Maejo Int. J. Sci. Technol. **2019**, *13*(03), 257-265

Maejo International Journal of Science and Technology

ISSN 1905-7873
Available online at www.mijst.mju.ac.th

Full Paper

A new study on absolute summability factors of infinite series

Hikmet S. Özarslan and Ahmet Karakaş *

Department of Mathematics, Erciyes University, Kayseri, 38039, Turkey

* Corresponding author, e-mail: ahmetkarakas1985@hotmail.com

Received: 29 May 2018 / Accepted: 18 December 2019 / Published: 27 December 2019

Abstract: We first give a new and more general definition on absolute matrix summability of infinite series. Then we generalise a known theorem on $\left|\bar{N}, p_n\right|_k$ summability factors of infinite series with this summability method by using an almost increasing sequence. This new theorem also includes several new results.

Keywords: summability factors, absolute matrix summability, almost increasing sequence, infinite series, Hölder inequality, Minkowski inequality

---

INTRODUCTION

Summability theory plays an important role in analysis, applied mathematics and engineering sciences. The aim of this theory is to bring a value to the indefinite divergent series. Various summability methods have been introduced by different researchers to find this value. Some of these methods are given by Cesàro [1], Abel [2], Nörlund [3], Riesz [4], matrix summability [5], etc.

A significant rise of the summability began in the latter part of the 19th century. In 1890, Cesàro published a paper on the multiplication of series [1]. Das gave the definition of absolute summability [6]. Then Kishore and Hotta defined the summability factor [7]. The definition of $\left|A\right|_k$ summability was given by Tanović-Miller [8]. Later Bor defined $\left|\bar{N}, p_n\right|_k$ and $\left|\bar{N}, p_n; \delta\right|_k$ summability of an infinite series [9, 10]. The definition of $\left|A, p_n; \delta\right|_k$ summability of an infinite series was defined by Özarslan and Öğdük [11]. In this paper a theorem on absolute matrix summability is obtained using a more general summability method.

Now we give some definitions related to the summability which are used in this article.

Definition 1 [12]. A positive sequence (r_n) is said to be almost increasing, if there exist a positive increasing sequence (d_n) and two positive constants K and L such that $Kd_n \le r_n \le Ld_n$. Obviously every increasing sequence is almost increasing sequence but the converse need not be true as can be seen from the example $b_n = n e^{(-1)^n}$.

Definition 2 [13]. Let $\sum a_n$ be a given infinite series with the partial sums (s_n). Let (p_n) be a sequence of positive numbers such that

$$P_n = \sum_{v=0}^{n} p_v \to \infty \quad \text{as} \quad n \to \infty, \; (P_{-i} = p_{-i} = 0, \; i \ge 1). \tag{1}$$

The sequence-to-sequence transformation

$$w_n = \frac{1}{P_n} \sum_{v=0}^{n} p_v s_v \tag{2}$$

defines the sequence (w_n) of the $(\bar{N}, p_n)$ means of the sequence (s_n), generated by the sequence of coefficients (p_n).

Definition 3 [9]. The series $\sum a_n$ is said to be summable $\left|\bar{N}, p_n\right|_k$, $k \ge 1$, if

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{k-1} |w_n - w_{n-1}|^k < \infty . \tag{3}$$

Definition 4 [14]. The series $\sum a_n$ is said to be summable $\left|\bar{N}, p_n, \beta; \delta\right|_k$, $k \ge 1$, $\delta \ge 0$ and β is a real number if

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{\beta(\delta k + k - 1)} |w_n - w_{n-1}|^k < \infty . \tag{4}$$

Definition 5 [15]. Let $A = (a_{nv})$ be a normal matrix, i.e. a lower triangular matrix of non-zero diagonal entries. Then A defines the sequence-to-sequence transformation, mapping the sequence $s = (s_n)$ to $As = (A_n(s))$, where

$$A_n(s) = \sum_{v=0}^{n} a_{nv} s_v, \; n = 0,1,... \tag{5}$$

The series $\sum a_n$ is said to be summable $\left|A, p_n\right|_k$, $k \ge 1$ if

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{k-1} |\bar{\Delta} A_n(s)|^k < \infty , \tag{6}$$

where

$$\bar{\Delta} A_n(s) = A_n(s) - A_{n-1}(s) . \tag{7}$$

Definition 6 [16]. The series $\sum a_n$ is summable $\left|A, p_n, \beta; \delta\right|_k$, $k \ge 1$, $\delta \ge 0$ and β is a real number if

$$\sum_{n=1}^{\infty}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\left|\bar{\Delta} A_n(s)\right|^k<\infty . \tag{8}$$

If we take $\beta=1$, then $|A,p_n,\beta;\delta|_k$ summability reduces to $|A,p_n;\delta|_k$ summability [11]. Also, if we take $\beta=1$ and $\delta=0$, then $|A,p_n,\beta;\delta|_k$ summability reduces to $|A,p_n|_k$ summability. Furthermore, if we take $\beta=1$, then $|\overline{N},p_n,\beta;\delta|_k$ summability reduces to $|\overline{N},p_n;\delta|_k$ summability [6]. Also, if we take $\beta=1$ and $\delta=0$, then $|\overline{N},p_n,\beta;\delta|_k$ summability reduces to $|\overline{N},p_n|_k$ summability.

KNOWN RESULT

Bor [17] has proved the following theorem for $|\overline{N},p_n|_k$ summability factors of infinite series.

Theorem 1. Let (p_n) be a sequence of positive numbers such that

$$P_n=O(n p_n) \quad as \quad n\to\infty . \tag{9}$$

If (X_n) is a positive monotonic non-decreasing sequence such that

$$|\lambda_m| X_m=O(1) \quad as \quad m\to\infty, \tag{10}$$

$$\sum_{n=1}^{m} n X_n\left|\Delta^2\lambda_n\right|=O(1) \quad as \quad m\to\infty, \tag{11}$$

$$\sum_{n=1}^{m}\frac{p_n}{P_n}|t_n|^k=O(X_m) \quad as \quad m\to\infty, \tag{12}$$

where $t_n=\frac{1}{n+1}\sum_{v=1}^{n} v a_v$, then the series $\sum a_n\lambda_n$ is summable $|\overline{N},p_n|_k$, $k\geq 1$.

MAIN RESULT

Much work on absolute matrix summability of infinite series has been done [18-29]. The aim of this paper is to generalise Theorem 1 to $|A,p_n,\beta;\delta|_k$ summability. We first introduce some further notations.

Given a normal matrix $A=(a_{nv})$, we associate two lower semi-matrices $\bar{A}=(\bar{a}_{nv})$ and $\hat{A}=(\hat{a}_{nv})$ as follows:

$$\bar{a}_{nv}=\sum_{i=v}^{n} a_{ni}, \quad n,v=0,1,... \tag{13}$$

and

$$\hat{a}_{00}=\bar{a}_{00}=a_{00}, \quad \hat{a}_{nv}=\bar{a}_{nv}-\bar{a}_{n-1,v}, \quad n=1,2,... \tag{14}$$

It may be noted that $\bar{A}$ and $\hat{A}$ are the well-known matrices of series-to-sequence and series-to-series transformations respectively. Then we have

$$A_n(s)=\sum_{v=0}^{n} a_{nv}s_v=\sum_{v=0}^{n}\bar{a}_{nv}a_v \tag{15}$$

and

Maejo Int. J. Sci. Technol. **2019**, *13*(03), 257-265

$$\bar{\Delta} A_n(s) = \sum_{v=0}^{n} \hat{a}_{nv} a_v . \tag{16}$$

Now we shall prove the following theorem.

Theorem 2. Let $A=(a_{nv})$ be a positive normal matrix such that

$$\bar{a}_{n0} = 1 , \quad n = 0,1,..., \tag{17}$$

$$a_{n-1,v} \geq a_{nv}, \text{ for } n \geq v+1, \tag{18}$$

$$a_{nn} = O\left(\frac{p_n}{P_n}\right), \tag{19}$$

$$\left|\hat{a}_{n,v+1}\right| = O\left(v\left|\Delta_v(\hat{a}_{nv})\right|\right). \tag{20}$$

Let (X_n) be an almost increasing sequence. If conditions (10)-(11) of Theorem 1 and

$$\sum_{n=1}^{m} \left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k} |t_n|^k = O(X_m) \quad as \quad m \to \infty, \tag{21}$$

$$\sum_{n=v+1}^{\infty} \left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1} |\Delta_v(\hat{a}_{nv})| = O\left(\left(\frac{P_v}{p_v}\right)^{\beta(\delta k+k-1)-k}\right) \tag{22}$$

are satisfied, then the series $\sum a_n \lambda_n$ is summable $|A, p_n, \beta; \delta|_k$, $k \geq 1$, $\delta \geq 0$ and $-\beta(\delta k + k - 1) + k > 0$.

We need the following lemma for the proof of Theorem 2.

Lemma 1 [30]**.** Under the conditions on (X_n) and (λ_n) which are taken in the statement of our theorem, we have the following:

$$n X_n |\Delta\lambda_n| = O(1) \quad as \quad n \to \infty, \tag{23}$$

$$\sum_{n=1}^{\infty} X_n |\Delta\lambda_n| < \infty . \tag{24}$$

PROOF OF THOREM 2

Let (I_n) denote A-transform of the series $\sum a_n \lambda_n$. Then we have by (15) and (16)

$$\bar{\Delta} I_n = \sum_{v=1}^{n} \hat{a}_{nv} a_v \lambda_v = \sum_{v=1}^{n} \frac{\hat{a}_{nv}\lambda_v}{v} v a_v .$$

Applying Abel's transformation to this sum, we obtain the following.

$$\bar{\Delta} I_n = \sum_{v=1}^{n-1} \Delta_v \left(\frac{\hat{a}_{nv}\lambda_v}{v}\right) \sum_{r=1}^{v} r a_r + \frac{\hat{a}_{nn}\lambda_n}{n} \sum_{r=1}^{n} r a_r$$

$$= \sum_{v=1}^{n-1} \Delta_v \left(\frac{\hat{a}_{nv}\lambda_v}{v}\right)(v+1)t_v + \frac{a_{nn}\lambda_n}{n}(n+1)t_n$$

$$= \frac{n+1}{n} a_{nn}\,\lambda_n\,t_n + \sum_{v=1}^{n-1}\frac{v+1}{v}\Delta_v(\hat{a}_{nv})\,\lambda_v\,t_v + \sum_{v=1}^{n-1}\frac{v+1}{v}\hat{a}_{n,v+1}\,\Delta\lambda_v\,t_v + \sum_{v=1}^{n-1}\frac{1}{v}\hat{a}_{n,v+1}\,\lambda_{v+1}\,t_v$$

$$= I_{n,1} + I_{n,2} + I_{n,3} + I_{n,4}\,.$$

Since $\left|I_{n,1}+I_{n,2}+I_{n,3}+I_{n,4}\right|^k \le 4^k\left(\left|I_{n,1}\right|^k+\left|I_{n,2}\right|^k+\left|I_{n,3}\right|^k+\left|I_{n,4}\right|^k\right)$, to complete the proof of Theorem 2 it is sufficient to show that

$$\sum_{n=1}^{\infty}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\left|I_{n,r}\right|^k < \infty, \quad \text{for } r=1,2,3,4. \qquad (25)$$

First, by using Abel's transformation, we have

$$\sum_{n=1}^{m}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\left|I_{n,1}\right|^k = O(1)\sum_{n=1}^{m}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} a_{nn}^k\left|\lambda_n\right|^k\left|t_n\right|^k$$

$$= O(1)\sum_{n=1}^{m}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\left(\frac{p_n}{P_n}\right)^k\left|\lambda_n\right|^{k-1}\left|\lambda_n\right|\left|t_n\right|^k$$

$$= O(1)\sum_{n=1}^{m}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k}\left|\lambda_n\right|\left|t_n\right|^k$$

$$= O(1)\sum_{n=1}^{m-1}\Delta\left|\lambda_n\right|\sum_{v=1}^{n}\left(\frac{P_v}{p_v}\right)^{\beta(\delta k+k-1)-k}\left|t_v\right|^k + O(1)\left|\lambda_m\right|\sum_{n=1}^{m}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k}\left|t_n\right|^k$$

$$= O(1)\sum_{n=1}^{m-1}\left|\Delta\lambda_n\right|X_n + O(1)\left|\lambda_m\right|X_m$$

$$= O(1) \quad as \quad m\to\infty$$

by virtue of the hypotheses of Theorem 2 and Lemma 1. Now applying Hölder's inequality with indices k and k', where $k>1$ and $\frac{1}{k}+\frac{1}{k'}=1$ as in $I_{n,1}$, we have

$$\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\left|I_{n,2}\right|^k = O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\left(\sum_{v=1}^{n-1}\left|\Delta_v(\hat{a}_{nv})\right|\left|\lambda_v\right|\left|t_v\right|\right)^k$$

$$= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\left(\sum_{v=1}^{n-1}\left|\Delta_v(\hat{a}_{nv})\right|\left|\lambda_v\right|^k\left|t_v\right|^k\right)\text{x}\left(\sum_{v=1}^{n-1}\left|\Delta_v(\hat{a}_{nv})\right|\right)^{k-1}$$

$$= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} a_{nn}^{k-1}\left(\sum_{v=1}^{n-1}\left|\Delta_v(\hat{a}_{nv})\right|\left|\lambda_v\right|^k\left|t_v\right|^k\right).$$

Now using the fact that $a_{nn} = O\left(\frac{p_n}{P_n}\right)$ by (19), we have

$$\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} \left| I_{n,2}\right|^k = O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1} \sum_{v=1}^{n-1}\left|\Delta_v(\hat{a}_{nv})\right|\left|\lambda_v\right|^k\left|t_v\right|^k$$

$$= O(1)\sum_{v=1}^{m}\left|\lambda_v\right|^k\left|t_v\right|^k \sum_{n=v+1}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1} \left|\Delta_v(\hat{a}_{nv})\right|$$

$$= O(1)\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\beta(\delta k+k-1)-k} \left|\lambda_v\right|^{k-1}\left|\lambda_v\right|\left|t_v\right|^k$$

$$= O(1)\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\beta(\delta k+k-1)-k} \left|\lambda_v\right|\left|t_v\right|^k$$

$$= O(1) \quad as \quad m\to\infty$$

by virtue of the hypotheses of Theorem 2 and Lemma 1.

Now using Hölder's inequality, we have

$$\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} \left| I_{n,3}\right|^k = O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} \left(\sum_{v=1}^{n-1}\left|\hat{a}_{n,v+1}\right|\left|\Delta\lambda_v\right|\left|t_v\right|\right)^k$$

$$= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} \left(\sum_{v=1}^{n-1} v\left|\Delta_v(\hat{a}_{nv})\right|\left|\Delta\lambda_v\right|\left|t_v\right|\right)^k$$

$$= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} \left(\sum_{v=1}^{n-1} v\left|\Delta_v(\hat{a}_{nv})\right|\left|\Delta\lambda_v\right|\left|t_v\right|^k\right)$$

$$\mathrm{x}\left(\sum_{v=1}^{n-1} v\left|\Delta_v(\hat{a}_{nv})\right|\left|\Delta\lambda_v\right|\right)^{k-1}$$

$$= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} a_{nn}^{k-1}\left(\sum_{v=1}^{n-1} v\left|\Delta_v(\hat{a}_{nv})\right|\left|\Delta\lambda_v\right|\left|t_v\right|^k\right).$$

By using (19), we have

$$\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} \left| I_{n,3}\right|^k = O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1} \sum_{v=1}^{n-1} v\left|\Delta_v(\hat{a}_{nv})\right|\left|\Delta\lambda_v\right|\left|t_v\right|^k$$

$$= O(1)\sum_{v=1}^{m} v\left|\Delta\lambda_v\right|\left|t_v\right|^k \sum_{n=v+1}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1} \left|\Delta_v(\hat{a}_{nv})\right|$$

$$= O(1)\sum_{v=1}^{m} v\left|\Delta\lambda_v\right|\left|t_v\right|^k \left(\frac{P_v}{p_v}\right)^{\beta(\delta k+k-1)-k}$$

$$= O(1)\sum_{v=1}^{m-1}\Delta\left(v\left|\Delta\lambda_v\right|\right)\sum_{r=1}^{v}\left(\frac{P_r}{p_r}\right)^{\beta(\delta k+k-1)-k} \left|t_r\right|^k$$

$$+ O(1)\, m\left|\Delta\lambda_m\right|\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\beta(\delta k+k-1)-k} \left|t_v\right|^k$$

$$= O(1)\sum_{v=1}^{m-1} v\,|\Delta^2\lambda_v|\,X_v + O(1)\sum_{v=1}^{m-1}|\Delta\lambda_v|\,X_v + O(1)\,m\,|\Delta\lambda_m|\,X_m$$
$$= O(1) \quad as \quad m \to \infty$$

by virtue of the hypotheses of Theorem 2 and Lemma 1.

Finally, as in $I_{n,1}$, we have

$$\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} |I_{n,4}|^k = O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\left(\sum_{v=1}^{n-1}\frac{1}{v}|\hat{a}_{n,v+1}||\lambda_{v+1}||t_v|\right)^k$$
$$= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\left(\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})||\lambda_{v+1}||t_v|\right)^k$$
$$= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\left(\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})||\lambda_{v+1}|^k|t_v|^k\right)$$
$$\mathrm{x}\left(\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})|\right)^{k-1}$$
$$= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} a_{nn}^{k-1}\left(\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})||\lambda_{v+1}|^{k-1}|\lambda_{v+1}||t_v|^k\right)$$
$$= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1}\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})||\lambda_{v+1}||t_v|^k$$
$$= O(1)\sum_{v=1}^{m}|\lambda_{v+1}||t_v|^k\sum_{n=v+1}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1}|\Delta_v(\hat{a}_{nv})|$$
$$= O(1)\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\beta(\delta k+k-1)-k}|\lambda_{v+1}||t_v|^k$$
$$= O(1) \quad as \quad m \to \infty$$

by virtue of the hypotheses of Theorem 2 and Lemma 1. This completes the proof of Theorem 2.

CONCLUSIONS

If we take $\beta = 1$, then we get a theorem dealing with $|A, p_n; \delta|_k$ summability. If we take (X_n) as a positive monotonic non-decreasing sequence, $\beta = 1$, $\delta = 0$ and $a_{nv} = \frac{p_v}{P_n}$, then we get Theorem 1. In this case condition (21) reduces to condition (12).

REFERENCES

1. E. Cesàro, "Sur la multiplication des séries", *Bull. Sci. Math.*, **1890**, *14*, 114-120.
2. N. H. Abel, "Untersuchungen über die Reihe: $1+\frac{m}{1}x+\frac{m(m-1)}{1.2}x^2+\frac{m(m-1)(m-2)}{1.2.3}x^3+\cdots$", *J. Reine Angew. Math.*, **1826**, *1*, 311-339.

3. N. E. Nörlund, "Sur une application des fonctions permutables", *Lunds Univ. Arssk.*, **1919**, *16*, 1-10.
4. M. Riesz, "Sur l'equivalence de certaines méthodes de sommation", *Proc. London Math. Soc.*, **1924**, *22*, 412-419.
5. G. M. Petersen, "Regular Matrix Transformations", McGraw-Hill, New York, **1966**.
6. G. Das, "Tauberian theorems for absolute Nörlund summability", *Proc. London Math. Soc.*, **1969**, *S3-19* , 357-384.
7. N. Kishore and G. C. Hotta, "On $\left|\bar{N}, p_n\right|$ summability factors", *Acta Sci. Math. (Szeged)*, **1970**, *31*, 9-12.
8. N. Tanović-Miller, "On strong summability", *Glas. Mat. Ser. III*, **1979**, *14*, 87-97.
9. H. Bor, "On two summability methods", *Math. Proc. Cambridge Philos. Soc.*, **1985**, *97*, 147-149.
10. H. Bor, "On local property of $\left|\bar{N}, p_n;\delta\right|_k$ summability of factored Fourier series", *J. Math. Anal. Appl.*, **1993**, *179*, 646-649.
11. H. S. Özarslan and H. N. Öğdük, "Generalizations of two theorems on absolute summability methods", *Aust. J. Math. Anal. Appl.*, **2004**, *1*, Art.no.13.
12. N. K. Bari and S. B. Stečkin, "Best approximations and differential properties of two conjugate functions", *Trudy Moskov. Mat. Obsc.*, **1956**, *5*, 483-522 (in Russian).
13. G. H. Hardy, "Divergent Series", Oxford University Press, Oxford, **1949**.
14. A. N. Gürkan, "Absolute summability methods of infinite series", *PhD Thesis*, **1998**, Erciyes University, Turkey.
15. W. T. Sulaiman, "Inclusion theorems for absolute matrix summability methods of an infinite series. IV", *Indian J. Pure Appl. Math.*, **2003**, *34*, 1547-1557.
16. H. S. Özarslan and A. Karakaş, "On generalized absolute matrix summability of infinite series", *Commun. Math. Appl.*, **2019**, *10*, 439-446.
17. H. Bor, "On absolute summability factors", *Proc. Amer. Math. Soc.*, **1993**, *118*, 71-75.
18. A. Karakaş, "On absolute matrix summability factors of infinite series", *J. Class. Anal.*, **2018**, *13*, 133-139.
19. H. S. Özarslan and H. N. Öğdük, "On absolute matrix summability methods", *Math. Commun.*, **2007**, *12*, 213-220.
20. H. S. Özarslan, "A note on $\left|A, p_n\right|_k$ summability factors", *Antarct. J. Math.*, **2010**, *7*, 23-30.
21. H. S. Özarslan, "A new application of almost increasing sequences", *Miskolc Math. Notes*, **2013**, *14*, 201-208.
22. H. S. Özarslan, "On generalized absolute matrix summability", *Asia Pacific J. Math.*, **2014**, *1*, 150-156.
23. H. S. Özarslan and M. Ö. Şakar, "A new application of absolute matrix summability", *Math. Sci. Appl. E-Notes*, **2015**, *3*, 36-43.
24. H. S. Özarslan, "A new application of absolute matrix summability", *C. R. Acad. Bulgare Sci.*, **2015**, *68*, 967-972.
25. H. S. Özarslan, "A new study on generalized absolute matrix summability", *Commun. Math. Appl.*, **2016**, *7*, 303-309.

26. H. S. Özarslan, "A new application of generalized almost increasing sequences", *Bull. Math. Anal. Appl.*, **2016**, *8*, 9-15.
27. H. S. Özarslan, "An application of δ-quasi monotone sequence", *Int. J. Anal. Appl.*, **2017**, *14*, 134-139.
28. H. S. Özarslan, "An application of absolute matrix summability using almost increasing and δ-quasi-monotone sequences", *Kyungpook Math. J.*, **2019**, *59*, 233-240.
29. H. S. Özarslan, "A new factor theorem for absolute matrix summability", *Quaest. Math.*, **2019**, *42*, 803-809.
30. S. M. Mazhar, "Absolute summability factors of infinite series", *Kyungpook Math. J.*, **1999**, *39*, 67-73.