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Full Paper

Multi-attribute decision-making problems based on aggregation operators with complex interval-valued T-spherical fuzzy information

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Abstract: The novel concept of the complex interval-valued T-spherical fuzzy set (CIVTSFS) is presented to handle the vagueness in the data. The proposed set takes the advantages of interval-valued spherical fuzzy sets and complex numbers to represent the information in terms of the interval-valued number of the truth, abstinence and falsity degrees. Various operation laws are stated to enrich the features of CIVTSFS. By utilising the proposed laws, several weighted operators are stated to aggregate the different preference of the experts towards the evaluation of the alternatives. Furthermore, a group decision-making algorithm is stated to solve the decision-making problems by utilising the uncertain data under CIVTSFS information. The presented algorithm is demonstrated through a numerical example and their advantages are indicated.

Keywords: complex T-spherical fuzzy set, aggregation operators, multi-attribute decision-making, interval-valued numbers

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INTRODUCTION

The decision-making procedure is one of the highly significant techniques in handling indefinite and awkward information or knowledge in real issues. In some decision-making procedures, owing to the convolution of the choice problems and the vagueness of the decision situation, it is challenging to convey the appropriate decision information accurately. For this, Zadeh [1] explored the hypothesis of the fuzzy set (FS). A FS incorporates the uncertain knowledge using a membership grade (MG) that measures the extent of how much a specific element belongs to a class in the range of 0 to 1. This versatile framework of FS is accompanied by a non-membership grade (NMG) together with the MG and the frame of Atanassov intuitionistic FS (IFS) [2] is developed where the two facts of the uncertain knowledge are discussed by the NMG and MG under the rule that the total sum of both must lie in the range 0 - 1. Some scholars have utilised the theory of IFS in different areas [3, 4]. The notion of IFS was extended to an interval-valued IFS by Atanassov [5] where the NMG and MG are represented in the form of interval numbers in the range 0 - 1. Some other recent work under uncertain circumstances can be found [6–8].

Yager [9] presented the frame of Pythagorean FS (PyFS) where the basic difference is the flexibility of the allocation of MG and NMG to elements in an uncertain environment. Lately, in an interval-valued PyFS developed and investigated by Garg [10], the MG and NMG have closed subintervals of $[0,1]$ which agrees to allow the square total sum of the upper parts in $[0,1]$. Yager [11] associated the NMG and MG of the pair of information with a varying parameter n and proposed the notion of q-rung orthopair FS (QROFS). This novel framework of QROFS was further extended to the interval-valued QROFS (IVQROFS) by Joshi et al. [12]. The applicability of these existing theories to the decision-making problems was addressed by several researchers [13-15 and references therein].

The above-mentioned theories are effectively applied in different fields but all of them have their limitation in their utilisation domains. As the IFS, PyFS and QROFS deal with only two facts of the opinion, they are unable to deal with more clear information. This led Cuong [16] to introduce the layout of the picture fuzzy set (PFS) which uses MG, NMG, abstinence grade (AG) and refusal grade (RG) to describe the opinion of human beings in the electoral process or any other decision-making process. Cuong also extended the frame of PFS to interval-valued PFS by expressing the NMG, MG abstinence and RG in the form of a closed subinterval of $[0,1]$. Sahu et al. [17] introduced some distance measures based on picture fuzzy and rough set theory and studied their applications in carrier selection of students. Some recent work on PFS and interval-valued PFS can be found [18-21]. Mahmood et al. [22] observed the shortcomings of the frame of PFSs and explored the theory of spherical FS (SFS) that allows the total sum of squares of the MG, NMG and AG in the range of 0 to 1.This structure of SFS was further extended to the idea of T-spherical fuzzy set (TSFS) by introducing a parameter t that allows the decision-makers to choose the values of memberships in the interval [0, 1]. Later on, this idea was generalised to interval-valued TSFS (IVTSFS) [23] that expresses the four types of grades in terms of closed subintervals of $[0,1]$. Some recent work on the SFS, TSFS and interval-valued TSFS can be found [24-30].

The fuzzy layouts discussed so far are taking the MG as the real unit interval, i.e. $[0,1]$. Ramot et al. [31] extended the range for assigning the MG from the real unit interval to the complex plan and by defining the frame of complex FS where the MG is described by a complex number. Alkouri and Salleh [32] associated a complex NMG (CNMG) with a complex MG (CMG) to present the framework of complex IFS. The theories of complex FS and complex IFS were investigated in dealing with problems of multi-attribute decision-making (MADM) by Liu et al. [33] and Rani and Garg [34] respectively. Garg and Rani [35] proposed the complex interval-valued IFS where some aggregation operators (AOs) were developed. Ullah et al. [36] enriched this framework of complex IFS by introducing the notion of complex PyFS where some information measures of complex PyFS were studied in pattern recognition. This notion of complex PyFS was further extended to the idea of complex QROFS (CQROFS) by Liu et al. [37, 38] where the applicability of the AOs of the CQROFSs were discussed in MADM problems. Rani and Garg [39] studied some preference relations based on complex IFSs for MADM problems. Ali et al. [40] investigated some Einstein operators based on complex interval-valued PyFS in the problems of green supplier chain management. Other works on this direction have been explored by various researchers [41-48 and references therein].

Ali et al. [49] investigated the novel concept of complex TSFS (CTSFS) and studied their purposes in MADM. A CTSFS allows the description of an object using CMG, complex AG (CAG), CNMG and complex RG (CRG) with a restriction that for some $n \in \mathbb{Z}^+$, $0 \leq sum(CMG^n, CAG^n, CNMG^n) \leq 1$. Further each of CMG, CAG, CNMG and CRG has further two aspects which are expressed by the amplitude and phase term of the CTSFSs. An interval-valued framework helps to model human opinion in a better way as representing uncertain events with crisp numbers as a replacement for intervals always leads to the loss of information. This is numerically proved by Ullah et al. [23]. Therefore, the present work aims to present the notion of complex IVTSFS (CIVTSFS) where the CMG, CAG, CNMG and CRG are described in the form of complex numbers, the amplitude and phase terms of which represent two aspects of the uncertain information. The key objectives are as follows:

1) To investigate the concept of CIVTSFSs and their fundamental laws;
2) To introduce some weighted operators to aggregate the CIVTSFS information;
3) To present a decision-making algorithm for solving the decision-making problems;
4) To validate the applicability of the work through some numerical examples and compare their performance with the existing studies.

PRELIMINARIES

In this section we aim to recall the notions of FS, complex FS, IVTSFS, and their basic operations. Mahmood et al. [22] initiated the notion of TSFS where the modelling of human opinion is coped with the help of four kinds of membership functions with no restrictions.

Definition 1 [22]. A TSFS has the following shape:

$$\mathcal{C} = \left\{\left(x, \left(\mathrm{m}_{\mathcal{C}}(x), \mathrm{i}_{\mathcal{C}}(x), \mathrm{n}_{\mathcal{C}}(x)\right)\right) : x \in X\right\}$$

where $\mathrm{m}_{\mathcal{C}}, \mathrm{i}_{\mathcal{C}}, \mathrm{n}_{\mathcal{C}}: X \longrightarrow [0,1]$ denotes the MG, AG and NMG of the element $x \in X$ such that $0 \leq \mathrm{m}_{\mathcal{C}}{}^n + \mathrm{i}_{\mathcal{C}}{}^n + \mathrm{n}_{\mathcal{C}}{}^n \leq 1$ for $n \in Z^+$. Further, the term $\hbar_{\mathcal{C}} = \sqrt[n]{1-(\mathrm{m}_{\mathcal{C}}{}^n + \mathrm{i}_{\mathcal{C}}{}^n + \mathrm{n}_{\mathcal{C}}{}^n)}$ is considered as RG. The triplet $\left(\mathrm{m}_{\mathcal{C}}(x), \mathrm{i}_{\mathcal{C}}(x), \mathrm{n}_{\mathcal{C}}(x)\right)$ is named a T-spherical fuzzy number (TSFN).

Remark 1 [22]. The notions of SFS, PFS, QROFS and IFS are the special cases of TSFSs.

Definition 2 [23]. An IVTSFS $\mathcal{C}$ over X is defined as

$$\mathcal{C} = \left\{\left(x, \left(\mathrm{m}_{\mathcal{C}}(x), \mathrm{i}_{\mathcal{C}}(x), \mathrm{n}_{\mathcal{C}}(x)\right)\right) : x \in X\right\}$$

where $\mathfrak{m}_{\mathcal{C}}(x) = [\mathfrak{m}^l_{\mathcal{C}}(x), \mathfrak{m}^u_{\mathcal{C}}(x)]$, $\mathfrak{i}_{\mathcal{C}}(x) = [\mathfrak{i}^l_{\mathcal{C}}(x), \mathfrak{i}^u_{\mathcal{C}}(x)]$ and $\mathfrak{n}_{\mathcal{C}}(x) = [\mathfrak{n}^l_{\mathcal{C}}(x), \mathfrak{n}^u_{\mathcal{C}}(x)]$ are closed subintervals of $[0,1]$ denoting the MG, AG and NMG of the element $x \in X$ such that $0 \leq \mathfrak{m}^{u\,n}_{\mathcal{C}} + \mathfrak{i}^{u\,n}_{\mathcal{C}} + \mathfrak{n}^{u\,n}_{\mathcal{C}} \leq 1$ for $n \in Z^+$. For the term $\hbar_{\mathcal{C}}(x)$, $\hbar_{\mathcal{C}}(x) = [\hbar^l_{\mathcal{C}}(x), \hbar^u_{\mathcal{C}}(x)]$ where $\hbar^u_{\mathcal{C}} = \sqrt[n]{1-(\mathfrak{m}^{u\,n}_{\mathcal{C}} + \mathfrak{i}^{u\,n}_{\mathcal{C}} + \mathfrak{n}^{u\,n}_{\mathcal{C}})}$ and $\hbar^l_{\mathcal{C}} = \sqrt[n]{1-({\mathfrak{m}^l_{\mathcal{C}}}^n + {\mathfrak{i}^l_{\mathcal{C}}}^n + {\mathfrak{n}^l_{\mathcal{C}}}^n)}$.
The term $\mathcal{C} = ([\mathfrak{m}^l_{\mathcal{C}}(x), \mathfrak{m}^u_{\mathcal{C}}(x)], [\mathfrak{i}^l_{\mathcal{C}}(x), \mathfrak{i}^u_{\mathcal{C}}(x)], [\mathfrak{n}^l_{\mathcal{C}}(x), \mathfrak{n}^u_{\mathcal{C}}(x)])$ is considered an interval-valued TSFN (IVTSFN).

For three IVTSFNs $\mathcal{C}$, $\mathcal{C}_1$ and $\mathcal{C}_2$ and real $\lambda > 0$, the basic operations [23] are stated as

1) $\mathcal{C}_1 \oplus \mathcal{C}_2 = \begin{pmatrix} [f_1(\mathfrak{m}^l_{\mathcal{C}_1}, \mathfrak{m}^l_{\mathcal{C}_2}), f_1(\mathfrak{m}^u_{\mathcal{C}_1}, \mathfrak{m}^u_{\mathcal{C}_2})], [g_1(\mathfrak{i}^l_{\mathcal{C}_1}, \mathfrak{i}^l_{\mathcal{C}_2}), g_1(\mathfrak{i}^u_{\mathcal{C}_1}, \mathfrak{i}^u_{\mathcal{C}_2})], \\ [g_1(\mathfrak{n}^l_{\mathcal{C}_1}, \mathfrak{n}^l_{\mathcal{C}_2}), g_1(\mathfrak{n}^u_{\mathcal{C}_1}, \mathfrak{n}^u_{\mathcal{C}_2})] \end{pmatrix}$
2) $\mathcal{C}_1 \otimes \mathcal{C}_2 = \begin{pmatrix} [g_1(\mathfrak{m}^l_{\mathcal{C}_1}, \mathfrak{m}^l_{\mathcal{C}_2}), g_1(\mathfrak{m}^u_{\mathcal{C}_1}, \mathfrak{m}^u_{\mathcal{C}_2})], \\ [g_1(\mathfrak{i}^l_{\mathcal{C}_1}, \mathfrak{i}^l_{\mathcal{C}_2}), g_1(\mathfrak{i}^u_{\mathcal{C}_1}, \mathfrak{i}^u_{\mathcal{C}_2})], [f_1(\mathfrak{n}^l_{\mathcal{C}_1}, \mathfrak{n}^l_{\mathcal{C}_2}), f_1(\mathfrak{n}^u_{\mathcal{C}_1}, \mathfrak{n}^u_{\mathcal{C}_2})] \end{pmatrix}$
3) $\lambda\mathcal{C} = ([f_2(\mathfrak{m}^l_{\mathcal{C}}), f_2(\mathfrak{m}^u_{\mathcal{C}})], [g_2(\mathfrak{i}^l_{\mathcal{C}}), g_2(\mathfrak{i}^u_{\mathcal{C}})], [g_2(\mathfrak{n}^l_{\mathcal{C}}), g_2(\mathfrak{n}^u_{\mathcal{C}})])$
4) $\mathcal{C}^\lambda = ([g_2(\mathfrak{m}^l_{\mathcal{C}}), g_2(\mathfrak{m}^u_{\mathcal{C}})], [g_2(\mathfrak{i}^l_{\mathcal{C}}), g_2(\mathfrak{i}^u_{\mathcal{C}})], [f_2(\mathfrak{n}^l_{\mathcal{C}}), f_2(\mathfrak{n}^u_{\mathcal{C}})])$

Here $f_1(x,y) = \sqrt[n]{x^n + y^n - x^n y^n}$, $g_1(x,y) = xy$, $f_2(x) = \sqrt[n]{1-(1-x^n)^\lambda}$, $g_2(x) = x^\lambda$.

Definition 3 [49]. A CTSFS has the following shape:

$$\mathcal{C} = \{(x, (\mathfrak{m}_{\mathcal{C}}(x), \mathfrak{i}_{\mathcal{C}}(x), \mathfrak{n}_{\mathcal{C}}(x))) : x \in X\}$$

where $\mathfrak{m}_{\mathcal{C}} = r_{\mathfrak{m}_{\mathcal{C}}}.e^{2\Pi i.\vartheta_{\mathfrak{m}_{\mathcal{C}}}(x)}$, $\mathfrak{i}_{\mathcal{C}} = r_{\mathfrak{i}_{\mathcal{C}}}.e^{2\Pi i.\vartheta_{\mathfrak{i}_{\mathcal{C}}}(x)}$ and $\mathfrak{n}_{\mathcal{C}} = r_{\mathfrak{n}_{\mathcal{C}}}.e^{2\Pi i.\vartheta_{\mathfrak{n}_{\mathcal{C}}}(x)}$ denote the CMG, CAG and CNMG of the element $x \in X$ such that $0 \leq r^n_{r_{\mathfrak{m}_{\mathcal{C}}}} + r^n_{\mathfrak{i}_{\mathcal{C}}} + r^n_{r_{\mathfrak{n}_{\mathcal{C}}}} \leq 1$ and $0 \leq \vartheta^n_{\mathfrak{m}_{\mathcal{C}}} + \vartheta^n_{\mathfrak{i}_{\mathcal{C}}} + \vartheta^n_{r_{\mathfrak{n}_{\mathcal{C}}}} \leq 1$ for some least $n \in Z^+$. Moreover, the term $\hbar_{\mathcal{C}}(x) = r_{\hbar_{\mathcal{C}}}.e^{2\Pi i.\vartheta_{\hbar_{\mathcal{C}}}(x)}$ is considered CRG of $x \in X$ such that $\hbar_{\mathcal{C}} = \sqrt[n]{1-(r^n_{r_{\mathfrak{m}_{\mathcal{C}}}} + r^n_{\mathfrak{i}_{\mathcal{C}}} + r^n_{r_{\mathfrak{n}_{\mathcal{C}}}})}$ and $\vartheta_{\hbar_{\mathcal{C}}} = \sqrt[n]{1-(\vartheta^n_{\mathfrak{m}_{\mathcal{C}}} + \vartheta^n_{\mathfrak{i}_{\mathcal{C}}} + \vartheta^n_{r_{\mathfrak{n}_{\mathcal{C}}}})}$. Further, $\mathcal{C} = (r_{\mathfrak{m}_{\mathcal{C}}}.e^{2\Pi i.\vartheta_{\mathfrak{m}_{\mathcal{C}}}(x)}, r_{\mathfrak{i}_{\mathcal{C}}}.e^{2\Pi i.\vartheta_{\mathfrak{i}_{\mathcal{C}}}(x)}, r_{\mathfrak{n}_{\mathcal{C}}}.e^{2\Pi i.\vartheta_{\mathfrak{n}_{\mathcal{C}}}(x)})$ is termed complex T-spherical fuzzy number (CTSFN).

COMPLEX INTERVAL-VALUED T-SPHERICAL FUZZY SET (CIVTSFS)

In this section we aim to develop the notion of CIVTSFS by taking the advantages of CTSFS and interval-valued numbers. The basic operations and the characteristics of the CIVTSFSs are also discussed.

Definition 4. A CIVTSFS defined over X is written as

$$\mathcal{C} = \{(x, (\mathfrak{m}_{\mathcal{C}}(x), \mathfrak{i}_{\mathcal{C}}(x), \mathfrak{n}_{\mathcal{C}}(x))) : x \in X\}$$

where $\mathfrak{m}_{\mathcal{C}}: X \longrightarrow [\mathfrak{m}^l_{\mathcal{C}}(x), \mathfrak{m}^u_{\mathcal{C}}(x)] = [(\mathfrak{r})^l_{\mathfrak{m}_{\mathcal{C}}} e^{2\Pi i.\vartheta^l_{\mathfrak{m}_{\mathcal{C}}}(x)}, (\mathfrak{r})^u_{\mathfrak{m}_{\mathcal{C}}} e^{2\Pi i.\vartheta^u_{\mathfrak{m}_{\mathcal{C}}}(x)}]$, $\mathfrak{i}_{\mathcal{C}}: X \rightarrow [\mathfrak{i}^l_{\mathcal{C}}(x), \mathfrak{i}^u_{\mathcal{C}}(x)] = [(\mathfrak{r})^l_{\mathfrak{i}_{\mathcal{C}}} e^{2\Pi i.\vartheta^l_{\mathfrak{i}_{\mathcal{C}}}(x)}, (\mathfrak{r})^u_{\mathfrak{i}_{\mathcal{C}}} e^{2\Pi i.\vartheta^u_{\mathfrak{i}_{\mathcal{C}}}(x)}]$ and $\mathfrak{n}_{\mathcal{C}}: X \longrightarrow [\mathfrak{n}^l_{\mathcal{C}}(x), \mathfrak{n}^u_{\mathcal{C}}(x)] = [(\mathfrak{r})^l_{\mathfrak{n}_{\mathcal{C}}} e^{2\Pi i.\vartheta^l_{\mathfrak{n}_{\mathcal{C}}}(x)}, (\mathfrak{r})^u_{\mathfrak{n}_{\mathcal{C}}} e^{2\Pi i.\vartheta^u_{\mathfrak{n}_{\mathcal{C}}}(x)}]$ denote the CMG, CAG and CNMG of the element $x \in X$ such that $0 \leq {(\mathfrak{r})^u_{\mathfrak{m}_{\mathcal{C}}}}^n + {(\mathfrak{r})^u_{\mathfrak{i}_{\mathcal{C}}}}^n + {(\mathfrak{r})^u_{\mathfrak{n}_{\mathcal{C}}}}^n \leq 1$ and $0 \leq {\vartheta^u_{\mathfrak{m}_{\mathcal{C}}(x)}}^n + {\vartheta^u_{\mathfrak{i}_{\mathcal{C}}(x)}}^n + {\vartheta^u_{\mathfrak{n}_{\mathcal{C}}(x)}}^n \leq 1$ for some least positive integers n. Moreover,

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the term $\hbar_{\mathcal{C}}(x) = \left[\hbar^{l}_{\mathcal{C}}(x), \hbar^{u}_{\mathcal{C}}(x)\right] = \left[(\text{ṛ})^{l}_{\hbar_{\mathcal{C}}}.\, e^{2\Pi i.\vartheta^{l}_{\hbar_{\mathcal{C}}}(x)}, (\text{ṛ})^{u}_{\hbar_{\mathcal{C}}}.\, e^{2\Pi i.\vartheta^{u}_{\hbar_{\mathcal{C}}}(x)}\right]$ is considered the CRG of $x \in X$ such that

$$\hbar_{\mathcal{C}} = \sqrt[n]{1 - \left(({(\text{ṛ})^{u}_{\text{ɱ}_{\mathcal{C}}}}^{n} + {(\text{ṛ})^{u}_{\text{ị}_{\mathcal{C}}}}^{n} + {(\text{ṛ})^{u}_{\text{ŋ}_{\mathcal{C}}}}^{n}\right)} \text{ and } \vartheta_{\hbar_{\mathcal{C}}(x)} = \sqrt[n]{1 - \left({\vartheta^{u}_{\text{ɱ}_{\mathcal{C}}(x)}}^{n} + {\vartheta^{u}_{\text{ị}_{\mathcal{C}}(x)}}^{n} + {\vartheta^{u}_{\text{ŋ}_{\mathcal{C}}(x)}}^{n}\right)}.$$

Further, the term $\mathcal{C} = \left(\begin{bmatrix}(\text{ṛ})^{l}_{\text{ɱ}_{\mathcal{C}}}.\, e^{2\Pi i.\vartheta^{l}_{\text{ɱ}_{\mathcal{C}}}}, \\ (\text{ṛ})^{u}_{\text{ɱ}_{\mathcal{C}}}.\, e^{2\Pi i.\vartheta^{u}_{\text{ɱ}_{\mathcal{C}}}}\end{bmatrix}, \begin{bmatrix}(\text{ṛ})^{l}_{\text{ị}_{\mathcal{C}}}.\, e^{2\Pi i.\vartheta^{l}_{\text{ị}_{\mathcal{C}}}}, \\ (\text{ṛ})^{u}_{\text{ị}_{\mathcal{C}}}.\, e^{2\Pi i.\vartheta^{u}_{\text{ị}_{\mathcal{C}}}}\end{bmatrix}, \begin{bmatrix}(\text{ṛ})^{l}_{\text{ŋ}_{\mathcal{C}}}.\, e^{2\Pi i.\vartheta^{l}_{\text{ŋ}_{\mathcal{C}}}}, \\ (\text{ṛ})^{u}_{\text{ŋ}_{\mathcal{C}}}.\, e^{2\Pi i.\vartheta^{u}_{\text{ŋ}_{\mathcal{C}}}}\end{bmatrix}\right)$ is termed complex interval-valued T-spherical fuzzy number (CIVTSFN).

Remark 2. The framework of CIVTSFS reduces to

1) IVTSFS [23] when $\vartheta^{l}_{\text{ɱ}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ɱ}_{\mathcal{C}}(x)} = \vartheta^{l}_{\text{ị}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ị}_{\mathcal{C}}(x)} = \vartheta^{l}_{\text{ŋ}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ŋ}_{\mathcal{C}}(x)} = 0$;
2) CTSFS [49] when $\text{ɱ}^{l}_{\mathcal{C}}(x) = \text{ɱ}^{u}_{\mathcal{C}}(x), \text{ị}^{l}_{\mathcal{C}}(x) = \text{ị}^{u}_{\mathcal{C}}(x)$ and $\text{ŋ}^{l}_{\mathcal{C}}(x) = \text{ŋ}^{u}_{\mathcal{C}}(x)$;
3) TSFS [22] when $\vartheta^{l}_{\text{ɱ}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ɱ}_{\mathcal{C}}(x)} = \vartheta^{l}_{\text{ị}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ị}_{\mathcal{C}}(x)} = \vartheta^{l}_{\text{ŋ}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ŋ}_{\mathcal{C}}(x)} = 0$, $\text{ɱ}^{l}_{\mathcal{C}}(x) = \text{ɱ}^{u}_{\mathcal{C}}(x)$, $\text{ị}^{l}_{\mathcal{C}}(x) = \text{ị}^{u}_{\mathcal{C}}(x)$ and $\text{ŋ}^{l}_{\mathcal{C}}(x) = \text{ŋ}^{u}_{\mathcal{C}}(x)$;
4) Complex interval-valued SFS (CIVSFS) when $n = 2$;
5) Interval-valued SFS (IVSFS) when $n = 2$ and $\vartheta^{l}_{\text{ɱ}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ɱ}_{\mathcal{C}}(x)} = \vartheta^{l}_{\text{ị}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ị}_{\mathcal{C}}(x)} = \vartheta^{l}_{\text{ŋ}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ŋ}_{\mathcal{C}}(x)} = 0$;
6) SFS [22] when $n = 2$, $\vartheta^{l}_{\text{ɱ}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ɱ}_{\mathcal{C}}(x)} = \vartheta^{l}_{\text{ị}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ị}_{\mathcal{C}}(x)} = \vartheta^{l}_{\text{ŋ}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ŋ}_{\mathcal{C}}(x)} = 0$, $\text{ɱ}^{l}_{\mathcal{C}}(x) = \text{ɱ}^{u}_{\mathcal{C}}(x), \text{ị}^{l}_{\mathcal{C}}(x) = \text{ị}^{u}_{\mathcal{C}}(x)$ and $\text{ŋ}^{l}_{\mathcal{C}}(x) = \text{ŋ}^{u}_{\mathcal{C}}(x)$;
7) Interval-valued PFS (IVPFS) when $n = 1$ and $\vartheta^{l}_{\text{ɱ}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ɱ}_{\mathcal{C}}(x)} = \vartheta^{l}_{\text{ị}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ị}_{\mathcal{C}}(x)} = \vartheta^{l}_{\text{ŋ}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ŋ}_{\mathcal{C}}(x)} = 0$;
8) PFS [16] when $n = 1$, $\vartheta^{l}_{\text{ɱ}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ɱ}_{\mathcal{C}}(x)} = \vartheta^{l}_{\text{ị}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ị}_{\mathcal{C}}(x)} = \vartheta^{l}_{\text{ŋ}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ŋ}_{\mathcal{C}}(x)} = 0$, $\text{ɱ}^{l}_{\mathcal{C}}(x) = \text{ɱ}^{u}_{\mathcal{C}}(x)$, $\text{ị}^{l}_{\mathcal{C}}(x) = \text{ị}^{u}_{\mathcal{C}}(x)$ and $\text{ŋ}^{l}_{\mathcal{C}}(x) = \text{ŋ}^{u}_{\mathcal{C}}(x)$;
9) Complex IVQROFS when $\vartheta^{l}_{\text{ị}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ị}_{\mathcal{C}}(x)} = 0$;
10) IVQROFS [12] when $\vartheta^{l}_{\text{ɱ}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ɱ}_{\mathcal{C}}(x)} = \vartheta^{l}_{\text{ị}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ị}_{\mathcal{C}}(x)} = \vartheta^{l}_{\text{ŋ}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ŋ}_{\mathcal{C}}(x)} = 0$ and $\text{ị}^{l}_{\mathcal{C}}(x) = \text{ị}^{u}_{\mathcal{C}}(x) = 0$;
11) CQROFS [38] when $\text{ɱ}^{l}_{\mathcal{C}}(x) = \text{ɱ}^{u}_{\mathcal{C}}(x), \text{ŋ}^{l}_{\mathcal{C}}(x) = \text{ŋ}^{u}_{\mathcal{C}}(x)$ and $\text{ị}^{l}_{\mathcal{C}}(x) = \text{ị}^{u}_{\mathcal{C}}(x) = 0$;
12) QROFS [11] when $\vartheta^{l}_{\text{ɱ}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ɱ}_{\mathcal{C}}(x)} = \vartheta^{l}_{\text{ị}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ị}_{\mathcal{C}}(x)} = \vartheta^{l}_{\text{ŋ}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ŋ}_{\mathcal{C}}(x)} = 0$, $\text{ɱ}^{l}_{\mathcal{C}}(x) = \text{ɱ}^{u}_{\mathcal{C}}(x), \text{ŋ}^{l}_{\mathcal{C}}(x) = \text{ŋ}^{u}_{\mathcal{C}}(x)$ and $\text{ị}^{l}_{\mathcal{C}}(x) = \text{ị}^{u}_{\mathcal{C}}(x) = 0$;
13) Complex interval-valued PyFS when $n = 2$ and $\vartheta^{l}_{\text{ị}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ị}_{\mathcal{C}}(x)} = 0$;
14) Interval-valued PyFS [10] when $n = 2$, $\vartheta^{l}_{\text{ɱ}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ɱ}_{\mathcal{C}}(x)} = \vartheta^{l}_{\text{ị}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ị}_{\mathcal{C}}(x)} = \vartheta^{l}_{\text{ŋ}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ŋ}_{\mathcal{C}}(x)} = 0$ and $\text{ị}^{l}_{\mathcal{C}}(x) = \text{ị}^{u}_{\mathcal{C}}(x) = 0$;
15) Complex PyFS [36] when $n = 2$, $\text{ɱ}^{l}_{\mathcal{C}}(x) = \text{ɱ}^{u}_{\mathcal{C}}(x), \text{ŋ}^{l}_{\mathcal{C}}(x) = \text{ŋ}^{u}_{\mathcal{C}}(x)$ and $\text{ị}^{l}_{\mathcal{C}}(x) = \text{ị}^{u}_{\mathcal{C}}(x) = 0$;
16) PyFS [9] when $n = 2$, $\vartheta^{l}_{\text{ɱ}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ɱ}_{\mathcal{C}}(x)} = \vartheta^{l}_{\text{ị}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ị}_{\mathcal{C}}(x)} = \vartheta^{l}_{\text{ŋ}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ŋ}_{\mathcal{C}}(x)} = 0$, $\text{ɱ}^{l}_{\mathcal{C}}(x) = \text{ɱ}^{u}_{\mathcal{C}}(x), \text{ŋ}^{l}_{\mathcal{C}}(x) = \text{ŋ}^{u}_{\mathcal{C}}(x)$ and $\text{ị}^{l}_{\mathcal{C}}(x) = \text{ị}^{u}_{\mathcal{C}}(x) = 0$;
17) Complex interval-valued IFS when $n = 1$ and $\vartheta^{l}_{\text{ị}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ị}_{\mathcal{C}}(x)} = 0$;
18) Interval-valued IFS [5] when $n = 1$, $\vartheta^{l}_{\text{ɱ}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ɱ}_{\mathcal{C}}(x)} = \vartheta^{l}_{\text{ị}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ị}_{\mathcal{C}}(x)} = \vartheta^{l}_{\text{ŋ}_{\mathcal{C}}(x)} = \vartheta^{u}_{\text{ŋ}_{\mathcal{C}}(x)} = 0$ and $\text{ị}^{l}_{\mathcal{C}}(x) = \text{ị}^{u}_{\mathcal{C}}(x) = 0$;
19) Complex IFS [32] when $n = 1$, $\text{ɱ}^{l}_{\mathcal{C}}(x) = \text{ɱ}^{u}_{\mathcal{C}}(x), \text{ŋ}^{l}_{\mathcal{C}}(x) = \text{ŋ}^{u}_{\mathcal{C}}(x)$ and $\text{ị}^{l}_{\mathcal{C}}(x) = \text{ị}^{u}_{\mathcal{C}}(x) = 0$;

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20) IFS [2] when $n=1$, $\vartheta_{\mathrm{m}_{\mathcal{C}}(x)}^{l}=\vartheta_{\mathrm{m}_{\mathcal{C}}(x)}^{u}=\vartheta_{\mathrm{i}_{\mathcal{C}}(x)}^{l}=\vartheta_{\mathrm{i}_{\mathcal{C}}(x)}^{u}=\vartheta_{\mathrm{n}_{\mathcal{C}}(x)}^{l}=\vartheta_{\mathrm{n}_{\mathcal{C}}(x)}^{u}=0$, $\mathrm{m}_{\mathcal{C}}^{l}(x)=\mathrm{m}_{\mathcal{C}}^{u}(x)$, $\mathrm{n}_{\mathcal{C}}^{l}(x)=\mathrm{n}_{\mathcal{C}}^{u}(x)$ and $\mathrm{i}_{\mathcal{C}}^{l}(x)=\mathrm{i}_{\mathcal{C}}^{u}(x)=0$.

Now we discuss the basic operations of CIVTSFSs followed by the investigation of their properties as follows.

Definition 5. For three CIVTSFNs $\mathcal{C}=\left([\mathrm{m}_{\mathcal{C}}^{l}(x),\mathrm{m}_{\mathcal{C}}^{u}(x)],[\mathrm{i}_{\mathcal{C}}^{l}(x),\mathrm{i}_{\mathcal{C}}^{u}(x)],[\mathrm{n}_{\mathcal{C}}^{l}(x),\mathrm{n}_{\mathcal{C}}^{u}(x)]\right)$, $\mathcal{C}_1=\left([\mathrm{m}_{\mathcal{C}_1}^{l}(x),\mathrm{m}_{\mathcal{C}_1}^{u}(x)],[\mathrm{i}_{\mathcal{C}_1}^{l}(x),\mathrm{i}_{\mathcal{C}_1}^{u}(x)],[\mathrm{n}_{\mathcal{C}_1}^{l}(x),\mathrm{n}_{\mathcal{C}_1}^{u}(x)]\right)$ and $\mathcal{C}_2=\left([\mathrm{m}_{\mathcal{C}_2}^{l}(x),\mathrm{m}_{\mathcal{C}_2}^{u}(x)],[\mathrm{i}_{\mathcal{C}_2}^{l}(x),\mathrm{i}_{\mathcal{C}_2}^{u}(x)],[\mathrm{n}_{\mathcal{C}_2}^{l}(x),\mathrm{n}_{\mathcal{C}_2}^{u}(x)]\right)$ and real $\lambda>0$, the basic operations are defined as

1) Complement: $\mathcal{C}'=\left([\mathrm{n}_{\mathcal{C}}^{l}(x),\mathrm{n}_{\mathcal{C}}^{u}(x)],[\mathrm{i}_{\mathcal{C}}^{l}(x),\mathrm{i}_{\mathcal{C}}^{u}(x)],[\mathrm{m}_{\mathcal{C}}^{l}(x),\mathrm{m}_{\mathcal{C}}^{u}(x)]\right)$ (1)

2) $$\mathcal{C}_1\oplus\mathcal{C}_2=\begin{pmatrix}\left[f_1\left((\mathrm{r})_{\mathrm{m}_{\mathcal{C}_1}}^{l},(\mathrm{r})_{\mathrm{m}_{\mathcal{C}_2}}^{l}\right)e^{2\pi i f_1\left((\vartheta)_{\mathrm{m}_{\mathcal{C}_1}}^{l},(\vartheta)_{\mathrm{m}_{\mathcal{C}_2}}^{l}\right)},f_1\left((\mathrm{r})_{\mathrm{m}_{\mathcal{C}_1}}^{u},(\mathrm{r})_{\mathrm{m}_{\mathcal{C}_2}}^{u}\right)e^{2\pi i f_1\left((\vartheta)_{\mathrm{m}_{\mathcal{C}_1}}^{u},(\vartheta)_{\mathrm{m}_{\mathcal{C}_2}}^{u}\right)}\right],\\ \left[g_1\left((\mathrm{r})_{\mathrm{i}_{\mathcal{C}_1}}^{l},(\mathrm{r})_{\mathrm{i}_{\mathcal{C}_2}}^{l}\right)e^{2\pi i\, g_2\left((\vartheta)_{\mathrm{i}_{\mathcal{C}_1}}^{l},(\vartheta)_{\mathrm{i}_{\mathcal{C}_2}}^{l}\right)},g_1\left((\mathrm{r})_{\mathrm{i}_{\mathcal{C}_1}}^{u},(\mathrm{r})_{\mathrm{i}_{\mathcal{C}_2}}^{u}\right)e^{2\pi i\, g_2\left((\vartheta)_{\mathrm{i}_{\mathcal{C}_1}}^{u},(\vartheta)_{\mathrm{i}_{\mathcal{C}_2}}^{u}\right)}\right],\\ \left[g_1\left((\mathrm{r})_{\mathrm{n}_{\mathcal{C}_1}}^{l},(\mathrm{r})_{\mathrm{n}_{\mathcal{C}_2}}^{l}\right)e^{2\pi i\, g_2\left((\vartheta)_{\mathrm{n}_{\mathcal{C}_1}}^{l},(\vartheta)_{\mathrm{n}_{\mathcal{C}_2}}^{l}\right)},g_1\left((\mathrm{r})_{\mathrm{n}_{\mathcal{C}_1}}^{u},(\mathrm{r})_{\mathrm{n}_{\mathcal{C}_2}}^{u}\right)e^{2\pi i\, g_2\left((\vartheta)_{\mathrm{n}_{\mathcal{C}_1}}^{u},(\vartheta)_{\mathrm{n}_{\mathcal{C}_2}}^{u}\right)}\right]\end{pmatrix} \quad (2)$$

3) $$\mathcal{C}_1\otimes\mathcal{C}_2=\begin{pmatrix}\left[g_1\left((\mathrm{r})_{\mathrm{m}_{\mathcal{C}_1}}^{l},(\mathrm{r})_{\mathrm{m}_{\mathcal{C}_2}}^{l}\right)e^{2\pi i f_1\left((\vartheta)_{\mathrm{m}_{\mathcal{C}_1}}^{l},(\vartheta)_{\mathrm{m}_{\mathcal{C}_2}}^{l}\right)},g_1\left((\mathrm{r})_{\mathrm{m}_{\mathcal{C}_1}}^{u},(\mathrm{r})_{\mathrm{m}_{\mathcal{C}_2}}^{u}\right)e^{2\pi i f_1\left((\vartheta)_{\mathrm{m}_{\mathcal{C}_1}}^{u},(\vartheta)_{\mathrm{m}_{\mathcal{C}_2}}^{u}\right)}\right],\\ \left[f_1\left((\mathrm{r})_{\mathrm{i}_{\mathcal{C}_1}}^{l},(\mathrm{r})_{\mathrm{i}_{\mathcal{C}_2}}^{l}\right)e^{2\pi i\, g_2\left((\vartheta)_{\mathrm{i}_{\mathcal{C}_1}}^{l},(\vartheta)_{\mathrm{i}_{\mathcal{C}_2}}^{l}\right)},f_1\left((\mathrm{r})_{\mathrm{i}_{\mathcal{C}_1}}^{u},(\mathrm{r})_{\mathrm{i}_{\mathcal{C}_2}}^{u}\right)e^{2\pi i\, g_2\left((\vartheta)_{\mathrm{i}_{\mathcal{C}_1}}^{u},(\vartheta)_{\mathrm{i}_{\mathcal{C}_2}}^{u}\right)}\right],\\ \left[f_1\left((\mathrm{r})_{\mathrm{n}_{\mathcal{C}_1}}^{l},(\mathrm{r})_{\mathrm{n}_{\mathcal{C}_2}}^{l}\right)e^{2\pi i\, g_2\left((\vartheta)_{\mathrm{n}_{\mathcal{C}_1}}^{l},(\vartheta)_{\mathrm{n}_{\mathcal{C}_2}}^{l}\right)},f_1\left((\mathrm{r})_{\mathrm{n}_{\mathcal{C}_1}}^{u},(\mathrm{r})_{\mathrm{n}_{\mathcal{C}_2}}^{u}\right)e^{2\pi i\, g_2\left((\vartheta)_{\mathrm{n}_{\mathcal{C}_1}}^{u},(\vartheta)_{\mathrm{n}_{\mathcal{C}_2}}^{u}\right)}\right]\end{pmatrix} \quad (3)$$

4) $$\lambda\mathcal{C}=\begin{pmatrix}\left[f_2\left((\mathrm{r})_{\mathrm{m}_{\mathcal{C}}}^{l}\right)e^{2\pi i\, f_2\left((\vartheta)_{\mathrm{m}_{\mathcal{C}}}^{l}\right)},\ f_2\left((\mathrm{r})_{\mathrm{m}_{\mathcal{C}}}^{u}\right)e^{2\pi i\, f_2\left((\vartheta)_{\mathrm{m}_{\mathcal{C}}}^{u}\right)}\right],\left[g_2\left((\mathrm{r})_{\mathrm{i}_{\mathcal{C}}}^{l}\right)e^{2\pi i g_2\left((\vartheta)_{\mathrm{i}_{\mathcal{C}}}^{l}\right)},\right.\\ \left.g_2\left((\mathrm{r})_{\mathrm{i}_{\mathcal{C}}}^{u}\right)e^{2\pi i g_2\left((\vartheta)_{\mathrm{i}_{\mathcal{C}}}^{u}\right)}\right],\left[g_2\left((\mathrm{r})_{\mathrm{n}_{\mathcal{C}}}^{l}\right)e^{2\pi i\, g_2\left((\vartheta)_{\mathrm{n}_{\mathcal{C}}}^{l}\right)},g_2\left((\mathrm{r})_{\mathrm{n}_{\mathcal{C}}}^{u}\right)e^{2\pi i g_2\left((\vartheta)_{\mathrm{n}_{\mathcal{C}}}^{u}\right)}\right]\end{pmatrix} \quad (4)$$

5) $$\mathcal{C}^{\lambda}=\begin{pmatrix}\left[g_2\left((\mathrm{r})_{\mathrm{m}_{\mathcal{C}}}^{l}\right)e^{2\pi i\, f_2\left((\vartheta)_{\mathrm{m}_{\mathcal{C}}}^{l}\right)},\ g_2\left((\mathrm{r})_{\mathrm{m}_{\mathcal{C}}}^{u}\right)e^{2\pi i\, f_2\left((\vartheta)_{\mathrm{m}_{\mathcal{C}}}^{u}\right)}\right],\left[f_2\left((\mathrm{r})_{\mathrm{i}_{\mathcal{C}}}^{l}\right)e^{2\pi i g_2\left((\vartheta)_{\mathrm{i}_{\mathcal{C}}}^{l}\right)}\right.\\ \left.f_2\left((\mathrm{r})_{\mathrm{i}_{\mathcal{C}}}^{u}\right)e^{2\pi i g_2\left((\vartheta)_{\mathrm{i}_{\mathcal{C}}}^{u}\right)}\right],\left[f_2\left((\mathrm{r})_{\mathrm{n}_{\mathcal{C}}}^{l}\right)e^{2\pi i\, g_2\left((\vartheta)_{\mathrm{n}_{\mathcal{C}}}^{l}\right)},f_2\left((\mathrm{r})_{\mathrm{n}_{\mathcal{C}}}^{u}\right)e^{2\pi i g_2\left((\vartheta)_{\mathrm{n}_{\mathcal{C}}}^{u}\right)}\right]\end{pmatrix} \quad (5)$$

where $f_1(x,y)=\sqrt[n]{x^n+y^n-x^ny^n}$, $g_1(x,y)=xy$, $f_2(x)=\sqrt[n]{1-(1-x^n)^{\lambda}}$, $g_2(x)=x^{\lambda}$.

Theorem 1. Let $\mathcal{C}=\left([\mathrm{m}_{\mathcal{C}}^{l}(x),\mathrm{m}_{\mathcal{C}}^{u}(x)],[\mathrm{i}_{\mathcal{C}}^{l}(x),\mathrm{i}_{\mathcal{C}}^{u}(x)],[\mathrm{n}_{\mathcal{C}}^{l}(x),\mathrm{n}_{\mathcal{C}}^{u}(x)]\right)$, $\mathcal{C}_1=\left([\mathrm{m}_{\mathcal{C}_1}^{l}(x),\mathrm{m}_{\mathcal{C}_1}^{u}(x)],[\mathrm{i}_{\mathcal{C}_1}^{l}(x),\mathrm{i}_{\mathcal{C}_1}^{u}(x)],[\mathrm{n}_{\mathcal{C}_1}^{l}(x),\mathrm{n}_{\mathcal{C}_1}^{u}(x)]\right)$ and $\mathcal{C}_2=\left([\mathrm{m}_{\mathcal{C}_2}^{l}(x),\mathrm{m}_{\mathcal{C}_2}^{u}(x)],[\mathrm{i}_{\mathcal{C}_2}^{l}(x),\mathrm{i}_{\mathcal{C}_2}^{u}(x)],[\mathrm{n}_{\mathcal{C}_2}^{l}(x),\mathrm{n}_{\mathcal{C}_2}^{u}(x)]\right)$ be three CIVTSFNs and $\lambda,\lambda_1,\lambda_2>0$. Then

$$\mathcal{C}_1\oplus\mathcal{C}_2=\mathcal{C}_2\oplus\mathcal{C}_1,\quad \mathcal{C}_1\otimes\mathcal{C}_2=\mathcal{C}_2\otimes\mathcal{C}_1,\quad \lambda(\mathcal{C}_1\oplus\mathcal{C}_2)=\lambda\mathcal{C}_1\oplus\lambda\mathcal{C}_2,\quad (\mathcal{C}_1\otimes\mathcal{C}_2)^{\lambda}={\mathcal{C}_1}^{\lambda}\otimes{\mathcal{C}_2}^{\lambda},$$
$$\lambda_1\mathcal{C}\oplus\lambda_2\mathcal{C}=(\lambda_1+\lambda_2)\mathcal{C},\quad \mathcal{C}^{\lambda_1}\otimes\mathcal{C}^{\lambda_2}=\mathcal{C}^{\lambda_1+\lambda_2}.$$

Proof. The proof of these results is straightforward.

Definition 6. Let $\mathcal{C}=\left([\mathrm{m}_{\mathcal{C}}^{l}(x),\mathrm{m}_{\mathcal{C}}^{u}(x)],[\mathrm{i}_{\mathcal{C}}^{l}(x),\mathrm{i}_{\mathcal{C}}^{u}(x)],[\mathrm{n}_{\mathcal{C}}^{l}(x),\mathrm{n}_{\mathcal{C}}^{u}(x)]\right)$ be a CIVTSFS. Then the score of $\mathcal{C}$ is defined as

$$Score(\mathcal{C})=\frac{1}{4}\begin{bmatrix}\left(\left((\mathrm{r})_{\mathrm{m}_{\mathcal{C}}}^{l}\right)^n\left(1-\left((\mathrm{r})_{\mathrm{i}_{\mathcal{C}}}^{l}\right)^n-\left((\mathrm{r})_{\mathrm{n}_{\mathcal{C}}}^{l}\right)^n\right)+\left((\mathrm{r})_{\mathrm{m}_{\mathcal{C}}}^{u}\right)^n\left(1-\left((\mathrm{r})_{\mathrm{i}_{\mathcal{C}}}^{u}\right)^n-\left((\mathrm{r})_{\mathrm{n}_{\mathcal{C}}}^{u}\right)^n\right)\right)+\\ \left(\left((\vartheta)_{\mathrm{m}_{\mathcal{C}}}^{l}\right)^n\left(1-\left((\vartheta)_{\mathrm{i}_{\mathcal{C}}}^{l}\right)^n-\left((\vartheta)_{\mathrm{n}_{\mathcal{C}}}^{l}\right)^n\right)+\left((\vartheta)_{\mathrm{m}_{\mathcal{C}}}^{u}\right)^n\left(1-\left((\vartheta)_{\mathrm{i}_{\mathcal{C}}}^{u}\right)^n-\left((\vartheta)_{\mathrm{n}_{\mathcal{C}}}^{u}\right)^n\right)\right)\end{bmatrix}. \quad (6)$$

It is clear that $Score(\mathcal{C})\in[0,1]$. Based on this score function, two CIVTSFNs $\mathcal{C}_1$ and $\mathcal{C}_2$ can be ranked as $\mathcal{C}_1>\mathcal{C}_2$ when $Score(\mathcal{C}_1)>Score(\mathcal{C}_2)$.

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WEIGHTED AVERAGING OPERATORS

In this section some averaging AOs of CIVTSFNs are developed. Throughout the study, $\omega_j \in [0,1]$, $\sum_{j=1}^{m} \omega_j = 1$ represents the weighted vector.

Definition 7. For some CIVTSFNs $\mathcal{C}_j$ $(j = 1,2,3 \ldots m)$, the complex interval-valued T-spherical fuzzy weighted average (CIVTSFWA) operator is of the form:

$$CIVTSFWA(\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3 \ldots \mathcal{C}_m) = \sum_{j=1}^{m} \omega_j \mathcal{C}_j \tag{7}$$

Theorem 2. For some CIVTSFNs $\mathcal{C}_j$ $(j = 1,2,3 \ldots m)$, their aggregated value using the CIVTSFWA operator is of the form:

$$CIVTSFWA(\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3 \ldots \mathcal{C}_m) = \begin{pmatrix} \left[f\left((r)^l_{m_{\mathcal{C}_j}}, (\vartheta)^l_{m_{\mathcal{C}_j}}\right), f\left((r)^u_{m_{\mathcal{C}_j}}, (\vartheta)^u_{m_{\mathcal{C}_j}}\right)\right], \\ \left[g\left((r)^l_{i_{\mathcal{C}}}, (\vartheta)^l_{i_{\mathcal{C}}}\right), g\left((r)^u_{i_{\mathcal{C}}}, (\vartheta)^u_{i_{\mathcal{C}}}\right)\right], \\ \left[g\left((r)^l_{\eta_{\mathcal{C}}}, (\vartheta)^l_{\eta_{\mathcal{C}}}\right), g\left((r)^u_{\eta_{\mathcal{C}}}\right), (\vartheta)^u_{\eta_{\mathcal{C}}}\right) \right] \end{pmatrix}. \tag{8}$$

where

$$f(x,y) = \sqrt[n]{1 - \prod_{j=1}^{m} (1 - x^n)^{\omega_j}}\, e^{2\Pi i \sqrt[n]{1 - \prod_{j=1}^{m} (1 - y^n)^{\omega_j}}}$$

and

$$g(x,y) = \prod_{j=1}^{m} (x)^{\omega_j}\, e^{2\Pi i . \prod_{j=1}^{m} (y)^{\omega_j}}$$

for $x, y \in [0,1]$.

Proof. The proof follows from Definition 5.

Theorem 3. The CIVTSFWA operators are likely to agree with the following attributes of the aggregation:

1) Idempotency: If $\mathcal{C}_j = \mathcal{C}$ for all j, then $CIVTSFWA(\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3 \ldots \mathcal{C}_m) = \mathcal{C}$. (9)
2) Boundedness: If $\mathcal{C}^-$ and $\mathcal{C}^+$ denote the least and greatest CIVTSFNs, then $\mathcal{C}^- \leq CIVTSFWA(\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3 \ldots \mathcal{C}_m) \leq \mathcal{C}^+$. (10)
3) Monotonicity: Let $\mathcal{C}_j$ and D_j be two collections of CIVTSFSs such that $m^l_{\mathcal{C}_j}(x) \leq m^l_{D_j}(x)$, $m^u_{\mathcal{C}_j}(x) \leq m^u_{D_j}(x)$, $i^l_{\mathcal{C}_j}(x) \geq i^l_{D_j}(x)$, $i^u_{\mathcal{C}_j}(x) \geq i^u_{D_j}(x)$, $\eta^l_{\mathcal{C}_j}(x) \geq \eta^l_{D_j}(x)$ and $\eta^u_{\mathcal{C}_j}(x) \geq \eta^u_{D_j}(x)$ $\forall$ j. Then $CIVTSFWA(\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3 \ldots \mathcal{C}_m) \leq CIVTSFWA(D_1, D_2, D_3 \ldots D_m)$. (11)

Proof. The proof is straightforward.

Sometimes in the aggregation process of uncertain information the position of the knowledge is vital, and to tackle this issue the hypothesis of ordered weighted averaging operators exists in the fuzzy environments. Therefore, we propose the notion of complex interval-valued T-spherical fuzzy ordered weighted average (CIVTSFOWA) operator.

Definition 8. For some CIVTSFNs $\mathcal{C}_j$ $(j = 1,2,3 \ldots m)$, the CIVTSFOWA operator is of the form:

$$CIVTSFOWA(\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3 \dots \mathcal{C}_m) = \sum_{\mathrm{j}=1}^{m} \mathrm{w}_{\mathrm{j}} \mathcal{C}_{\gamma(\mathrm{j})} \quad , \tag{12}$$

where $\mathcal{C}_{\gamma(\mathrm{j})}$ denotes the j^{th} largest value of $\mathcal{C}_{\mathrm{j}}$.

We use Definitions 5 and 8 to obtain the following result.

Theorem 4. For some CIVTSFNs $\mathcal{C}_{\mathrm{j}}$ $(\mathrm{j} = 1, 2, 3 \dots m)$, their aggregated value using the CIVTSFOWA operator is of the form:

$$CIVTSFOWA(\mathcal{C}_1, \mathcal{C}_2, \dots, \mathcal{C}_m) = \begin{pmatrix} \left[f\left((\mathrm{r}_{\gamma(\mathrm{j})})^{l}_{\mathrm{m}_{\mathcal{C}_{\mathrm{j}}}}, (\vartheta_{\gamma(\mathrm{j})})^{l}_{\mathrm{m}_{\mathcal{C}_{\mathrm{j}}}} \right), f\left((\mathrm{r}_{\gamma(\mathrm{j})})^{u}_{\mathrm{m}_{\mathcal{C}_{\mathrm{j}}}}, (\vartheta_{\gamma(\mathrm{j})})^{u}_{\mathrm{m}_{\mathcal{C}_{\mathrm{j}}}} \right) \right], \\ \left[g\left((\mathrm{r}_{\gamma(\mathrm{j})})^{l}_{\mathrm{i}_{\mathcal{C}}}, (\vartheta_{\gamma(\mathrm{j})})^{l}_{\mathrm{i}_{\mathcal{C}}} \right), g\left((\mathrm{r}_{\gamma(\mathrm{j})})^{u}_{\mathrm{i}_{\mathcal{C}}}, (\vartheta_{\gamma(\mathrm{j})})^{u}_{\mathrm{i}_{\mathcal{C}}} \right) \right], \\ \left[g\left((\mathrm{r}_{\gamma(\mathrm{j})})^{l}_{\mathrm{n}_{\mathcal{C}}}, (\vartheta_{\gamma(\mathrm{j})})^{l}_{\mathrm{n}_{\mathcal{C}}} \right), g\left((\mathrm{r}_{\gamma(\mathrm{j})})^{u}_{\mathrm{n}_{\mathcal{C}}} \right), (\vartheta_{\gamma(\mathrm{j})})^{u}_{\mathrm{n}_{\mathcal{C}}}) \right] \end{pmatrix}. \tag{13}$$

Proof. The proof can be easily obtained from the operational laws as defined in Definition 5 so it is omitted here.

Definition 9. For some CIVTSFNs $\mathcal{C}_{\mathrm{j}}$ $(\mathrm{j} = 1, 2, 3 \dots m)$, the complex interval-valued T-spherical fuzzy hybrid average (CIVTSFHA) operator is of the form:

$$CIVTSFHA(\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3 \dots \mathcal{C}_m) = \sum_{\mathrm{j}=1}^{m} \mathrm{w}_{\mathrm{j}} \dot{\mathcal{C}}_{\gamma(\mathrm{j})} \tag{14}$$

Here $\dot{\mathcal{C}}_{\mathrm{j}}$ can be computed as $\dot{\mathcal{C}}_{\mathrm{j}} = m\mathcal{w}\mathcal{C}_{\mathrm{j}}$ and $\dot{\mathcal{C}}_{\alpha(\mathrm{j})}$ denotes the j^{th} largest value of $\dot{\mathcal{C}}_{\mathrm{j}}$. Further, $\mathcal{w} = (\mathcal{w}_1, \mathcal{w}_2, \mathcal{w}_3 \dots \mathcal{w}_n)^T$ is the weighted vector of $\mathcal{C}_{\mathrm{j}}$. Now we use Definitions 5 and 8 to obtain the following result, the proof of which is omitted as it is similar to that of Theorem 2.

Theorem 5. For some CIVTSFNs $\mathcal{C}_{\mathrm{j}}$ $(\mathrm{j} = 1, 2, 3 \dots m)$, their aggregated value using the CIVTSFHA operator is of the form:

$$CIVTSFHA(\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3 \dots \mathcal{C}_m) = \begin{pmatrix} \left[f\left((\dot{\mathrm{r}}_{\gamma(\mathrm{j})})^{l}_{\mathrm{m}_{\mathcal{C}_{\mathrm{j}}}}, (\dot{\vartheta}_{\gamma(\mathrm{j})})^{l}_{\mathrm{m}_{\mathcal{C}_{\mathrm{j}}}} \right), f\left((\dot{\mathrm{r}}_{\gamma(\mathrm{j})})^{u}_{\mathrm{m}_{\mathcal{C}_{\mathrm{j}}}}, (\dot{\vartheta}_{\gamma(\mathrm{j})})^{u}_{\mathrm{m}_{\mathcal{C}_{\mathrm{j}}}} \right) \right], \\ \left[g\left((\dot{\mathrm{r}}_{\gamma(\mathrm{j})})^{l}_{\mathrm{i}_{\mathcal{C}}}, (\dot{\vartheta}_{\gamma(\mathrm{j})})^{l}_{\mathrm{i}_{\mathcal{C}}} \right), g\left((\dot{\mathrm{r}}_{\gamma(\mathrm{j})})^{u}_{\mathrm{i}_{\mathcal{C}}}, (\dot{\vartheta}_{\gamma(\mathrm{j})})^{u}_{\mathrm{i}_{\mathcal{C}}} \right) \right], \\ \left[g\left((\dot{\mathrm{r}}_{\gamma(\mathrm{j})})^{l}_{\mathrm{n}_{\mathcal{C}}}, (\dot{\vartheta}_{\gamma(\mathrm{j})})^{l}_{\mathrm{n}_{\mathcal{C}}} \right), g\left((\dot{\mathrm{r}}_{\gamma(\mathrm{j})})^{u}_{\mathrm{n}_{\mathcal{C}}} \right), (\dot{\vartheta}_{\gamma(\mathrm{j})})^{u}_{\mathrm{n}_{\mathcal{C}}}) \right] \end{pmatrix}. \tag{15}$$

Proof. The proof follows from Definition 5.

WEIGHTED GEOMETRIC OPERATORS

In this section the geometric AOs of CIVTSFNs are discussed.

Definition 10. For some CIVTSFNs $\mathcal{C}_{\mathrm{j}}$ $(\mathrm{j} = 1, 2, 3 \dots m)$, the complex interval-valued T-spherical fuzzy weighted geometric (CIVTSFWG) operator is of the form:

$$CIVTSFWG(\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3 \dots \mathcal{C}_m) = \sum_{\mathrm{j}=1}^{m} \mathcal{C}_{\mathrm{j}}^{\mathrm{w}_{\mathrm{j}}} \tag{16}$$

Theorem 6. For some CIVTSFNs $\mathcal{C}_{\mathrm{j}}$ $(\mathrm{j} = 1, 2, 3 \dots m)$, their aggregated value using the CIVTSFWG operator is of the form:

$$CIVTSFWG(\mathcal{C}_1,\mathcal{C}_2,\mathcal{C}_3 \ldots \mathcal{C}_m) = \begin{pmatrix} \left[g\left((\mathrm{r})^{l}_{\mathrm{m}_{\mathcal{C}_{\mathrm{j}}}},(\vartheta)^{l}_{\mathrm{m}_{\mathcal{C}_{\mathrm{j}}}}\right), g\left((\mathrm{r})^{u}_{\mathrm{m}_{\mathcal{C}_{\mathrm{j}}}},(\vartheta)^{u}_{\mathrm{m}_{\mathcal{C}_{\mathrm{j}}}}\right)\right], \\ \left[f\left((\mathrm{r})^{l}_{\mathrm{i}_{\mathcal{C}}},(\vartheta)^{l}_{\mathrm{i}_{\mathcal{C}}}\right), f\left((\mathrm{r})^{u}_{\mathrm{i}_{\mathcal{C}}},(\vartheta)^{u}_{\mathrm{i}_{\mathcal{C}}}\right)\right], \\ \left[f\left((\mathrm{r})^{l}_{\mathrm{n}_{\mathcal{C}}},(\vartheta)^{l}_{\mathrm{n}_{\mathcal{C}}}\right), f\left((\mathrm{r})^{u}_{\mathrm{n}_{\mathcal{C}}}\right),(\vartheta)^{u}_{\mathrm{n}_{\mathcal{C}}})\right] \end{pmatrix}. \quad (17)$$

Proof. The proof can be easily obtained from the operational laws as defined in Definition 5 so it is omitted here.

Remark 3. The CIVTSFWG operator is likely to agree with following attributes of the aggregation, i.e. Idempotency, Boundedness and Monotonicity as discussed in Theorem 3.

Definition 11. For some CIVTSFNs $\mathcal{C}_{\mathrm{j}}$ ($\mathrm{j} = 1,2,3 \ldots m$), the complex interval-valued T-spherical fuzzy ordered weighted geometric (CIVTSFOWG) operator is of the form:

$$CIVTSFOWG(\mathcal{C}_1,\mathcal{C}_2,\mathcal{C}_3 \ldots \mathcal{C}_m) = \sum_{\mathrm{j}=1}^{m} \mathcal{C}_{\gamma(\mathrm{j})}^{\mathrm{w}_{\mathrm{j}}}, \quad (18)$$

where $\mathcal{C}_{\gamma(\mathrm{j})}$ denotes the j^{th} largest value of $\mathcal{C}_{\mathrm{j}}$.

Theorem 7. For a number of CIVTSFNs $\mathcal{C}_{\mathrm{j}}$ ($\mathrm{j} = 1,2,3 \ldots m$), their aggregated value using CIVTSFOWG operator is of the form:

$$CIVTSFOWG(\mathcal{C}_1,\mathcal{C}_2,\ldots,\mathcal{C}_m) = \begin{pmatrix} \left[g\left(\left(\mathrm{r}_{\gamma(\mathrm{j})}\right)^{l}_{\mathrm{m}_{\mathcal{C}_{\mathrm{j}}}},\left(\vartheta_{\gamma(\mathrm{j})}\right)^{l}_{\mathrm{m}_{\mathcal{C}_{\mathrm{j}}}}\right), g\left(\left(\mathrm{r}_{\gamma(\mathrm{j})}\right)^{u}_{\mathrm{m}_{\mathcal{C}_{\mathrm{j}}}},\left(\vartheta_{\gamma(\mathrm{j})}\right)^{u}_{\mathrm{m}_{\mathcal{C}_{\mathrm{j}}}}\right)\right], \\ \left[f\left(\left(\mathrm{r}_{\gamma(\mathrm{j})}\right)^{l}_{\mathrm{i}_{\mathcal{C}}},\left(\vartheta_{\gamma(\mathrm{j})}\right)^{l}_{\mathrm{i}_{\mathcal{C}}}\right), f\left(\left(\mathrm{r}_{\gamma(\mathrm{j})}\right)^{u}_{\mathrm{i}_{\mathcal{C}}},\left(\vartheta_{\gamma(\mathrm{j})}\right)^{u}_{\mathrm{i}_{\mathcal{C}}}\right)\right], \\ \left[f\left(\left(\mathrm{r}_{\gamma(\mathrm{j})}\right)^{l}_{\mathrm{n}_{\mathcal{C}}},\left(\vartheta_{\gamma(\mathrm{j})}\right)^{l}_{\mathrm{n}_{\mathcal{C}}}\right), f\left(\left(\mathrm{r}_{\gamma(\mathrm{j})}\right)^{u}_{\mathrm{n}_{\mathcal{C}}}\right),\left(\vartheta_{\gamma(\mathrm{j})}\right)^{u}_{\mathrm{n}_{\mathcal{C}}})\right] \end{pmatrix}. \quad (19)$$

Proof. The proof is straightforward.

In Definition 12 we present the geometric hybrid AO for CIVTSFNs.

Definition 12. For some CIVTSFNs $\mathcal{C}_{\mathrm{j}}$ ($\mathrm{j} = 1,2,3 \ldots m$), the complex interval-valued T-spherical fuzzy hybrid geometric (CIVTSFHG) operator is of the form:

$$CIVTSFHG(\mathcal{C}_1,\mathcal{C}_2,\mathcal{C}_3 \ldots \mathcal{C}_m) = \sum_{\mathrm{j}=1}^{m} \dot{\mathcal{C}}_{\gamma(\mathrm{j})}^{\mathrm{w}_{\mathrm{j}}}. \quad (20)$$

Here $\dot{\mathcal{C}}_{\mathrm{j}}$ can be computed as $\dot{\mathcal{C}}_{\mathrm{j}} = m w \mathcal{C}_{\mathrm{j}}$ and $\dot{\mathcal{C}}_{\alpha(\mathrm{j})}$ denotes the j^{th} largest value of $\dot{\mathcal{C}}_{\mathrm{j}}$. Further, $w = (w_1, w_2, w_3 \ldots w_n)^T$ is the weighted vector of $\mathcal{C}_{\mathrm{j}}$.

Theorem 8. For some CIVTSFNs $\mathcal{C}_{\mathrm{j}}$ ($\mathrm{j} = 1,2,3 \ldots m$), their aggregated value using the CIVTSFHG operator is of the form:

$$CIVTSFHG(\mathcal{C}_1,\mathcal{C}_2,\ldots,\mathcal{C}_m) = \begin{pmatrix} \left[g\left(\left(\dot{\mathrm{r}}_{\gamma(\mathrm{j})}\right)^{l}_{\mathrm{m}_{\mathcal{C}_{\mathrm{j}}}},\left(\dot{\vartheta}_{\gamma(\mathrm{j})}\right)^{l}_{\mathrm{m}_{\mathcal{C}_{\mathrm{j}}}}\right), g\left(\left(\dot{\mathrm{r}}_{\gamma(\mathrm{j})}\right)^{u}_{\mathrm{m}_{\mathcal{C}_{\mathrm{j}}}},\left(\dot{\vartheta}_{\gamma(\mathrm{j})}\right)^{u}_{\mathrm{m}_{\mathcal{C}_{\mathrm{j}}}}\right)\right], \\ \left[f\left(\left(\dot{\mathrm{r}}_{\gamma(\mathrm{j})}\right)^{l}_{\mathrm{i}_{\mathcal{C}}},\left(\dot{\vartheta}_{\gamma(\mathrm{j})}\right)^{l}_{\mathrm{i}_{\mathcal{C}}}\right), f\left(\left(\dot{\mathrm{r}}_{\gamma(\mathrm{j})}\right)^{u}_{\mathrm{i}_{\mathcal{C}}},\left(\dot{\vartheta}_{\gamma(\mathrm{j})}\right)^{u}_{\mathrm{i}_{\mathcal{C}}}\right)\right], \\ \left[f\left(\left(\dot{\mathrm{r}}_{\gamma(\mathrm{j})}\right)^{l}_{\mathrm{n}_{\mathcal{C}}},\left(\dot{\vartheta}_{\gamma(\mathrm{j})}\right)^{l}_{\mathrm{n}_{\mathcal{C}}}\right), f\left(\left(\dot{\mathrm{r}}_{\gamma(\mathrm{j})}\right)^{u}_{\mathrm{n}_{\mathcal{C}}}\right),\left(\dot{\vartheta}_{\gamma(\mathrm{j})}\right)^{u}_{\mathrm{n}_{\mathcal{C}}})\right] \end{pmatrix}. \quad (21)$$

Proof. The proof follows from Definition 5.

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PROPOSED MADM ALGORITHM AND NUMERICAL EXAMPLE

MADM Algorithm

In this section we present an MADM algorithm for decision-making problems. For this, we consider an MADM which consists of a finite number of alternatives $A = \{A_1, A_2, A_3 \dots A_n\}$ which are evaluated under the different attributes (factors) $K = \{K_1, K_2, K_3 \dots K_m\}$. The target of the problem is to find the most suitable alternative(s). To access it completely, an expert was invited to access each alternative and provide his ratings in terms of CIVTSFNs. Let $\omega_j \in [0,1], \sum_{j=1}^{m} \omega_j = 1$ represent the weighted vector of the attributes. Now, under each attribute, we aggregated the given collective data by using the proposed series of the operators such as CIVTSFWA or CIVTSFWG. Finally, the defuzzified value of the aggregated number was obtained by utilising the score function and hence the given numbers were ranked. In a nutshell, the steps of the proposed MADM algorithm are written briefly as follows.

- **Step 1:** Arrange the expert information as decision-matrix where each information is represented as CIVTSFNs.
- **Step 2:** If the given attributes are of different kinds, namely the cost type and the benefit type, then we can normalise the information by converting the cost-type attributes rating into the benefit type using complementary laws as stated in Eq. (1).
- **Step 3:** Utilise the weighted vector ω_j to aggregate the information into the collective value by using either of the stated operators such as CIVTSFWA, CIVTSFOWA, CIVTSFHA, CIVTSFWG, CIVTSFOWG and CIVTSFHG.
- **Step 4:** Compute the score value of the aggregated numbers obtained through Step 3 by using Eq. (6). Based on these numbers, we rank the numbers and hence find the most suitable alternative(s).

Illustrative Example

To demonstrate the working of the stated MADM algorithm above, we consider the study related to the process of evaluation of an investment policy by a multinational company. In the upcoming financial year, the company needs to launch its new policy and select an optimum policy keeping in mind the current growth of the company. The policy-making department of the company sets 4 possible investment policies to be launched after initial scrutiny. Among these four policies, the governing board needs to approve one. To do so, the proposed approach of MADM is followed. The four possible policies are taken as alternatives: Invest in South Asia (A_1), Invest in China (A_2), Invest in Africa (A_3) and Invest in UAE (A_4). The factors by which the members of the governing board of the company assess the policies are considered as:

H_1) People's interest H_2) Government rules

H_3) Comfort zone H_4) Market competition

Our goal is to find the most optimum policy based on the given attributes where the weighted vector is taken as $(0.22, 0.34, 0.27, 0.17)^T$. To find the best alternative, the steps of the proposed MADM algorithm are implemented as follows.

Step 1: An expert has provided the ratings of the given alternatives in terms of CIVTSFNs as a decision matrix given in Table 1.

Step 2: As all the attributes are of profit type, there is no need for normalisation.

Step 3: Without loss of generality, we take the CIVTSFWA and CIVTSFWG operators to aggregate the ratings. The results corresponding to them are listed in Tables 2 and 3 respectively.

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Table 1. Input decision matrix

	H_1	H_2	H_3	H_4
A_1	$\left([0.3.e^{2\pi i\,(0.5)}, 0.5.e^{2\pi i\,(0.7)}], [0.4.e^{2\pi i\,(0.6)}, 0.6.e^{2\pi i\,(0.9)}], [0.2.e^{2\pi i\,(0.3)}, 0.6.e^{2\pi i\,(0.5)}]\right)$	$\left([0.1.e^{2\pi i\,(0.7)}, 0.5.e^{2\pi i\,(0.8)}], [0.3.e^{2\pi i\,(0.2)}, 0.6.e^{2\pi i\,(0.5)}], [0.4.e^{2\pi i\,(0.4)}, 0.7.e^{2\pi i\,(0.6)}]\right)$	$\left([0.2.e^{2\pi i\,(0.3)}, 0.7.e^{2\pi i\,(0.4)}], [0.4.e^{2\pi i\,(0.6)}, 0.6.e^{2\pi i\,(0.8)}], [0.3.e^{2\pi i\,(0.7)}, 0.5.e^{2\pi i\,(0.9)}]\right)$	$\left([0.4.e^{2\pi i\,(0.6)}, 0.9.e^{2\pi i\,(0.9)}], [0.5.e^{2\pi i\,(0.7)}, 0.6.e^{2\pi i\,(0.8)}], [0.3.e^{2\pi i\,(0.4)}, 0.4.e^{2\pi i\,(0.7)}]\right)$
A_2	$\left([0.4.e^{2\pi i\,(0.3)}, 0.5.e^{2\pi i\,(0.7)}], [0.2.e^{2\pi i\,(0.6)}, 0.7.e^{2\pi i\,(0.7)}], [0.6.e^{2\pi i\,(0.3)}, 0.7.e^{2\pi i\,(0.6)}]\right)$	$\left([0.4.e^{2\pi i\,(0.4)}, 0.7.e^{2\pi i\,(0.5)}], [0.3.e^{2\pi i\,(0.3)}, 0.6.e^{2\pi i\,(0.4)}], [0.5.e^{2\pi i\,(0.1)}, 0.8.e^{2\pi i\,(0.8)}]\right)$	$\left([0.4.e^{2\pi i\,(0.1)}, 0.8.e^{2\pi i\,(0.2)}], [0.2.e^{2\pi i\,(0.4)}, 0.4.e^{2\pi i\,(0.7)}], [0.1.e^{2\pi i\,(0.1)}, 0.6.e^{2\pi i\,(0.3)}]\right)$	$\left([0.3.e^{2\pi i\,(0.4)}, 0.8.e^{2\pi i\,(0.5)}], [0.4.e^{2\pi i\,(0.6)}, 0.6.e^{2\pi i\,(0.7)}], [0.2.e^{2\pi i\,(0.3)}, 0.2.e^{2\pi i\,(0.7)}]\right)$
A_3	$\left([0.8.e^{2\pi i\,(0.5)}, 0.8.e^{2\pi i\,(0.8)}], [0.0.e^{2\pi i\,(0.2)}, 0.3.e^{2\pi i\,(0.6)}], [0.5.e^{2\pi i\,(0.5)}, 0.6.e^{2\pi i\,(0.7)}]\right)$	$\left([0.2.e^{2\pi i\,(0.2)}, 0.7.e^{2\pi i\,(0.3)}], [0.1.e^{2\pi i\,(0.1)}, 0.2.e^{2\pi i\,(0.5)}], [0.4.e^{2\pi i\,(0.1)}, 0.8.e^{2\pi i\,(0.2)}]\right)$	$\left([0.7.e^{2\pi i\,(0.2)}, 0.9.e^{2\pi i\,(0.4)}], [0.3.e^{2\pi i\,(0.4)}, 0.3.e^{2\pi i\,(0.7)}], [0.4.e^{2\pi i\,(0.6)}, 0.8.e^{2\pi i\,(0.8)}]\right)$	$\left([0.3.e^{2\pi i\,(0.4)}, 0.4.e^{2\pi i\,(0.8)}], [0.4.e^{2\pi i\,(0.6)}, 0.8.e^{2\pi i\,(0.7)}], [0.4.e^{2\pi i\,(0.7)}, 0.4.e^{2\pi i\,(0.9)}]\right)$
A_4	$\left([0.2.e^{2\pi i\,(0.3)}, 0.2.e^{2\pi i\,(0.7)}], [0.5.e^{2\pi i\,(0.4)}, 0.5.e^{2\pi i\,(0.5)}], [0.2.e^{2\pi i\,(0.4)}, 0.6.e^{2\pi i\,(0.6)}]\right)$	$\left([0.5.e^{2\pi i\,(0.6)}, 0.7.e^{2\pi i\,(0.9)}], [0.3.e^{2\pi i\,(0.4)}, 0.6.e^{2\pi i\,(0.7)}], [0.3.e^{2\pi i\,(0.6)}, 0.3.e^{2\pi i\,(0.7)}]\right)$	$\left([0.2.e^{2\pi i\,(0.1)}, 0.8.e^{2\pi i\,(0.3)}], [0.3.e^{2\pi i\,(0.2)}, 0.4.e^{2\pi i\,(0.8)}], [0.1.e^{2\pi i\,(0.5)}, 0.3.e^{2\pi i\,(0.8)}]\right)$	$\left([0.1.e^{2\pi i\,(0.3)}, 0.9.e^{2\pi i\,(0.6)}], [0.3.e^{2\pi i\,(0.4)}, 0.7.e^{2\pi i\,(0.8)}], [0.5.e^{2\pi i\,(0.3)}, 0.6.e^{2\pi i\,(0.4)}]\right)$

Table 2. Aggregated information using CIVTSFWA operator

	Aggregated information about alternatives
For A_1	$\left([0.30e^{2\pi i(0.61)}, 0.72.\,e^{2\pi i(0.78)}], [0.38e^{2\pi i(0.42)}, 0.60e^{2\pi i(0.7)}], [0.30e^{2\pi i(0.44)}, 0.56e^{2\pi i(0.66)}]\right)$
For A_2	$\left([0.39e^{2\pi i(0.36)}, 0.74e^{2\pi i(0.57)}], [0.26e^{2\pi i(0.42)}, 0.56e^{2\pi i(0.58)}], [0.29e^{2\pi i(0.15)}, 0.57e^{2\pi i(0.56)}]\right)$
For A_3	$\left([0.66e^{2\pi i(0.4)}, 0.80e^{2\pi i(0.7)}], [0.0e^{2\pi i(0.23)}, 0.31e^{2\pi i(0.6)}], [0.42e^{2\pi i(0.32)}, 0.67e^{2\pi i(0.49)}]\right)$
For A_4	$\left([0.40e^{2\pi i(0.5)}, 0.77e^{2\pi i(0.8)}], [0.34e^{2\pi i(0.33)}, 0.53e^{2\pi i(0.69)}], [0.22e^{2\pi i(0.46)}, 0.39e^{2\pi i(0.64)}]\right)$

Table 3. Aggregated information using CIVTSFWG operator

	Aggregated information about alternatives
For A_1	$\left([0.19e^{2\pi i(0.50)}, 0.61e^{2\pi i(0.66)}], [0.41e^{2\pi i(0.59)}, 0.60e^{2\pi i(0.8)}], [0.34e^{2\pi i(0.57)}, 0.61e^{2\pi i(0.78)}]\right)$
For A_2	$\left([0.38e^{2\pi i(0.26)}, 0.69e^{2\pi i(0.42)}], [0.31e^{2\pi i(0.52)}, 0.61e^{2\pi i(0.66)}], [0.49e^{2\pi i(0.26)}, 0.71e^{2\pi i(0.71)}]\right)$
For A_3	$\left([0.41e^{2\pi i(0.28)}, 0.70e^{2\pi i(0.48)}], [0.30e^{2\pi i(0.46)}, 0.58e^{2\pi i(0.64)}], [0.43e^{2\pi i(0.58)}, 0.75e^{2\pi i(0.77)}]\right)$
For A_4	$\left([0.24e^{2\pi i(0.28)}, 0.57e^{2\pi i(0.59)}], [0.39e^{2\pi i(0.38)}, 0.58e^{2\pi i(0.74)}], [0.36e^{2\pi i(0.53)}, 0.50e^{2\pi i(0.71)}]\right)$

Step 4: By using Eq. (6), the computed score values are 0.1022, 0.0590, 0.1294, 0.1245 by CIVTSFWA operator and 0.0312, 0.0319, 0.0348, 0.0220 by CIVTSFWG operator. From these values, the ranking order of the given alternative is observed as $A_3 > A_4 > A_1 > A_2$ through CIVTSFWA operator and as $A_3 > A_2 > A_1 > A_4$ through CIVTSFWG operator. Hence from the analysis, we find that A_3 is the most optimum policy for the investment.

Effect of n on Ranking Analysis

Ullah et al. [36] pointed out that for larger n, the ranking results may alter. Upon checking we observe that the ranking arrangement drastically changes at $n = 7$ in the case of the CIVTSWA operator, although no significant change occurs in the case of the CIVTSFWG operator. At $n = 7$, the ranking results are provided in Table 4. Another change in the ranking pattern occurs at $n = 15$. Again, the ranking result using the CIVTSFWA operator alters as the alternative A_1 becomes more prominent than A_3 while no significant change occurs in the ranking result by the CIVTSFWG operator. The changed ranking pattern at $n = 15$ is given in Table 5.

Table 4. Ranking results for $n = 7$

CIVTSFWA operator	$A_4 > A_3 > A_1 > A_2$
CIVTSFWG operator	$A_3 > A_2 > A_1 > A_4$

Table 5. Ranking results for $n = 15$

CIVTSFWA operator	$A_4 > A_1 > A_3 > A_2$
CIVTSFWG operator	$A_3 > A_2 > A_1 > A_4$

We further varied the value of n to a higher level and did not observe any further change in the ranking pattern, so $n = 15$ may be regarded as a stability value in this case. The decision-maker can select the suitable value of n as per his/her choice.

CONCLUSIONS

The complex FS was introduced to deal with complex information under uncertainty with the help of real and complex membership grades (CMGs). To handle uncertain information, we have introduced the notion of complex interval-valued T-spherical fuzzy set (CIVTSFS) to increase the possibility of taking the grades of memberships in the form of intervals instead of crisp numbers. Some basic operations of the proposed set have been defined and examined to the effect that several existing theories are considered as a special case of it. To aggregate the different elements of the collective CIVTSFNs, we have stated a series of weighted averaging and geometric operators, namely CIVTSFWA, CIVTSFOWA, CIVTSFHA, CIVTSFWG, CIVTSFOWG and CIVTSFHG. The CIVTSFS is then the generalisation of all the previous fuzzy layouts and can be applied to problems discussed in the literature with more reliability.

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