_{ji\mu U}], [a_{ji\eta L}, a_{ji\eta U}], [a_{jivL}, a_{jivU}])$
(v)

$$A \oplus B = \left\{ \begin{array}{c} [((\mu_{aij\mu L})^2 + (\mu_{bij\mu L})^2 - (\mu_{aij\mu L})^2(\mu_{bij\mu L})^2)^{\frac{1}{2}}, ((\mu_{aij\mu U})^2 + (\mu_{bij\mu U})^2 - (\mu_{aij\mu U})^2(\mu_{bij\mu U})^2)^{\frac{1}{2}}], \\ [\mu_{aijvL}\mu_{bijvL}, \mu_{aijvU}\mu_{bijvU}], \\ \begin{bmatrix} ((1-(\mu_{bij\mu L})^2)(\mu_{aij\eta L})^2 + (1-(\mu_{aij\mu L})^2)(\mu_{bij\eta L})^2 - (\mu_{aij\eta L})^2(\mu_{bij\eta L})^2)^{1/2} \\ ((1-(\mu_{bij\mu U})^2)(\mu_{aij\eta U})^2 + (1-(\mu_{aij\mu U})^2)(\mu_{bij\eta U})^2 - (\mu_{aij\eta U})^2(\mu_{bij\eta U})^2)^{1/2} \end{bmatrix} \end{array} \right\}$$

(vi)

$$A \otimes B = \left\{ \begin{array}{c} [\mu_{aij\mu L}\mu_{bij\mu L}, \mu_{aij\mu U}\mu_{bij\mu U}], [((\mu_{aijvL})^2 + (\mu_{bijvL})^2 - (\mu_{aijvL})^2(\mu_{bijvL})^2)^{\frac{1}{2}}, \\ ((\mu_{aijvU})^2 + (\mu_{bijvU})^2 - (\mu_{aijvU})^2(\mu_{bijvU})^2)^{1/2}], \\ \begin{bmatrix} ((1-(\mu_{bijvL})^2)(\mu_{aij\eta L})^2 + (1-(\mu_{aijvL})^2)(\mu_{bij\eta L})^2 - (\mu_{aij\eta L})^2(\mu_{bij\eta L})^2)^{1/2} \\ ((1-(\mu_{bijvU})^2)(\mu_{aij\eta U})^2 + (1-(\mu_{aijvU})^2)(\mu_{bij\eta U})^2 - (\mu_{aij\eta U})^2(\mu_{bij\eta U})^2)^{1/2} \end{bmatrix} \end{array} \right\}$$

(vii)

$$k.A = \begin{Bmatrix} [(1-(1-(\mu_{aij\mu L})^2)^k)^{1/2}, (1-(1-(\mu_{aij\mu U})^2)^k)^{1/2}], [(\mu_{aijvL})^k, (\mu_{aijvU})^k], \\ [((1-(\mu_{aij\mu L})^2)^k - (1-(\mu_{aij\mu L})^2-(\mu_{aij\eta L})^2)^k)^{1/2}, \\ ((1-(\mu_{aij\mu U})^2)^k - (1-(\mu_{aij\mu U})^2-(\mu_{aij\eta U})^2)^k)^{1/2}] \end{Bmatrix}$$

(viii)

$$A^k = \begin{Bmatrix} [(\mu_{aij\mu L})^k, (\mu_{aij\mu U})^k], [(1-(1-(\mu_{aijvL})^2)^k)^{1/2}, (1-(1-(\mu_{aijvU})^2)^k)^{1/2}], \\ [((1-(\mu_{aijvL})^2)^k - (1-(\mu_{aijvL})^2-(\mu_{aij\eta L})^2)^k)^{1/2}, \\ ((1-(\mu_{aijvU})^2)^k - (1-(\mu_{aijvU})^2-(\mu_{aij\eta U})^2)^k)^{1/2}] \end{Bmatrix},$$

where A^c and A^T are the complement and transpose of A respectively, and k is any scalar value.

Theorem 1. Let A, B be two IVSFM of order $m \times n$. Then
(i) $(A \vee B)^c = A^c \wedge B^c$
(ii) $(A \wedge B)^c = A^c \vee B^c$.

Proof: (i) Let $A = ([a_{ij\mu L}, a_{ij\mu U}], [a_{ij\eta L}, a_{ij\eta U}], [a_{ijvL}, a_{ijvU}])$,
$B = ([b_{ij\mu L}, b_{ij\mu U}], [b_{ij\eta L}, b_{ij\eta U}], [b_{ijvL}, b_{ijvU}])$,
$A^c = (< [a_{ijvL}, a_{ijvU}], [a_{ij\eta L}, a_{ij\eta U}], [a_{ij\mu L}, a_{ij\mu U}] >)$,
$B^c = (< [b_{ijvL}, b_{ijvU}], [b_{ij\eta L}, b_{ij\eta U}], [b_{ij\mu L}, b_{ij\mu U}] >)$. Then
$A^c \wedge B^c = ([min(a_{ijvL}, b_{ijvL}), min(a_{ijvU}, b_{ijvU})][min(a_{ij\eta L}, b_{ij\eta L}), min(a_{ij\eta U}, b_{ij\eta U})]$
$[max(a_{ij\mu L}, b_{ij\mu L}), max(a_{ij\mu U}, b_{ij\mu U})])$,
$A \vee B$=$([max(a_{ijvL}, b_{ijvL}), max(a_{ijvU}, b_{ijvU})][min(a_{ij\eta L}, b_{ij\eta L}), min(a_{ij\eta U}, b_{ij\eta U})]$
$[min(a_{ij\mu L}, b_{ij\mu L}), min(a_{ij\mu U}, b_{ij\mu U})])$,
$(A \vee B)^c$=$([min(a_{ijvL}, b_{ijvL}), min(a_{ijvU}, b_{ijvU})][min(a_{ij\eta L}, b_{ij\eta L}), min(a_{ij\eta U}, b_{ij\eta U})]$
$[max(a_{ij\mu L}, b_{ij\mu L}), max(a_{ij\mu U}, b_{ij\mu U})])$
= $A^c \wedge B^c$.

The proof of part (ii) can be done on similar lines.

Theorem 2. Suppose A, B and C are three IVSFM and $A \leq C$ and $B \leq C$. Then $A \vee B \leq C$.

Proof: Let $A = ([a_{ij\mu L}, a_{ij\mu U}], [a_{ij\eta L}, a_{ij\eta U}], [a_{ijvL}, a_{ijvU}])$,
$B = ([b_{ij\mu L}, b_{ij\mu U}], [b_{ij\eta L}, b_{ij\eta U}], [b_{ijvL}, b_{ijvU}])$,
$C = ([c_{ij\mu L}, c_{ij\mu U}], [c_{ij\eta L}, c_{ij\eta U}], [c_{ijvL}, c_{ijvU}])$.
If $A \leq C$, then $a_{ij\mu L} \leq c_{ij\mu L}, a_{ij\mu U} \leq c_{ij\mu U}, a_{ij\eta L} \leq c_{ij\eta L}, a_{ij\eta U} \leq c_{ij\eta U}, a_{ijvL} \geq c_{ijvL}, a_{ijvU} \geq c_{ijvU}$ for all i,j.
And if $B \leq C$, then $b_{ij\mu L} \leq c_{ij\mu L}, b_{ij\mu U} \leq c_{ij\mu U}, b_{ij\eta L} \leq c_{ij\eta L}, b_{ij\eta U} \leq c_{ij\eta U}, b_{ijvL} \geq c_{ijvL}, b_{ijvU} \geq c_{ijvU}$ for all i,j.
Now max $(a_{ij\mu L}, b_{ij\mu L}) \leq c_{ij\mu L}$, max $(a_{ij\mu U}, b_{ij\mu U}) \leq c_{ij\mu U}$,
min $(a_{ij\eta L}, b_{ij\eta L}) \leq c_{ij\eta L}$, min $(a_{ij\eta U}, b_{ij\eta U}) \leq c_{ij\eta U}$,
min $(a_{ijvL}, b_{ijvL}) \geq c_{ijvL}$, min $(a_{ijvU}, b_{ijvU}) \geq c_{ijvU}$.
Thus, $A \vee B \leq C$ using Definition 7.

Theorem 3. Suppose A, B and C are three IVSFM and $A \leq B$. Then $A \vee C \leq B \vee C$.

Proof: Let $A = (\langle a_{ij\mu L}, a_{ij\mu U}\rangle, \langle a_{ij\eta L}, a_{ij\eta U}\rangle, \langle a_{ijvL}, a_{ijvU}\rangle)$,
$B = (\langle b_{ij\mu L}, b_{ij\mu U}\rangle, \langle b_{ij\eta L}, b_{ij\eta U}\rangle, \langle b_{ijvL}, b_{ijvU}\rangle)$ and
$C = (\langle c_{ij\mu L}, c_{ij\mu U}\rangle, \langle c_{ij\eta L}, c_{ij\eta U}\rangle, \langle c_{ijvL}, c_{ijvU}\rangle)$ be three IVSFM of the same order $m \times n$.
If $A \leq B$, then $a_{ij\mu L} \leq b_{ij\mu L}, a_{ij\mu U} \leq b_{ij\mu U}, a_{ij\eta L} \leq b_{ij\eta L}, a_{ij\eta U} \leq b_{ij\eta U}, a_{ijvL} \geq b_{ijvL}, a_{ijvU} \geq b_{ijvU}$. Now $\max(a_{ij\mu L}, c_{ij\mu L}) \leq \max(b_{ij\mu L}, c_{ij\mu L})$, $\max(a_{ij\mu U}, c_{ij\mu U}) \leq \max(b_{ij\mu U}, c_{ij\mu U})$,
$\min(a_{ij\eta L}, c_{ij\eta L}) \leq \min(b_{ij\eta L}, c_{ij\eta L})$, $\min(a_{ij\eta U}, c_{ij\eta U}) \leq \min(b_{ij\eta U}, c_{ij\eta U})$,
$\min(a_{ijvL}, c_{ijvL}) \geq \min(b_{ijvL}, c_{ijvL})$, $\min(a_{ijvU}, c_{ijvU}) \geq \min(b_{ijvU}, c_{ijvU})$ for all *i, j*.
Therefore, $A \vee C \leq B \vee C$.

Theorem 4. Let *A*, *B* and *C* be three IVSFM of the same order, and $C \leq A$ and $C \leq B$. Then $C \leq A \wedge B$.

Proof: Proof of the above result directly follows from Theorem 3.

Theorem 5. Suppose *A*, *B* and *C* are three IVSFM of the same order, and if $A \leq B$, $A \leq C$ and $B \wedge C = 0$, then $A = 0$.

Proof: The proof directly follows from Definition 7 and Theorem 4.

Theorem 6. Let *A*, *B* and *C* be three IVSFM of the same order of $A \leq B$. Then $A \wedge C \leq B \wedge C$.

Proof: If $A \leq B$, then
$a_{ij\mu L} \leq b_{ij\mu L}, a_{ij\mu U} \leq b_{ij\mu U}, a_{ij\eta L} \leq b_{ij\eta L}, a_{ij\eta U} \leq b_{ij\eta U}, a_{ijvL} \geq b_{ijvL}, a_{ijvU} \geq b_{ijvU}$.
Now $\min[a_{ij\mu L}, c_{ij\mu L}] \leq \min[b_{ij\mu L}, c_{ij\mu L}]$, $\min(a_{ij\mu U}, c_{ij\mu U}) \leq \min(b_{ij\mu U}, c_{ij\mu U})$,
$\min(a_{ij\eta L}, c_{ij\eta L}) \leq \min(b_{ij\eta L}, c_{ij\eta L})$, $\min(a_{ij\eta U}, c_{ij\eta U}) \leq \min(b_{ij\eta U}, c_{ij\eta U})$,
$\max(a_{ijvL}, c_{ijvL}) \geq \max(b_{ijvL}, c_{ijvL})$, $\max(a_{ijvU}, c_{ijvU}) \geq \max(b_{ijvU}, c_{ijvU})$ for all *i* , *j*.
So $A \wedge C \leq B \wedge C$.

Theorem 7. Let *A*, *B* and *C* be three IVSFM of the same order, and if $A \leq B$ and $B \wedge C = 0$, then $A \wedge C = 0$.

Proof: By Theorem 6 the proof is straightforward.

DETERMINANT AND ADJOINT OF IVSFM

In this section we define determinant and adjoint of the IVSFM.

Definition 9. Consider the IVSFM $A = ([a_{ij\mu L}, a_{ij\mu U}], [a_{ij\eta L}, a_{ij\eta U}], [a_{ijvL}, a_{ijvU}])$ of order *k*. Then the determinant of *A* is denoted by $|A|$ and define by

$$|A| = \begin{pmatrix} \vee_{h\in H_k} ([a_{1h(1)\mu L}, a_{1h(1)\mu U}] \wedge [a_{2h(2)\mu L}, a_{2h(2)\mu U}] \ldots \wedge [a_{kh(k)\mu L}, a_{kh(k)\mu U}]), \\ \wedge_{h\in H_k} ([a_{1h(1)\eta L}, a_{1h(1)\eta U}] \wedge [a_{2h(2)\eta L}, a_{2h(2)\eta U}] \ldots \wedge [a_{kh(k)\eta L}, a_{kh(k)\eta U}]), \\ \wedge_{h\in H_k} ([a_{1h(1)vL}, a_{1h(1)vU}] \vee [a_{2h(2)vL}, a_{2h(2)vU}] \ldots \vee [a_{kh(k)vL}, a_{kh(k)vU}]), \end{pmatrix},$$

where H_k denotes the symmetric group of all permutations on the indices $\{1, 2, 3 \ldots\ldots k\}$.

Example 1. Consider IVSFM of order 3:

$$A = \begin{pmatrix} [0.85,0.95][0.10,0.15][0.05,0.15] & [0.55,0.65][0.25,0.30][0.25,0.30] & [0.13,0.19][0.69,0.79][0.22,0.27] \\ [0.75,0.85][0.15,0.20][0.15,0.20] & [0.47,0.61][0.33,0.41][0.27,0.36] & [0.31,0.42][0.43,0.52][0.30,0.36] \\ [0.56,0.75][0.20,0.25][0.20,0.25] & [0.7,0.88][0.15,0.22][0.10,0.22] & [0.22,0.31][0.53,0.63][0.32,0.38] \end{pmatrix}.$$

To find the determinant of A, we need to find out all the permutations on $\{1,2,3\}$. The permutations on $\{1,2,3\}$ are

$\psi_1 = \begin{pmatrix} 1\,2\,3 \\ 1\,2\,3 \end{pmatrix}$, $\psi_2 = \begin{pmatrix} 1\,2\,3 \\ 1\,3\,2 \end{pmatrix}$, $\psi_3 = \begin{pmatrix} 1\,2\,3 \\ 2\,1\,3 \end{pmatrix}$ $\psi_4 = \begin{pmatrix} 1\,2\,3 \\ 2\,3\,1 \end{pmatrix}$, $\psi_5 = \begin{pmatrix} 1\,2\,3 \\ 3\,1\,2 \end{pmatrix}$, $\psi_6 = \begin{pmatrix} 1\,2\,3 \\ 3\,2\,1 \end{pmatrix}$.

The membership degree of $|A|$ is

$$
\begin{aligned}
&([a_{1\psi_1(1)\mu L}, a_{1\psi_1(1)\mu U}] \wedge [a_{2\psi_1(2)\mu L}, a_{2\psi_1(2)\mu U}] \wedge [a_{3\psi_1(3)\mu L}, a_{3\psi_1(3)\mu U}]) \\
&\vee ([a_{1\psi_2(1)\mu L}, a_{1\psi_2(1)\mu U}] \wedge [a_{2\psi_2(2)\mu L}, a_{2\psi_2(2)\mu U}] \wedge [a_{3\psi_2(3)\mu L}, a_{3\psi_2(3)\mu U}]) \\
&\vee ([a_{1\psi_3(1)\mu L}, a_{1\psi_3(1)\mu U}] \wedge [a_{2\psi_3(2)\mu L}, a_{2\psi_3(2)\mu U}] \wedge [a_{3\psi_3(3)\mu L}, a_{3\psi_3(3)\mu U}]) \\
&\vee ([a_{1\psi_4(1)\mu L}, a_{1\psi_4(1)\mu U}] \wedge [a_{2\psi_4(2)\mu L}, a_{2\psi_4(2)\mu U}] \wedge [a_{3\psi_4(3)\mu L}, a_{3\psi_4(3)\mu U}]) \\
&\vee ([a_{1\psi_5(1)\mu L}, a_{1\psi_5(1)\mu U}] \wedge [a_{2\psi_5(2)\mu L}, a_{2\psi_5(2)\mu U}] \wedge [a_{3\psi_5(3)\mu L}, a_{3\psi_5(3)\mu U}]) \\
&\vee ([a_{1\psi_6(1)\mu L}, a_{1\psi_6(1)\mu U}] \wedge [a_{2\psi_6(2)\mu L}, a_{2\psi_6(2)\mu U}] \wedge [a_{3\psi_6(3)\mu L}, a_{3\psi_6(3)\mu U}])
\end{aligned}
$$

$$
\begin{aligned}
&= ([a_{11\mu L},\ a_{11\mu U}] \wedge [a_{22\mu L},\ a_{22\mu U}] \wedge [a_{33\mu L},\ a_{33\mu U}]) \\
&\vee ([a_{11\mu L},\ a_{11\mu U}] \wedge [a_{23\mu L}, a_{23\mu U}] \wedge [a_{32\mu L},\ a_{32\mu U}]) \\
&\vee ([a_{12\mu L},\ a_{12\mu U}] \wedge [a_{21\mu L},\ a_{21\mu U}] \wedge [a_{33\mu L},\ a_{33\mu U}]) \\
&\vee ([a_{12\mu L},\ a_{12\mu U}] \wedge [a_{23\mu L},\ a_{23\mu U}] \wedge [a_{31\mu L},\ a_{31\mu U}]) \\
&\vee ([a_{13\mu L},\ a_{13\mu U}] \wedge [a_{21\mu L},\ a_{21\mu U}] \wedge [a_{32\mu L},\ a_{32\mu U}]) \\
&\vee ([a_{13\mu L},\ a_{13\mu U}] \wedge [a_{22\mu L}, a_{22\mu U}] \wedge [a_{31\mu L}, a_{31\mu U}])
\end{aligned}
$$

$$
\begin{aligned}
&= ([0.85, 0.95] \wedge [0.47, 0.61] \wedge [0.22, 0.31]) \\
&\vee ([0.85, 0.95] \wedge [0.31, 0.42] \wedge [0.7, 0.88]) \\
&\vee ([0.55, 0.65] \wedge [0.75, 0.85] \wedge [0.22, 0.31]) \\
&\vee ([0.55, 0.65] \wedge [0.31, 0.42] \wedge [0.65, 0.75]) \\
&\vee ([0.13, 0.19] \wedge [0.75, 0.85] \wedge [0.7, 0.88]) \\
&\vee ([0.13, 0.19] \wedge [0.47, 0.61] \wedge [0.65, 0.75])
\end{aligned}
$$

$$
\begin{aligned}
&= [0.22, 0.31] \vee [0.31, 0.42] \vee [0.22, 0.31] \vee [0.31, 0.42] \vee [0.7, 0.19] \vee [0.13, 0.19] \\
&= [0.31, 0.42].
\end{aligned}
$$

Similarly, the neutrality degree of $|A|$ is

$$
\begin{aligned}
&([0.10, 0.15] \wedge [0.33, 0.41] \wedge [0.53, 0.63]) \\
&\wedge ([0.10, 0.15] \wedge [0.43, 0.52] \wedge [0.15, 0.22]) \\
&\wedge ([0.25, 0.30] \wedge [0.15, 0.20] \wedge [0.53, 0.63]) \\
&\wedge ([0.25, 0.30] \wedge [0.43, 0.52] \wedge [0.20, 0.25]) \\
&\wedge ([0.69, 0.79] \wedge [0.15, 0.20] \wedge [0.15, 0.22]) \\
&\wedge ([0.69, 0.79] \wedge [0.33, 0.41] \wedge [0.20, 0.25])
\end{aligned}
$$

$=[0.10, 0.15] \wedge [0.10, 0.15] \wedge [0.15, 0.20] \wedge [0.20, 0.25] \wedge [0.15, 0.22] \wedge [0.20, 0.25]$
$= [0.10, 0.15]$.

Now the non-membership degree of $|A|$ is

$$
\begin{aligned}
&([0.05, 0.15] \vee [0.27, 0.36] \vee [0.32, 0.38]) \\
&\wedge ([0.05, 0.15] \vee [0.30, 0.35] \vee [0.10, 0.22]) \\
&\wedge ([0.25, 0.30] \vee [0.15, 0.20] \vee [0.32, 0.38]) \\
&\wedge ([0.25, 0.30] \vee [0.30, 0.36] \vee [0.20, 0.25]) \\
&\wedge ([0.22, 0.27] \vee [0.15, 0.20] \vee [0.10, 0.22]) \\
&\wedge ([0.22, 0.27] \vee [0.27, 0.36] \vee [0.20, 0.25]).
\end{aligned}
$$

$= [0.32, 0.38] \vee [0.30, 0.36] \vee [0.32, 0.38] \vee [0.30, 0.36] \vee [0.22, 0.27] \vee [0.27, 0.36]$
$= [0.22, 0.27]$.

$|A|$= ([0.31, 0.42] [0.10, 0.15] [0.22, 0.27]).

Definition 10. Suppose IVSFM $A = ([a_{ij\mu L} a_{ij\mu U}], [a_{ij\eta L}, a_{ij\eta U}], [a_{ijvL}, a_{ijvU}])$ of order x. Then the adjoint of A is denoted by $Adj.(A)$ and defined by

Q= $([q_{ij\mu}, q_{ij\eta}, q_{ijv}])$= $Adj(A)$,

where $q_{ij\mu}=\vee_{\delta\in S_{x_jx_i}}\wedge_{u\in x_j} a_{u\delta(u)\mu}$,

$q_{ij\eta}=\wedge_{\delta\in S_{x_jx_i}}\wedge_{u\in x_j} a_{u\delta(u)\eta}$,

$q_{ijv}=\wedge_{\delta\in S_{x_jx_i}}\vee_{u\in x_j} a_{u\delta(u)v}$.

Here x_j= {1,2........x}-{j} and $S_{x_jx_i}$ represents the set of all permutations of set x_j over set x_i.

Example 2. Consider IVSFM of order 3:

$$A = \begin{pmatrix} [0.85,0.95][0.10,0.15][0.05,0.15] & [0.55,0.65][0.25,0.30][0.25,0.30] & [0.13,0.19][0.69,0.79][0.22,0.27] \\ [0.75,0.85][0.15,0.20][0.15,0.20] & [0.47,0.61][0.33,0.41][0.27,0.36] & [0.31,0.42][0.43,0.52][0.30,0.36] \\ [0.56,0.75][0.20,0.25][0.20,0.25] & [0.7,0.88][0.15,0.22][0.10,0.22] & [0.22,0.31][0.53,0.63][0.32,0.38] \end{pmatrix}.$$

For j=1 and i=1, x_j= {1,2,3}-{1}= {2,3} and x_i= {1,2,3}-{1}= {2,3}. The permutations of x_i over x_j are

$$\begin{pmatrix} 2 & 3 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}.$$

Now

$(a_{22\mu} \wedge a_{33\mu}) \vee (a_{23\mu} \wedge a_{32\mu})$

$= ([0.47,0.61] \wedge [0.22,0.31]) \vee ([0.31,0.42] \wedge [0.7,0.88])$

$= [0.22,0.31] \vee [0.31,0.42]$ =[0.31, 0.42],

$(a_{22\eta} \wedge a_{33\eta}) \wedge (a_{23\eta} \wedge a_{32\eta})$

$= ([0.33,0.41] \wedge [0.53,0.63]) \wedge ([0.43,0.52] \wedge [0.15,0.22])$

$= [0.33,0.41] \wedge [0.15,0.22]$ =[0.15, 0.22],

and $(a_{22v} \vee a_{33v}) \wedge (a_{23v} \vee a_{32v})$

$= ([0.27,0.36] \vee [0.32,0.38]) \wedge ([0.30,0.36] \vee [0.10,0.22])$

$= [0.32,0.38] \wedge [0.30,0.36]$ = [0.30, 0.36].

Similarly, we can find other values of $Adj.(A)$. Thus,

$$Adj.A = \begin{pmatrix} [0.31,0.42][0.15,0.22][0.30,0.36] & [0.22,0.31][0.15,0.22][0.22,0.27] & [0.31,0.42][0.25,0.30][0.27,0.36] \\ [0.31,0.42][0.15,0.20][0.30,0.36] & [0.22,0.31][0.10,0.15][0.22,0.27] & [0.31,0.42][0.10,0.15][0.20,0.25] \\ [0.7,0.85][0.15,0.20][0.15,0.22] & [0.55,0.65][0.10,0.15][0.10,0.22] & [0.55,0.65][0.10,0.15][0.25,0.30] \end{pmatrix}.$$

APPLICATION OF IVSFM IN DECISION-MAKING

In this section we propose the score function and discuss the application of IVSFM in decision-making.

Definition 11. Let N= $([\alpha,\beta],[\gamma,\delta],[\tau,\theta])$ be an interval-valued spherical fuzzy number. Then the score function for interval-valued spherical fuzzy number is

$$E(N) = \frac{1+\alpha^2+\beta^2-\gamma^2-\delta^2-(\frac{\tau}{2})^2-(\frac{\theta}{2})^2}{6}\left|\alpha+\beta-\gamma-\delta-\frac{\tau}{2}-\frac{\theta}{2}\right| \in [-1,1],$$

with accuracy function $F(N) = \frac{\alpha^2+\beta^2+\gamma^2+\delta^2+\tau^2+\theta^2}{2} \in [0,1]$.

For any two interval-valued spherical fuzzy numbers N_1 and N_2,

i. If $E(N_1) > E(N_2)$, then $N_1 > N_2$

ii. If $E(N_1) < E(N_2)$, then $N_1 < N_2$

iii. If $E(N_1) = E(N_2)$, then

(a) If $F(N_1) > F(N_2)$, then $N_1 > N_2$

(b) If $F(N_1) < F(N_2)$, then $N_1 < N_2$

(c) If $F(N_1) = F(N_2)$, then $N_1 = N_2$.

One can verify the following properties for the proposed score function:

Property 1. Let $N = ([1,1],[0,0][0,0])$ be an interval-valued spherical fuzzy number; then $E(N) = 1$.

Property 2. Let $N = ([0,0],[0,0][0,0])$ be an interval-valued spherical fuzzy number; then $E(N) = 0$.

Problem in Career Placement Assessment

The following is how we localise the interval-valued spherical fuzzy relation concept. Let $V=\{v_1, v_2, v_3, v_4 \ldots\ldots v_i\}$, $U=\{u_1, u_2, u_3, u_4 \ldots\ldots u_i\}$ and $W=\{w_1, w_2, w_3, w_4 \ldots\ldots w_i\}$ be the set of subjects related to courses, finite set of courses and finite set of applicants respectively. Suppose we have the relations $R_1(W \rightarrow V)$ and $R_2(W \rightarrow V)$ such that

$$R_1 = \{((w,v), \mu_{R_1}(w,v), \nu_{R_1}(w,v), \eta_{R_1}(w,v)) | (w,v) \in (W \times V)\},$$

$$R_2 = \{((v,u), \mu_{R_2}(v,u), \nu_{R_2}(v,u), \eta_{R_2}(v,u)) | (v,u) \in (V \times U)\},$$

where $\mu_{R_1}(w,v)$ represents the degree with which the applicant, *w*, passes the related subject requirement, v; $\nu_{R_1}(w,v)$ represents the degree with which the applicant, w, does not pass the related subject requirement; and $\eta_{R_1}(w,v)$ represents the degree with which the applicant may pass or may not pass the related subject requirement. Similarly, $\mu_{R_2}(v,u)$ represents the degree with which the related subject requirement, v, determines the courses u; $\nu_{R_2}(v,u)$ represents the degree with which the related subject requirement, v, does not determine the course, u; and $\eta_{R_2}(v,u)$ represents the degree with which the related subject requirement, v, may or may not determine the course. The composition, *T*, of R_1 and R_2 is given as $T = R_1 \odot R_2$. This describes the state in which the applicants, w_i, with respect to the related subjects requirement, v_j, fit the courses, u_k. Thus,

$$\mu_T(w_i, u_k) = \bigvee_{v_j \in V} \{\mu_{R_1}(w_i, v_j) \wedge \mu_{R_2}(v_j, u_k)\},$$

$$\nu_T(w_i, u_k) = \bigwedge_{v_j \in V} \{\nu_{R_1}(w_i, v_j) \vee \nu_{R_2}(v_j, u_k)\},$$

$$\eta_T(w_i, u_k) = \bigwedge_{v_j \in V} \{\eta_{R_1}(w_i, v_j) \wedge \eta_{R_2}(v_j, u_k)\}$$

for all $w_i \in W$ and $u_k \in U$ where *i, j* and *k* take value from 1,2...n. The values of $\mu_{R_1 \odot R_2}(w_i, u_k)$, $\nu_{R_1 \odot R_2}(w_i, u_k)$ and $\eta_{R_1 \odot R_2}(w_i, u_k)$ of the composition $T = R_1 \odot R_2$ are as follows: If the value of T is maximised, the career placement can be achieved. This value is computed from R_1 and R_2 for the placement of w_i into any u_k relative to v_j, and it is maximised by using the proposed score function (Definition 11).

Example 3. Let $W = \{Tom, Ankit, Sahil, Aashima\}$ be a set of applicants for the course placements; $V = \{English\ Lang., Maths, Biology, Physics, Chemistry\}$ be a set of related subject requirements for the set of courses, and $U = \{medicine, pharmacy, surgery, anatomy\}$ be a set of courses. The following results are then obtained:

• A hypothetical relation R_1 $(W \rightarrow V)$ is given in Table 1;

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• A hypothetical relation R_2 ($V \to U$) is given in Table 2;
• The composition relation T ($W \to U$) = $T = R_1 \odot R_2$ is given in Table 3;
• The degree of affiliation between the set of applicants W_i to the set of courses U_i is calculated in Table 4 by using the proposed score function (Definition 11).

Table 1 shows the relation between W and course placement V in the form of interval-valued spherical fuzzy number. It defines how strongly the set of applicants and that of related subjects are related. Table 2 shows the relation between the set of related subjects V and that of courses U.

Table 1. Hypothetical relation R_1 ($W \to V$)

W/V	English Lang.	Mathematics	Biology	Physics	Chemistry
Tom	[0.85,0.93] [0.10,0.13] [0.05,0.15]	[0.20,0.25] [0.65,0.75] [0.20,0.25]	[0.6,0.7] [0.1,0.2][0.4,0.5]	[0.7,0.8] [0.2,0.3][0.3,0.4]	[0.21,0.26] [0.64,0.74] [0.21,0.26]
Ankit	[0.55,0.65] [0.25,0.30] [0.25,0.30]	[0.85,0.9] [0.1,0.15] [0.07,0.17]	[0.3,0.6][0.2,0.3][0.3,0.4]	[0.5,0.6][0.1,0.2][0.3,0.4]	[0.76,0.88] [0.15,0.20] [0.14,0.21]
Sahil	[0.65,0.75] [0.2,0.25] [0.25,0.30]	[0.10,0.15] [0.8,0.9] [0.08,0.15]	[0.4,0.5][0.8,0.9][0.6,0.7]	[0.6,0.8][0.3,0.4][0.2,0.4]	[0.82,0.93] [0.12,0.17] [0.09,0.17]
Aashima	[0.55,0.65] [0.25,0.30] [0.25,0.30]	[0.15,0.19] [0.7,0.8] [0.13,0.20]	[0.8,0.9][0.2,0.3][0.1,0.2]	[0.68,0.79] [0.19,0.24] [0.19,0.24]	[0.65,0.75] [0.20,0.25] [0.20,0.25]

Table 2. Hypothetical relation R_2 ($V \to U$)

V/U	Medicine	Pharmacy	Surgery	Anatomy
English Lang.	[0.31,0.42][0.15,0.22] [0.30,0.36]	[0.55,0.65][0.10,0.15] [0.25,0.30]	[0.56,0.66] [0.29,0.36] [0.24,0.30]	[0.44,0.51][0.43,0.52] [0.26,.034]
Mathematics	[0.22,0.31][0.10,0.15] [0.10,0.15]	[0.31,0.42][0.15,0.20] [0.3,0.36]	[0.59,0.68] [0.27,0.33] [0.25,0.32]	[0.54,0.62][0.33,0.41] [0.26,0.33]
Biology	[0.22,0.31][0.15,0.22] [0.22,0.27]	[0.7,0.85][0.15,0.20] [0.15,0.22]	[0.55,0.63] [0.32,0.31] [0.25,0.32]	[0.48,0.57][0.37,0.45] [0.24,0.30]
Physics	[0.31,0.42] [0.25,0.30] [0.27,0.36]	[0.58,0.67][0.3,0.37] [0.25,0.32]	[0.6,0.8] [0.29,0.36] [0.24,0.30]	[0.85,.93][.10,.13] [0.05,0.15]
Chemistry	[0.7,0.85][0.15,0.20] [0.15,0.20]	[0.19,0.24][0.67,0.77] [0.19,0.24]	[0.19,0.24] [0.67,0.77] [0.19,0.24]	[0.65,0.75][0.20,0.25] [0.25,0.30]

Table 3. Composition relation *T*: $(W \rightarrow U) = R_1 \odot R_2$

W/U	Medicine	Pharmacy	Surgery	Anatomy
Tom	[0.31,0.42][0.10,0.13] [0.20,0.25]	[0.7,0.85][0.1,0.15] [0.1,0.15]	[0.7,0.85][0.10,0.15] [0.10,0.15]	[0.65,0.75] [0.10,0.15] [0.13,0.20]
Ankit	[0.58,0.67][0.1,0.13] [0.4,0.5]	[0.55,0.65][0.1,0.15] [0.3,0.4]	[.58,.67][.1,.15] [.19,.24]	[0.7,0.85][0.10,0.15] [0.15,0.22]
Sahil	[0.6,.8][0.1,0.13] [.21,.26]	[0.59,0.68][0.1,0.15] [0.19,0.24]	[0.6,0.8][0.12,0.17] [0.19,0.24]	[0.55,0.65][0.19,0.24] [0.2,0.25]
Aashima	[0.7,0.8] [0.1,0.13] [0.25,0.3]	[0.65,0.75][0.1,0.15] [0.25,0.3]	[0.6,0.8][0.1,0.3] [0.2,0.4]	[0.68,0.79][0.2,0.25] [0.19,0.24]

Now to calculate degree of closeness value (Table 4), we use the score function (Definition 11) for finding the weights (score value) of Table 3:

$$E(N) = \frac{1+\alpha^2+\beta^2-\gamma^2-\delta^2-(\frac{\tau}{2})^2-(\frac{\theta}{2})^2}{6}\left|\alpha+\beta-\gamma-\delta-\frac{\tau}{2}-\frac{\theta}{2}\right|.$$

Suppose N_1= [0.31, 0.42] [0.10,0.13] [0.20,0.25]; here α=0.31, β=0.42, γ=0.10, δ=0.13, τ =0.20, θ=0.25. Then E (N_1) = 0.0681. Also, suppose N_2= [0.7,0.85] [0.1,0.15] [0.1,0.15]; here α=0.7, β=0.85, γ=0.1, δ=0.15, τ =0.1, θ=0.15. Then E (N_2) = 0.425. Similarly, we can calculate other weights (score values) for Table 3.

Table 4. Degree of closeness value

W/U	Medicine	Pharmacy	Surgery	Anatomy
Tom	0.0681	**0.425**	**0.425**	0.1335
Ankit	0.1573	0.163	0.226	**0.401**
Sahil	**0.3031**	0.2354	0.2883	0.145
Aashima	**0.3424**	0.2791	0.2788	0.2630

According to the analysis of score value, i.e. degree of closeness value, given in Table 4, Tom is suited to studying either medicine or surgery and Ankit is suited to studying anatomy. Sahil is suited to studying medicine while Aashima is suited to studying only medicine.

COMPARATIVE STUDY AND ADVANTAGES

Ejegwa [30] worked on the Pythagorean fuzzy set and its application in career placements based on academic performance using max–min–max composition. The information was taken in a Pythagorean fuzzy sense in previous papers on Pythagorean fuzzy decision-making. When there are additional kinds of uncertainty in data, present strategies are ineffective in dealing with them. In these instances data should be gathered or displayed in the form of an interval-valued spherical fuzzy meaning. In such instances the currently developed process plays an important role in a successful conclusion.

Silambarasan [24] worked on the SFM, in which the membership, neutrality and non-membership degree have a point. However, in some cases it is difficult to measure the degree of membership, neutrality and non-membership values as a point. In those cases we consider the membership, neutrality and non-membership values as an interval, and it is practically useful in the case of real-life problems. So our study is an extension of the study of Silambarasan [24].

However, in the current technique the matrix entries that are considered are interval-valued spherical fuzzy values. Over a set of universes, IVSFM is extracted from them. The final matrix is then calculated using the proposed score function between two IVSFM, yielding a decision. It is not necessary to perform many different sorts of computation to implement the steps of this approach. As a result, developing algorithms and computer programming for this method is relatively simple. Furthermore, the data points used here are capable of tolerating a wider range of information ambiguity. This study can be thought of as one in an advanced spherical fuzzy sense because the interval-valued spherical fuzzy concept is a generalisation of the spherical fuzzy concept. The use of IVSFM has the following advantages:

i. In the case that information cannot be fully captured by the existing matrices due to each of their flaws, the void is filled by the IVSFM.

ii. The limitation of the FM, IFM, PFM and SFM conditions in the literature at hand is that they prevent experts and decision-makers from assigning membership, neutrality and non-membership degrees by their personal preferences. The membership, neutrality and non-membership degrees can all be described broadly as interval numbers in this work.

iii. The implementation of the IVSFM and the approach suggested for the problems of career placement assessment demonstrate how well and consistently the proposed work addresses the extended framework. The observations indicate that the IVSFM is the most generalised structure among all fuzzy matrix models.

The detailed analysis presented in Table 5 further compares the proposed work with existing research available in the literature.

Table 5. Analysis of proposed work and existing work in literature

Characteristic method	Whether consider membership degree (MD)	Whether consider MD more flexible, i.e IVMD	Whether consider MD or NMD	Whether consider MD or NMD more flexible, i.e IVMD or IVNMD	Whether consider MD, neutrality and NMD degree	Whether consider MD neutrality and NMD degree more Flexible, i.e IVMD, interval-valued neutrality degree and IVNMD
Thomason [5]	✓	X	X	X	X	X
Pal [10]	✓	✓	X	X	X	X
Pal et al. [14]	✓	✓	✓	X	X	X
Khan and Pal [17]	✓	✓	✓	✓	X	X
Dogra and Pal [19]	✓	✓	✓	✓	✓	X
Silambarasan [24]	✓	✓	✓	✓	✓	X
Proposed work	✓	✓	✓	✓	✓	✓

Note: MD=membership degree, IVMD=interval-valued membership degree, NMD=non-membership degree, IVNMD= interval-valued non-membership degree

CONCLUSIONS

We have defined the interval-valued spherical fuzzy matrix. Important definitions and theorems are defined with their proofs. The procedure of determinant and adjoint of the IVSFM is developed. Such investigations can be seen as an extension of studies on spherical fuzzy matrices.

Additionally, we study the application of IVSFM in decision-making processes. A score function is introduced to address decision-making challenges. A limitation of IVSFM is related to the representation of degrees of membership, neutrality and non-membership as interval numbers. The limitation arises when the sum of the upper degree of membership, neutrality and upper degree of non-membership exceeds the interval [0, 1]. Exceeding this interval can lead to inconsistencies in calculations and may affect the accuracy and reliability of the results obtained using IVSFM. Therefore, careful consideration and management of this limitation are necessary to ensure the validity of the analysis conducted using IVSFM.

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