

Technical Note

Determination of normal height of Chornohora summits by precise modern measurement technique

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Received: 14 April 2019 / Accepted: 18 May 2020 / Published: 11 June 2020

Abstract: The heights of the two Ukrainian peaks - Hoverla and Pip Ivan located in the Chornohora mountain range - were determined by using Global Navigation Satellite System observations and a global quasi-geoid model. The height of Hoverla, the highest Ukrainian summit, 2056.0 ± 0.5 m, and that of Pip Ivan is 2019.4 ± 0.5 m. The results are several metres lower than the official altitudes.

Keywords: Chornohora mountain, Global Navigation Satellite System, mountain height determination, Hoverla, Pip Ivan

INTRODUCTION

The highest peak of the Chornohora Mountains, a part of the Outer Eastern Carpathians, the highest mountain range in the Ukraine, is Hoverla (2,061 m, Figures 1, 2) [1]. Pip Ivan (2,022 m) is the third highest Ukrainian peak, after Hoverla and Brebeneskul [2]. Only six Ukrainian mountains reach over 2,000 m and all of them are located in the Chornohora Mountains. The other three are Petros, Rebra and Hutyn Tomnatyk [4]. The Chornohora Mountains are a widely studied area in the Earth sciences [5–11], but the authors of this paper did not find any elaborations concerning the determination of the height of any of these Ukrainian peaks.

Formerly the Chornohora mountain range was the historical border of Galicia and Lodomeria and the Polish-Czechoslovak border [12]. Currently it is the border between two Ukrainian districts - Transcarpathian and Ivano-Frankivsk. The reason for our research was a certain inaccuracy and uncertainty regarding the heights of Hoverla and Pip Ivan based on the historical cartographic and geographic materials.

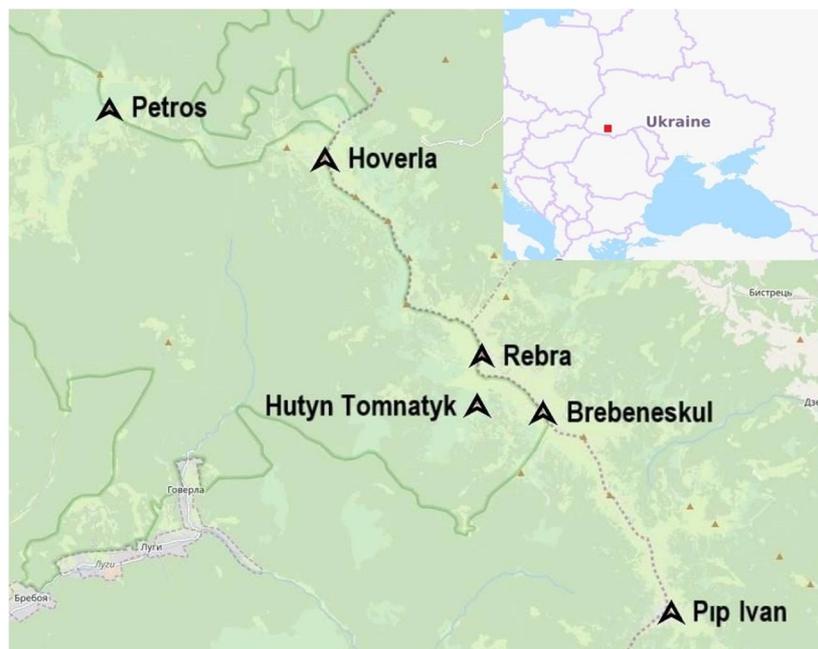


Figure 1. Location of the Ukrainian two-thousanders [3]



Figure 2. Summits of Hoverla (left) and Pip Ivan (right)

CHORNOHORA HEIGHTS

When analysing historical cartographic materials concerning the heights of the Chornohora peaks, Austria-Hungarian maps from 1855-1914 [13] and Polish maps from 1930 – 1933 [13] were found. In addition, the Chornohora's peak heights are given in a book [14]. This book contains three different heights of Hoverla, given in metres, feet and fathoms, while the height of Pip Ivan appears only once. Table 1 contains a summary from all the above-mentioned source materials.

The heights presented in Table 1 vary between 2050.7 - 2058.0 m (difference 7.3 m) for Hoverla and 2013.1 - 2026.0 m (difference 12.9 m) for Pip Ivan. The height of Hoverla expressed in feet (7,200 ft = 2,276.0 m) is most likely incorrect. The altitude differences are results of the adopted reference level, measurement accuracy and the selected method of unit conversion (Figure 3).

Table 1. Historical heights of Hoverla and Pip Ivan

Source		Hoverla		Pip Ivan	
Name	Year	Non- metric	Metric	Non-metric	Metric
Austria-Hungarian map [13]	1855	1,081.3 vf ¹	2,050.7 m	1,061,5 vf	2,013.1 m
	1873	1,082.0 vf	2,052.0 m	1,062.0 vf	2,014.1 m
	1876	-	2,058.0 m	-	2,026.0 m
Słownik geograficzny Królestwa Polskiego (Geographical dictionary of the Kingdom of Poland) [14]	1880	-	2,058.0 m	-	2,026.0 m
		1,081.3 vf	2,050.7 m	-	
		7,200 ft ²	2,276.0 m	-	
Austria-Hungarian map [13]	1914	-	2,057.0 m	-	2,026.0 m
Polish map [13]	1930	-	2,058.0 m	-	2,026.0 m
Polish map [13]	1933	-	2,058.0 m	-	2,022.0 m
This paper's results	2018	-	2,055.0 m	-	2,019.4 m

¹ Vienna fathom = 1.896484 m [14], ² Viennese foot = 0.3161 m [14]

**Figure 3.** Part of Austria-Hungarian map of Hoverla and Pip Ivan from 1855 and 1914 [13]

Figure 4 presents historical pictures of Pip Ivan. The left picture shows the triangulation tower where probably the heights were referenced before World War II. The right picture shows the triangulation tower with the Bilyi Slon observatory in the background. The height of the triangulation tower in Figure 4 is about 3 - 4 m and probably some of the determined Pip Ivan heights were not reduced to the terrain. On the Hoverla top there is a concrete monument (Figure 7B) which is also a former triangulation point. The heights presented in Table 1 might also refer to the top of the monument.

METHODS OF PEAK HEIGHT DETERMINATION

There were three different methods of determining mountain heights above sea level or other adopted reference surface: the geodetic method, the barometric method and the boiling point method [17]. The geodetic method is based on levelling [18] and triangulation (angular-linear intersections). The barometric method is based on the air pressure difference between sea level and the top of the mountain; this method is rarely used, rather as a scientific issue [19]. The boiling point method (also



Figure 4. Historical plane surveyor's table on triangulation tower and observatory on top of Pip Ivan [15, 16]

called thermometric) is based on the rise of the boiling point of water with the increase in atmospheric pressure. The most accurate of the three is the geodetic method, which is applied in practical mountain height determination. For example, the first determination of Mount Everest after World War II was made by a combination of triangulation and trigonometric levelling with included deflection of the plumb-line [20, 21].

In the 20th century satellite techniques become the simplest, fastest and most accurate way of height determination [22–26]. Global Navigation Satellite System (GNSS) provides the surveyors with three dimensional coordinates in the geocentric reference frame, usually World Geodetic System 1984. Thus, satellite techniques provide ellipsoidal heights. In practice the height above sea level is used. There are a few methods for determination of orthometric (normal) heights. In the case of GNSS observations where the ellipsoidal height is known, the most common is the use of the geoid (quasi-geoid) model [22]. Depending on the area of elaboration, a local or global model of geoid (quasi-geoid) is used. The local geoid (quasi-geoid) is usually more accurate than the global one. Nowadays, global geopotential models with a centimetre accuracy are available, e.g. Earth Gravitational Model 2008 (EGM2008) [27], which are sufficient for determining the height of mountain peaks. As research shows, heights determined by a satellite technique might vary up to several metres from the currently known heights obtained by classic measurements [28]. In 2005 Mount Everest's height was determined by various kinds of geodetic techniques such as GNSS, gravimetric and trigonometric levelling [29, 30].

Determination of orthometric (normal) height is burdened by an ellipsoidal height error, as well as the geoid (quasi-geoid) model error, according to the law of propagation of errors:

$$m_H = \sqrt{m_h^2 + m_N^2} \quad (1)$$

where m_H is normal height error, m_h is ellipsoidal height error and m_N is geoid (quasi-geoid) model error. Currently GNSS height error and geoid (quasi-geoid) model error are at level of a few cm. Former surveys showed even a 1-m geoid (quasi-geoid) error. It should also be borne in mind that in mountainous areas geoid error can take extreme values [31] due to the mass of the mountains. Figure

5 presents height anomaly at the Chornohora mountains region on the basis of the EGM2008 geopotential model.

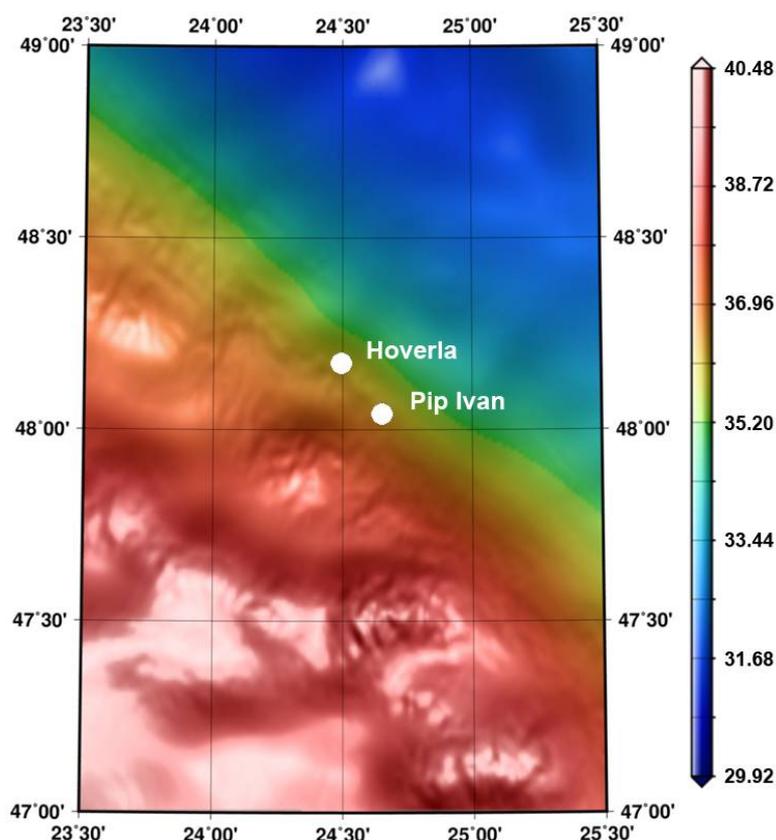


Figure 5. EGM2008 height anomaly at Chornohora area (scale in metre) [32]

METHODOLOGY

The GNSS technique was chosen to determine the height of the summit of Pip Ivan and Hoverla. Although the GNSS technique is very convenient and relatively precise in height determination, it gives results in ellipsoidal coordinates, so a determined height is referred to ellipsoid, not to the mean sea level (reference level - geoid in orthometric height system, and quasi-geoid in a normal height system) [33]. The difference between the ellipsoid and geoid is the geoid height N in the orthometric height system. The difference between ellipsoidal height and height above sea level is called height anomaly in the normal height system (Figure 6). The height anomaly in the normal height system, usually marked as ζ , is in the range of -100 to 100 m over the whole Earth. In the Chornohora region the height anomaly is about 36 m (Figure 5).

The normal height (H) determination consists of the ellipsoidal height (h) measurement, usually with GNSS techniques, and the determination of height anomaly according to a simple formula:

$$H = h - \zeta . \quad (2)$$

The height anomaly ζ may be obtained from global or regional geoid models.

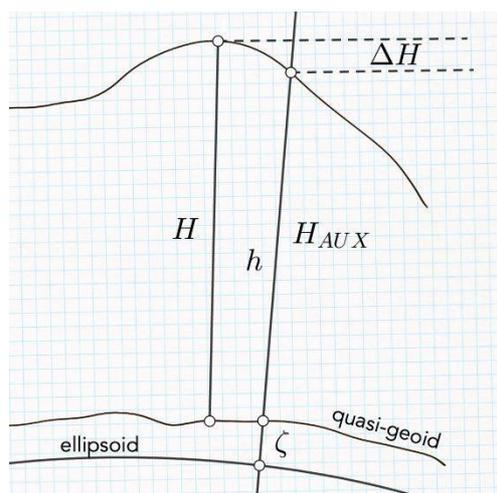


Figure 6. Ellipsoidal height (h), normal height (H, H_{AUX}) and height anomaly (ζ)

MEASUREMENTS

The measurements were carried out by a two two-person measuring team with geodetic instruments (GNSS receiver, precision leveller). The base of border mark no. 16 on Pip Ivan (Figures 7D, 8) on the west side of the observatory was adopted as the peak. It is the same border mark as shown in Figure 4. In the case of Hoverla, a concrete mark with a notched cross was adopted as the highest point (Figures 7C, 8). The highest summit points are with high sky obstructions on Pip Ivan and in the location of the main tourist route on Hoverla. Therefore, for the GNSS measurements session, auxiliary points with maximum exposure of the horizon were chosen. The auxiliary points were located a few metres next to the highest points (Hoverla - Figure 7A, Pip Ivan - Figure 8E).

GNSS measurements at auxiliary points using a Leica GS16 were made on 13 September 2018 during a single 5-hour measuring session, between 8:00 - 13:00 GMT. The observations were made with a 1-sec. interval and 0° elevation cut-off angle. After the GNSS session, the height difference between the auxiliary and highest points was determined by geometric levelling using a Leica Sprinter leveller. The normal height system is legally operative on Ukrainian territory; thus the EGM2008 quasi-geoid model and height anomaly values are used for the calculations.

RESULTS

The geodetic coordinates of the auxiliary points were determined using Precise Point Positioning (PPP) technique in the International Terrestrial Reference Frame (ITRF2014 realization). Based on ellipsoidal coordinates (φ, λ) and the EGM2008 model, the height anomaly of these points was calculated using the EGM2008 Harmonic Synthesis Program [27]. The normal heights (above sea level) of the auxiliary points were calculated from equation (2). The distance between the auxiliary and highest points was a few metres so height anomalies were adopted as being the same. The results are presented in Table 2.

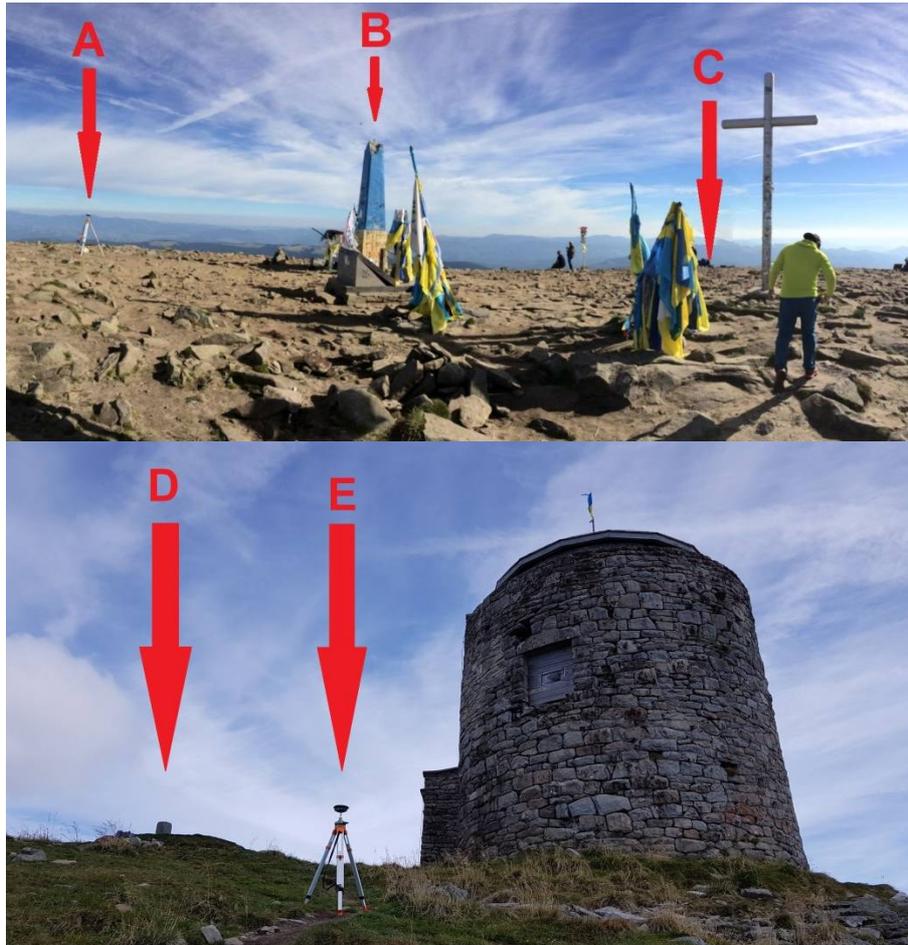


Figure 7. Hoverla (top) and Pip Ivan (bottom) summits during the experiment



Figure 8. Geodetic marks on the top of Pip Ivan (left) and Hoverla (right)

Table 2. Results of peak height determination

	h (m)	ζ (m)	dH (m)	H (m)
Pip Ivan	2053.527	36.805	2.634	2019.356
Hoverla	2092.239	36.507	0.228	2055.960

Note: h - ellipsoidal height of the auxiliary point, ζ - height anomaly, dH - difference from levelling between auxiliary point and geodetic mark, H - summit normal height

For result verification, differential processing of the Pip Ivan-Hoverla baseline was also performed. The differences of vector components dx , dy , dz from the differential method, compared with those obtained by PPP technique, did not exceed 2 cm. In accordance with the measurement method and the geoid model used, the accuracy of the height determination can be estimated as the sum of three error components: auxiliary point height from GNSS measurement (m_h) = 0.05 m, height anomaly from EGM2008 model (m_ζ) = 0.50 m [34], and height difference from levelling (m_{dH}) = 0.001 m. According to the uncertainty propagation, the summit height determination error may be represented by equation (3) - a modified formula of (1):

$$m_H = \sqrt{m_h^2 + m_\zeta^2 + m_{dH}^2} . \quad (3)$$

The obtained accuracy of the summit height determination (m_H) is about 0.5 m. Among all of the errors - GNSS measurement, levelling and height anomaly model - the last one is significant; the others may be considered negligibly small without affecting the accuracy of the height determination.

CONCLUSIONS

The PPP method is accurate enough for ellipsoidal height determination. A comparison of the PPP results with post-processed Pip Ivan-Hoverla baseline shows compatibility at a level of several centimetres. Global geoid models for the Chornohora Mountains enable us to determine the summit heights with an accuracy of 0.5 m.

ACKNOWLEDGEMENT

The article was created as part of the statutory research of AGH No. 16.16.150.545.

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