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Full Paper

Effects of wave solutions on shallow-water equation, optical-fibre equation and electric-circuit equation

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Abstract: The fractional non-linearity of space-and-time of Estevez-Mansfield-Clarkson equation, Ablowitz-Kaup-Newell-Segur equation and modified Korteweg-de Vries equation reveals the effects on shallow-water waves, optical-fibre waves and electric-circuit waves respectively. Using the fractional derivative of Jumarie's Riemann-Liouville and a combination of Kudryashov method and the process of establishing the answer in finite series, the procedure is called the transformation of fractional non-linearity of partial differential equations into the non-linearity of ordinary differential equations. The newly discovered analytical solutions take the form of exponential functions, which ultimately leads to the occurrence of physical wave effects. These effects are manifested in kink and periodic waves, and they are separately depicted by 2-D, 3-D and contour graphs.

Keywords: wave solutions, fractional Estevez-Mansfield-Clarkson equation, fractional Ablowitz-Kaup-Newell-Segur equation, fractional modified Korteweg-de Vries equation, Kudryashov method

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INTRODUCTION

The non-linear evolution equations are highly essential equations used in real-world situations in applied mathematics, applied science and engineering, such as plasma waves, water waves, capillary-gravity waves, optical fibres, electric circuits, plasma physics, fluid mechanics, chemical kinematics and chemical physics. Researchers are now interested in exploring numerical solutions [1, 2] or exact solutions and putting the outcomes from the aforementioned studies into practice. To explore the effects of these factors further, it is required to investigate the method to solve the fractional non-linearity of partial differential equations (PDEs). Many new powerful methods of seeking new results of the analytical wave study have been implemented by mathematicians, such as first integral method [3, 4], fractional sub-equation method [5, 6], Poincaré-Lighthill-Kuo method [7, 8], *G′/G*-expansion method [9, 10], modified Kudryashov method [11, 12], generalised Kudryashov method [13, 14] and simple equation method [15, 16].

Estevez-Mansfield-Clarkson (EMC) equation (shallow-water equation) is a fourth-order non-linearity of PDE derived from Mansfield and Clarkson's dispersion of patterns in liquid drop in 1997 [17], which describes the influence of shallow-water wave and has the following form with the constant λ:

$$u_{yyyt} + \lambda u_y u_{yt} + \lambda u_{yy} u_t + u_{tt} = 0. \tag{1}$$

The fourth-order non-linear Ablowitz-Kaup-Newell-Segur (AKNS) equation (optical-fibre equation) was established in 1970 by Ablowitz, Kaup, Newell and Segur [18], who were motivated by the non-linear optical fibre. The AKNS equation with the parameter μ is shown as

$$4u_{xt} + u_{xxxt} + 8u_x u_{xt} + 4u_{xx} u_y - \mu u_{xx} = 0. \tag{2}$$

In 1985 the modified Korteweg-de Vries (mKdV) equation (electric-circuit equation) was developed from the third-order KdV equation, which was reconstructed by Diederik Johannes Korteweg and Gustav de Vries [19, 20]. The mKdV equation has been applied in the study of electric circuits and is given by

$$u_t - \gamma u^2 u_x + u_{xxx} = 0, \tag{3}$$

where γ is a constant.

In this work we have found new analytical wave solutions of the fractional non-linearity of space-and-time EMC equation, the fractional non-linearity of space-and-time AKNS equation and the fractional non-linearity of space-and-time mKdV equation by using the fractional derivative of Jumarie's Riemann-Liouville and the algorithm of Kudryashov method. Moreover, we have shown the wave effects in contour graphs, 2D graphs and 3D graphs.

Definition 1. The fractional derivative of Jumarie's Riemann-Liouville [21] is given as follows.
If $\alpha = 0$,

$$D_t^\alpha f(t) = f(t),$$

if $0 < \alpha < 1$,

$$D_t^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t (1-\psi)^{-\alpha} \left[f(\psi) - f(0) \right] d\psi,$$

if $n \le \alpha < n+1$ and $n \ge 1$,

$$D_t^\alpha f(t) = \frac{d^n}{dt^n} D_t^{\alpha-n} f(t),$$

where α is the fractional derivative order.

The following basic features of the fractional derivatives of Jumarie's Riemann-Liouville were found in 2009 [22]:

$$D_t^\alpha t^m = \frac{\Gamma(m+1)}{\Gamma(m-\alpha+1)}, \quad m \ge 0, \tag{5}$$

$$D_t^\alpha \left[f(t) h(t) \right] = f(t) D_t^\alpha h(t) + h(t) D_t^\alpha f(t), \tag{6}$$

$$D_t^{\alpha} f\left[h(t)\right] = D_t^{\alpha} f\left[h(t)\right]\left[h'(t)\right]^{\alpha}$$
$$= f_h'\left[h(t)\right] D_t^{\alpha} h(t). \tag{7}$$

ALGORITHM OF KUDRYASHOV METHOD

This algorithm goes through how to solve the fractional non-linearity of PDEs using Kudryashov method [23]. The basic structure of the fractional non-linearity of PDEs may be represented as

$$A\left(u, D_x^{\alpha} u, D_y^{\alpha} u, D_t^{\alpha} u, D_x^{2\alpha} u, D_y^{\alpha} D_x^{\alpha} u, D_t^{\alpha} D_x^{\alpha} u, \ldots\right) = 0, 0 < \alpha \le 1, t > 0, \tag{8}$$

where A represents a polynomial in $u(x, y, t)$ and its fractional derivatives. The processes involved in this technique are illustrated here.

First Step: Transformation of fractional non-linearity of PDE to non-linearity of ordinary differential equation (ODE)

Set a solution and use the wave transformation:

$$u(x, y, t) = U(\Psi),\ \Psi = \frac{kx^{\alpha}}{\Gamma(\alpha+1)} + \frac{ly^{\alpha}}{\Gamma(\alpha+1)} - \frac{ct^{\alpha}}{\Gamma(\alpha+1)}, \tag{9}$$

where constants k and l are non-zero, Ψ is a wave transformation term and c is a constant of the wave speed. When c is larger than 0, the wave is travelling in the direction of positive motion, and when c is less than 0, the wave is travelling in the direction of negative motion. Converting Eq. (8) by Eq. (9) to the non-linearity of ODE,

$$B\left(U, U', U'', U''', \ldots\right) = 0, \tag{10}$$

where B represents a polynomial in $U(\Psi)$ and its derivatives.

Second Step: Assumption of the solution

Set the solution of Eq. (10) as a finite series:

$$U = \sum_{i=0}^{M} a_i P^i, \tag{11}$$

where a_i are constants with $a_M \neq 0$ and the Kudryashov method is used to determine P:

$$P' = P^2 - P. \tag{12}$$

The solution of Eq. (12) is

$$P = \frac{1}{1 + de^{\Psi}}, \tag{13}$$

where d is an integrating constant.

Third Step: Identification of M

Find the integer M by finding the balance between the derivative of the highest order term and the non-linear variable in Eq. (10).

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Fourth Step: Obtaining the solutions

To generate the parameters $a_i\left(i=1,2,3,\ldots,M\right)$ and c, collect the coefficients of all terms that have the same order of $P^i\left(i=1,2,3,\ldots\right)$ and set them to zero. Substitute all parameters in Eq. (11); the analytical wave solutions to Eq. (8) are constructed.

APPLICATIONS

The effects of the fractional non-linearity of the three space-and-time equations, i.e. EMC equation, AKNS equation and mKdV equation are explored.

Fractional Non-Linearity of Space-and-Time EMC Equation

The following is the fourth-order fractional non-linearity of space-and-time EMC equation [11, 16]:

$$D_y^{3\alpha} D_t^{\alpha} u + \lambda D_y^{\alpha} u D_y^{\alpha} D_t^{\alpha} u + \lambda D_y^{2\alpha} u D_t^{\alpha} u + D_t^{2\alpha} u = 0,\ 0 < \alpha \le 1, t > 0, \tag{14}$$

where $u=u(x,y,t)$ and λ is a constant. Identifying the solution and carrying out the transformation by the first step, Eq. (14) is transformed into the non-linearity of ODE:

$$-l^3 U^{(4)} - 2\lambda l^2 U' U'' + cU'' = 0. \tag{15}$$

We then obtain Eq. (16) by integrating Eq. (15) with zero constant:

$$-l^3 U''' - l^2 \lambda (U')^2 + cU' = 0. \tag{16}$$

From the second step, Eq. (11) is used to write the solution. Next step, the derivative of the highest order term and the non-linear term of Eq. (16) are balanced, $M=1$. Eq. (11) turns into

$$U(\Psi) = a_0 + a_1 P(\Psi). \tag{17}$$

Eq. (16) is substituted with Eq. (17). In the fourth step we aggregate all the same power terms of $P(\Psi)$ and put zero in each of the coefficients:

$$P^1(\Psi)\ :\ a_1 l^3 - a_1 c = 0, \tag{18}$$

$$P^2(\Psi)\ :\ -7a_1 l^3 - a_1^2 l^2 \lambda + a_1 c = 0, \tag{19}$$

$$P^3(\Psi)\ :\ 12a_1 l^3 + 2a_1^2 l^2 \lambda = 0, \tag{20}$$

$$P^4(\Psi)\ :\ -6a_1 l^3 - a_1^2 l^2 \lambda = 0. \tag{21}$$

Solving the system of Eqs. (18) - (21), we get

$$a_1 = -\frac{6l}{\lambda},\ c = l^3. \tag{22}$$

Considering Eqs. (9), (13), (17) and (22), the analytical solutions of the fractional non-linearity of space-and-time EMC equations are given as

$$u(x,y,t) = a_0 - \frac{6l}{\lambda\left(1+de^{\Psi}\right)}, \tag{23}$$

where d is an integrating constant and $\Psi = \frac{kx^{\alpha}}{\Gamma(\alpha+1)} + \frac{ly^{\alpha}}{\Gamma(\alpha+1)} - \frac{l^3 t^{\alpha}}{\Gamma(\alpha+1)}$.

For the fractional EMC equation, a comparison of solutions is shown in Table 1, using the G'/G-expansion method [24] and Kudryashov method. It is quite clear that the structure of our solutions is less complicated.

Following that, the sample graph of the wave effect of the fractional non-linearity of space-and-time EMC equation is obtained by adjusting specific parameters. Figure 1 shows the exact exponential solutions in the form of kink wave influence, changing from one state to another, when we set $a_0 = 0,\ \lambda = -1,\ l = 1,\ k = 1,\ d = 1,\ \alpha = 0.5,\ 10 \le x \le 200,\ 10 \le y \le 200,\ t = 100$ (Figure 1a) and 300 (Figure 1b) in Eq. (23). Figure 2 shows the periodic wave influence, the wave form that repeats at regular intervals, when we set the parameters in Eq. (23) as $a_0 = 0,\ \lambda = -1,\ l = 1,\ k = 1,\ d = -1,\ \alpha = 0.5,\ 0 \le x \le 300,\ 0 \le y \le 300,\ t = 100$ (Figure 2a) and $t = 300$ (Figure 2b).

Table 1. Comparing differences of exact solutions to the fractional EMC equation

G'/G **- expansion method**	**Kudryashov method**
Case 1: $\lambda^2 - 4\mu > 0$, $u = a_0 + \frac{6l}{\beta}\left(-\frac{\lambda}{2} + \frac{\sqrt{c/l^3}}{2}\left(\frac{c_1 \sinh\left(\frac{\sqrt{c/l^3}}{2}\zeta\right) + c_2 \cosh\left(\frac{\sqrt{c/l^3}}{2}\zeta\right)}{c_1 \cosh\left(\frac{\sqrt{c/l^3}}{2}\zeta\right) + c_2 \sinh\left(\frac{\sqrt{c/l^3}}{2}\zeta\right)}\right)\right)$,	$u = a_0 - \frac{6l}{\lambda\left(1 + de^{\Psi}\right)}$, where $\Psi = \frac{kx^\alpha}{\Gamma(\alpha+1)} + \frac{ly^\alpha}{\Gamma(\alpha+1)} - \frac{l^3 t^\alpha}{\Gamma(\alpha+1)}$.
Case 2: $\lambda^2 - 4\mu < 0$, $u = a_0 + \frac{6l}{\beta}\left(-\frac{\lambda}{2} + \frac{\sqrt{-c/l^3}}{2}\left(\frac{-c_1 \sin\left(\frac{\sqrt{-c/l^3}}{2}\zeta\right) + c_2 \cos\left(\frac{\sqrt{-c/l^3}}{2}\zeta\right)}{c_1 \cos\left(\frac{\sqrt{-c/l^3}}{2}\zeta\right) + c_2 \sin\left(\frac{\sqrt{-c/l^3}}{2}\zeta\right)}\right)\right)$,	
Case 3: $\lambda^2 - 4\mu = c = 0$, $u = a_0 + \frac{6l}{\beta}\left(-\frac{\lambda}{2} + \frac{c_2}{c_1 + c_2\zeta}\right)$,	
where $\zeta = \frac{kx^\alpha}{\Gamma(\alpha+1)} + \frac{ly^\alpha}{\Gamma(\alpha+1)} - \frac{ct^\alpha}{\Gamma(\alpha+1)}$.	

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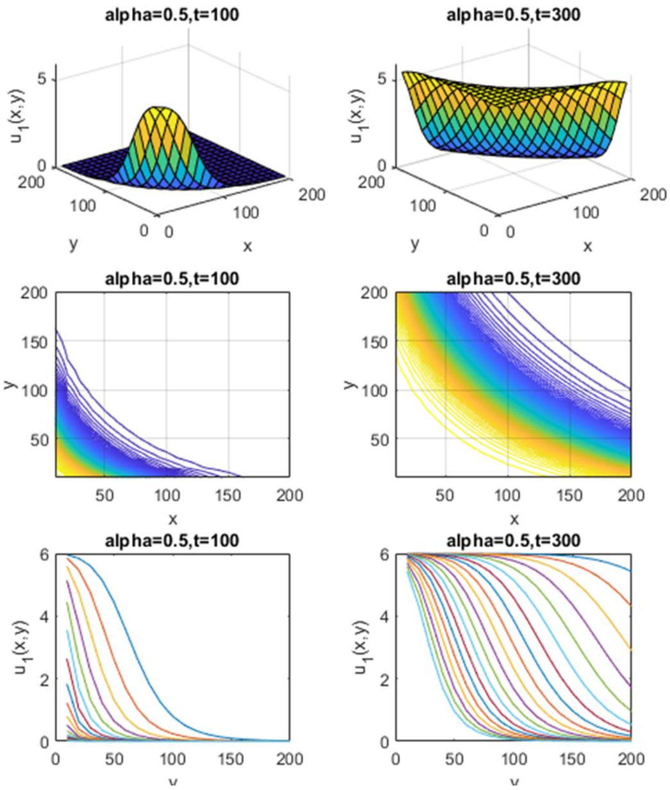
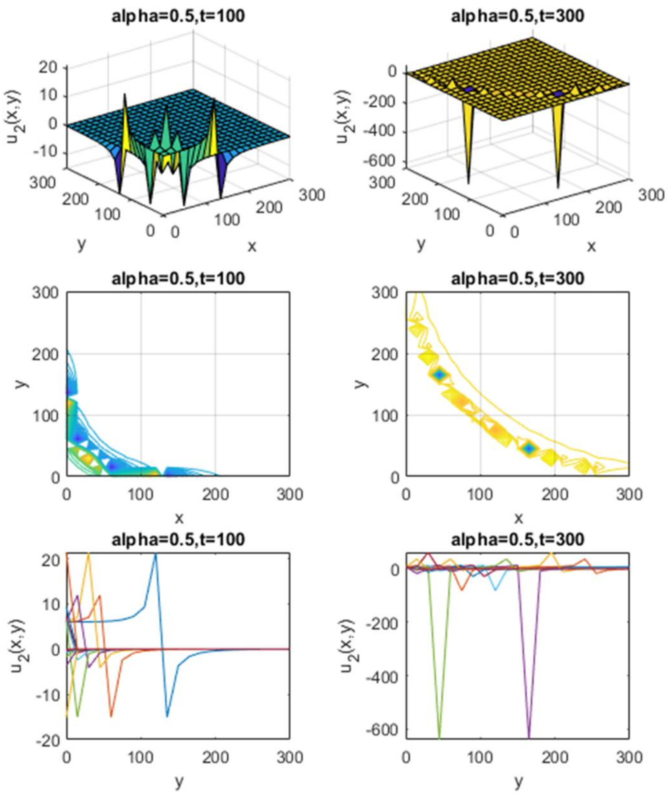


(a) (b)

Figure 1. 3-D, contour and 2-D kink graphs of Eq. (23)



(a) (b)

Figure 2. 3-D, contour and 2-D periodic graphs of Eq. (23)

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Fractional Non-Linearity of Space-and-Time AKNS Equation

The fourth-order fractional non-linearity of space-and-time AKNS equation is defined [11, 16]:

$$4D_x^{\alpha} D_t^{\alpha} u + D_x^{3\alpha} D_t^{\alpha} u + 8D_x^{\alpha} u D_x^{\alpha} D_y^{\alpha} u + 4D_x^{2\alpha} u D_x^{\alpha} u - \mu D_x^{2\alpha} u = 0, 0 < \alpha \leq 1, t > 0, \tag{24}$$

where $u = u(x, y, t)$ and μ is a constant. Transforming Eq. (24) by Eq. (9), we get

$$-(4c + \mu k)U'' - ck^2 U^{(4)} + 12klU'U'' = 0. \tag{25}$$

Integrating Eq. (25) and setting to zero, we get

$$-(4c + \mu k)U' - ck^2 U''' + 6kl(U')^2 = 0. \tag{26}$$

Balancing Eq. (26), $M = 1$, Eq. (11) is transformed into

$$U(\Psi) = a_0 + a_1 P(\Psi). \tag{27}$$

Replacing Eq. (26) by Eq. (27) and collecting the coefficients of all terms with the same power and setting them to zero, we get

$$P^1(\Psi) \ : \ 4ca_1 + a_1\mu k + ca_1 k^2 = 0, \tag{28}$$

$$P^2(\Psi) \ : \ -4ca_1 - a_1\mu k - 7c_1 k^2 + 6ka_1^2 l = 0, \tag{29}$$

$$P^3(\Psi) \ : \ 12ca_1 k^2 - 12ka_1^2 l = 0, \tag{30}$$

$$P^4(\Psi) \ : \ -6ca_1 k^2 + 6ka_1^2 l = 0. \tag{31}$$

Solving the system of Eqs. (28) - (31), we obtain

$$a_1 = \frac{-\mu k^2}{l(4+k^2)}, \ c = \frac{-\mu k}{(4+k^2)}. \tag{32}$$

The analytical solutions of the fractional non-linearity of space-and-time AKNS equations are described as

$$u(x, y, t) = a_0 - \frac{\mu k^2}{l(4+k^2)(1+de^{\Psi})}, \tag{33}$$

where d is an integrating constant and $\Psi = \dfrac{kx^{\alpha}}{\Gamma(\alpha+1)} + \dfrac{ly^{\alpha}}{\Gamma(\alpha+1)} + \dfrac{\mu kt^{\alpha}}{(4+k^2)\Gamma(\alpha+1)}$.

The Kudryashov method and G'/G-expansion method [24] are compared in Table 2 for their ability to solve the fractional AKNS problem. The solution using the Kudryashov method is an easier form to work with than that using the G'/G-expansion method.

To obtain a kink graph of the wave influence of the fractional non-linearity of the space-and-time AKNS equation, we set $a_0 = 0, \ \mu = -1, \ l = 1, \ k = 1, \ d = 1, \ \alpha = 0.5, \ 1 \leq x \leq 5, \ 1 \leq y \leq 5, \ t = 100$ (Figure 3a) and t = 1000 (Figure 3b). In Figure 4a the periodic wave influence is shown by setting $a_0 = 0, \ \mu = -1, \ l = 1, \ k = 1, \ d = -1, \ \alpha = 0.5, \ 0 \leq x \leq 5, \ 0 \leq y \leq 5, \ t = 100$. It is shown again with $a_0 = 0, \ \mu = -1, \ l = 1, \ k = 1, \ d = -1, \ \alpha = 0.5, \ 0 \leq x \leq 5, \ 0 \leq y \leq 5, \ t = 200$ in Figure 4b.

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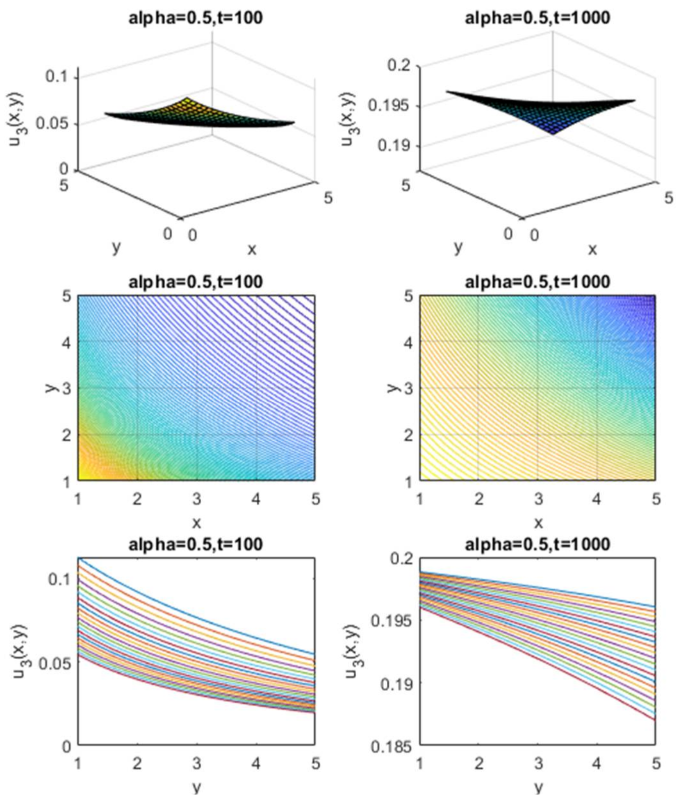
Table 2. Comparing differences of exact solutions to the fractional AKNS equation

G'/G **-expansion method**	**Kudryashov method**
Case 1: $\lambda^2 - 4\mu > 0$, $$u = a_0 + \frac{6l}{\beta}\left(-\frac{\lambda}{2} + \frac{\sqrt{c/l^3}}{2}\left(\frac{c_1 \sinh\left(\frac{\sqrt{c/l^3}}{2}\zeta\right) + c_2 \cosh\left(\frac{\sqrt{c/l^3}}{2}\zeta\right)}{c_1 \cosh\left(\frac{\sqrt{c/l^3}}{2}\zeta\right) + c_2 \sinh\left(\frac{\sqrt{c/l^3}}{2}\zeta\right)}\right)\right),$$ Case 2: $\lambda^2 - 4\mu < 0$, $$u = a_0 + \frac{6l}{\beta}\left(-\frac{\lambda}{2} + \frac{\sqrt{-c/l^3}}{2}\left(\frac{-c_1 \sin\left(\frac{\sqrt{-c/l^3}}{2}\zeta\right) + c_2 \cos\left(\frac{\sqrt{-c/l^3}}{2}\zeta\right)}{c_1 \cos\left(\frac{\sqrt{-c/l^3}}{2}\zeta\right) + c_2 \sin\left(\frac{\sqrt{-c/l^3}}{2}\zeta\right)}\right)\right),$$ Case 3: $\lambda^2 - 4\mu = c = 0$, $$u = a_0 + \frac{6l}{\beta}\left(-\frac{\lambda}{2} + \frac{c_2}{c_1 + c_2\zeta}\right),$$ where $\zeta = \frac{kx^\alpha}{\Gamma(\alpha+1)} + \frac{ly^\alpha}{\Gamma(\alpha+1)} - \frac{ct^\alpha}{\Gamma(\alpha+1)}$.	$$u = a_0 - \frac{\mu k^2}{l(4+k^2)(1+de^{\Psi})},$$ where $$\Psi = \frac{kx^\alpha}{\Gamma(\alpha+1)} + \frac{ly^\alpha}{\Gamma(\alpha+1)} + \frac{\mu k t^\alpha}{(4+k^2)\Gamma(\alpha+1)}.$$



(a) (b)

Figure 3. 3-D, contour and 2-D kink graphs of Eq. (33)

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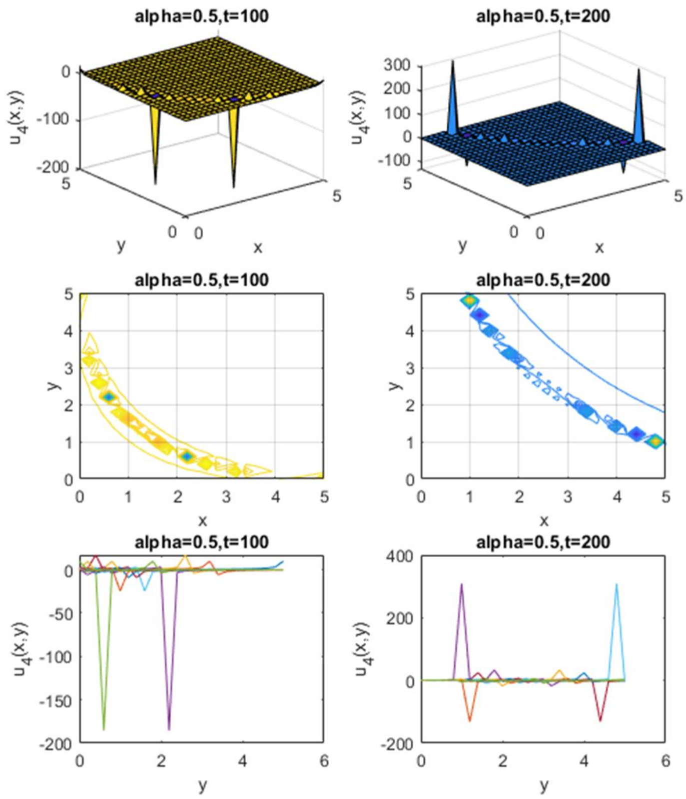

(a) (b)

Figure 4. 3-D, contour and 2-D periodic graphs of Eq. (33)

Fractional Non-Linearity of Space-and-Time mKdV Equation

The third-order fractional non-linearity of space-and-time mKdV equation [19, 20] is defined as

$$D_t^{\alpha}u-\gamma u^2 D_x^{\alpha}u+D_x^{3\alpha}u=0, 0<\alpha\le 1, t>0, \tag{34}$$

where $u=u(x,t)$ and γ is a constant. Eq. (34) is converted into an ODE by Eq. (9) without considering the variable y:

$$-cU'-\gamma kU^2U'+k^3U'''=0. \tag{35}$$

By integrating Eq. (35) with a constant of zero, we get

$$-cU-\frac{\gamma kU^3}{3}+k^3U''=0. \tag{36}$$

Balancing Eq. (36), we found $M=1$. Eq. (11) is evolved into

$$U(\Psi)=a_0+a_1P(\Psi). \tag{37}$$

Eq. (36) is replaced with Eq. (37). When we group all the same power terms of $P(\Psi)$ and put zero in each of the coefficients, we get

$$P^0(\Psi)\ :\ -ca_0-\frac{\gamma ka_0^3}{3}=0, \tag{38}$$

$$P^1(\Psi)\ :\ -ca_1-\gamma ka_0^2a_1+k^3a_1=0, \tag{39}$$

$$P^2(\Psi)\ :\ -\gamma ka_0a_1^2-3k^3a_1=0, \tag{40}$$

$$P^3(\Psi)\ :\ -\frac{\gamma ka_1^3}{3}+2k^3a_1=0. \tag{41}$$

The system of Eqs. (38) - (41) is solved and the result is

$$a_0=\sqrt{\frac{3k^2}{2\gamma}},\ a_1=-\sqrt{\frac{6k^2}{\gamma}},c=-\frac{k^3}{2}, \tag{42}$$

$$a_0=-\sqrt{\frac{3k^2}{2\gamma}},\ a_1=\sqrt{\frac{6k^2}{\gamma}},c=-\frac{k^3}{2}. \tag{43}$$

The following are the analytical wave solutions to the fractional non-linearity of space-and-time mKdV equations:

$$u(x,t)=\sqrt{\frac{3k^2}{2\gamma}}-\sqrt{\frac{6k^2}{\gamma}}\left(\frac{1}{1+de^{\Psi}}\right), \tag{44}$$

$$u(x,t)=-\sqrt{\frac{3k^2}{2\gamma}}+\sqrt{\frac{6k^2}{\gamma}}\left(\frac{1}{1+de^{\Psi}}\right), \tag{45}$$

where d is an integrating constant and $\Psi=\frac{kx^\alpha}{\Gamma(\alpha+1)}+\frac{k^3t^\alpha}{2\Gamma(\alpha+1)}$.

Table 3 presents a comparison of the solutions to the fractional mKdV equation obtained by the use of generalised G'/G-expansion approach [25] and Kudryashov method. The solutions of the Kudryashov method appear to be simpler than those of the generalised G'/G-expansion approach.

Table 3. Comparing differences of exact solutions to the fractional mKdV equation

Generalised G'/G-expansion method	**Kudryashov method**
$u_1=\pm\frac{3\delta}{A\sqrt{6\delta}}\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2A}\Phi\right),u_2=\pm\frac{3\delta}{A\sqrt{6\delta}}\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2A}\Phi\right),$	$u=\sqrt{\frac{3k^2}{2\gamma}}-\sqrt{\frac{6k^2}{\gamma}}\left(\frac{1}{1+de^{\Psi}}\right),$
$u_3=\pm\frac{3i\delta}{A\sqrt{6\delta}}\sqrt{\Omega}\cot\left(\frac{\sqrt{-\Omega}}{2A}\Phi\right),u_4=\pm\frac{3i\delta}{A\sqrt{6\delta}}\sqrt{\Omega}\tan\left(\frac{\sqrt{-\Omega}}{2A}\Phi\right),$	$u=-\sqrt{\frac{3k^2}{2\gamma}}+\sqrt{\frac{6k^2}{\gamma}}\left(\frac{1}{1+de^{\Psi}}\right)$
$u_5=\pm\frac{6\delta\psi}{A\sqrt{6\delta}}\left(\frac{C_2}{C_1+C_2\Phi}\right),u_6=\pm\frac{1}{A\sqrt{6\delta}}\left(-3\delta\left(B-2\sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{A}\Phi\right)\right)\right),$	where
$u_7=\pm\frac{1}{A\sqrt{6\delta}}\left(-3\delta\left(B-2\sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{A}\Phi\right)\right)\right),u_8=\pm\frac{1}{A\sqrt{6\delta}}\left(-3\delta\left(B-2i\sqrt{\Delta}\cot\left(\frac{\sqrt{-\Delta}}{A}\Phi\right)\right)\right),$	$\Psi=\frac{kx^\alpha}{\Gamma(\alpha+1)}+\frac{k^3t^\alpha}{2\Gamma(\alpha+1)}.$
$u_9=\pm\frac{1}{A\sqrt{6\delta}}\left(-3\delta\left(B+2i\sqrt{\Delta}\tan\left(\frac{\sqrt{-\Delta}}{A}\Phi\right)\right)\right),$	
where $\Phi=x+\frac{\delta}{2A^2}\left(B^2+4E\psi\right)t,\psi=A-C,\Omega=B^2+4E(A-C),\Delta=\psi E.$	

Next, we choose the parameters $a_0=0,\gamma=1,\ d=1,\ k=1,\ \alpha=0.5,1\le x\le 20$ and $1\le t\le 30$ to get the kink wave influence of Eq. (44) and Eq. (45), which are shown in Figure 5 and Figure 6 respectively.

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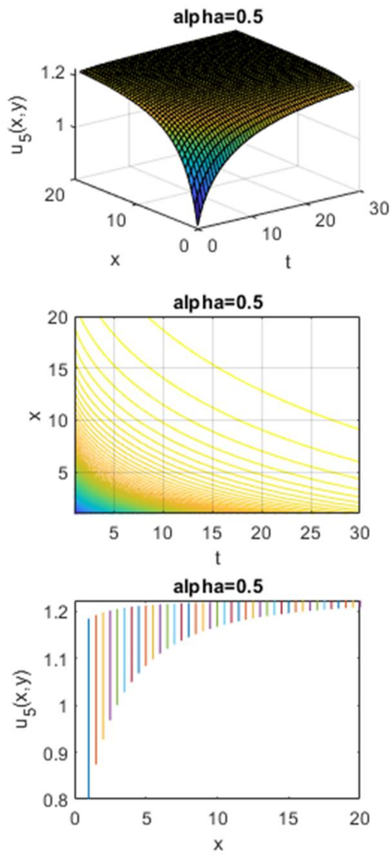

Figure 5. 3-D, contour and 2-D kink graphs of Eq. (44)

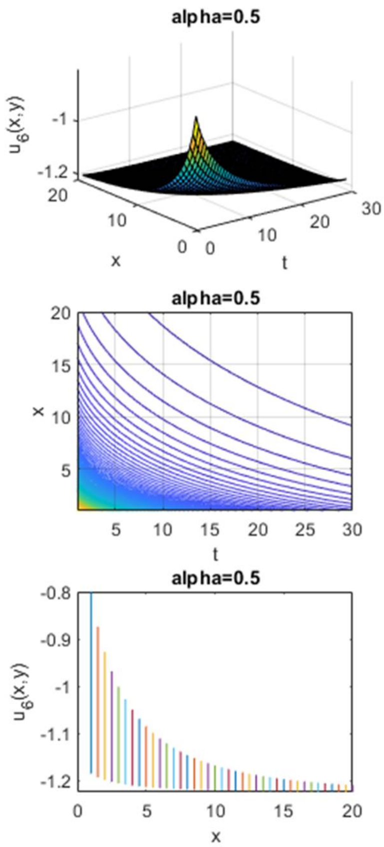

Figure 6. 3-D, contour and 2-D kink graphs of Eq. (45)

CONCLUSIONS

The fractional non-linearity of space-and-time EMC equation, space-and-time AKNS equation and space-and-time mKdV equation has been solved for the analytical wave solutions by a powerful method, namely Kudryashov method. As a direct consequence of this, the new analytical solutions to these equations take the form of exponential functions and the influences of the wave solutions on the fractional EMC equation (shallow-water equation) and fractional AKNS equation (optical-fibre equation) are shown as a kink wave and a periodic wave, and the influence of the wave solutions on the fractional mKdV equation (electric-circuit equation) is shown as a kink wave. Moreover, the exact solutions by the Kudryashov method to all of these equations have a form that is easier to understand than the solution obtained through the earlier technique.

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