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Full Paper

Multifarious correlations for *p*-adic gamma function and weighted *q*-Daehee polynomials

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Abstract: Diverse relations and identities for *p*-adic gamma function and *p*-adic Euler constant by means of weighted *p*-adic *q*-integral on $\mathbf{Z}_p$ and Mahler expansion of the function are investigated. Then several correlations and formulas including the *p*-adic gamma function, weighted *q*-Daehee polynomials and weighted *q*-Daehee polynomials of the second kind are derived. An intriguing representation of the *p*-adic Euler constant via the weighted *q*-Daehee polynomials and numbers is also provided.

Keywords: *p*-adic numbers, *p*-adic gamma function, *p*-adic Euler constant, Mahler expansion, *q*-Daehee polynomials, *p*-adic *q*-integral

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INTRODUCTION

The set of all *p*-adic numbers is a counter-intuitive arithmetic system which was first introduced by Kummer [1] around 1850. Then Hensel [2], a German mathematician, initially improved the *p*-adic number system during a study concerned with the advancement of algebraic numbers in power series. With these developments of the aforementioned numbers, many physicists and mathematicians started to research novel scientific tools by utilising their useful and effective properties. Several consequences of this research have emerged in mathematics and physics, such as *p*-adic analysis, algebraic geometry, complex systems, dynamical systems, genetic codes, string theory, *p*-adic quantum mechanics, quantum field theory and representation theory [1-18]. One of the most significant concepts of the mentioned investigations is *p*-adic gamma function, which was originally introduced by Morita [17] in 1975. Intensive research activity in areas such as *p*-adic gamma function is primarily motivated by the importance of *p*-adic analysis. Hence the past forty years have played host to numerous extensive studies and investigations regarding the p-adic gamma function and its generalisations [3, 7, 9-11, 17, 18].

The Daehee polynomials $D_n(x)$ are defined by the following exponential generating function:

$$\sum_{n=0}^{\infty} D_n(x)\frac{t^n}{n!} = \frac{\log(1+t)}{t}(1+t)^x. \quad (1)$$

In the case where $x = 0$, one can get $D_n(0) := D_n$, standing for n-th Daehee number [4, 6-8, 12, 14].

Let p be a prime number. For any non-zero integer a, let $ord_p a$ be the highest power of p that divides a, i.e. the greatest m such that $a \equiv 0(mod p^m)$, where the notation $a \equiv b(mod c)$ means c divides $a - b$. Note that $ord_p 0 = \infty$. The p-adic norm of x is given by $|x|_p = p^{-ord_p x}$ with $x \neq 0$ and $|0|_p = 0$ [3, 13, 18].

Here are some common number sets: $\mathbb{N}$ denotes the set of all natural numbers, $\mathbb{Q}_p$ denotes the field of all p-adic numbers, $\mathbb{Z}_p$ denotes the ring of all p-adic integers and $\mathbb{C}_p$ denotes the completion of the algebraic closure of $\mathbb{Q}_p$.

The q-number is defined by $[n]_q = \frac{q^n-1}{q-1}$. The symbol q can be considered either as indeterminate, a complex number with $0 < |q| < 1$, or a p-adic number with $|q-1|_p < p^{-\frac{1}{p-1}}$ so that $q^x = \exp(x\log q)$ for $|x|_p \leq 1$.

For $f \in UD(\mathbb{Z}_p) = \{f | f$ is uniformly differentiable function at a point $a \in \mathbb{Z}_p\}$, the weighted q-Volkenborn integral or the weighted p-adic q-integral on $\mathbb{Z}_p$ of a function $f \in UD(\mathbb{Z}_p)$ is defined [6] as follows:

$$I_q^{(\alpha,\beta)}(f) = \int_{\mathbb{Z}_p} q^{-\beta x} f(x) d\mu_{q^\alpha}(x) = \lim_{N\to\infty} \frac{1}{[p^N]_{q^\alpha}} \sum_{k=0}^{p^N-1} f(k) q^{(\alpha-\beta)k}. \quad (2)$$

Remark 1. *When $\alpha = 1$ and $\beta = 0$ in (2), we attain the p-adic q-integral defined by Kim [13].*
Remark 2. *As q approaches 1 in (2), we obtain the familiar p-adic integral [12].*

Suppose that $f_1(x) = f(x+1)$. We then see that [6]

$$q^{\alpha-\beta} I_q^{(\alpha,\beta)}(f_1) = I_q^{(\alpha,\beta)}(f) + (q-1)\frac{[\alpha]_q}{\alpha}(\alpha-\beta)f(0) + \frac{[\alpha]_q}{\alpha}\frac{q-1}{\log q} f'(0). \quad (3)$$

The weighted q-Daehee numbers $D_{n,q}^{(\alpha,\beta)}$ and the weighted q-Daehee polynomials $D_{n,q}^{(\alpha,\beta)}(x)$ are defined [6] as follows:

$$\sum_{n=0}^{\infty} D_{n,q}^{(\alpha,\beta)} \frac{t^n}{n!} = \frac{(q-1)\frac{[\alpha]_q}{\alpha}(\alpha-\beta) + \frac{[\alpha]_q}{\alpha}\frac{q-1}{\log q}\log(1+t)}{tq^{\alpha-2\beta} + q^{\alpha-2\beta} - 1} = \int_{\mathbb{Z}_p} q^{-\beta y}(1+t)^y d\mu_{q^\alpha}(y) \quad (4)$$

and

$$\sum_{n=0}^{\infty} D_{n,q}^{(\alpha,\beta)}(x) \frac{t^n}{n!} = \int_{\mathbb{Z}_p} q^{-\beta y}(1+t)^{x+y} d\mu_{q^\alpha}(y). \quad (5)$$

It is obvious that $\lim_{q\to 1} D_{n,q} := D_n$ and $\lim_{q\to 1} D_{n,q}(x) := D_n(x)$.

The weighted q-Daehee numbers $\widehat{D}_{n,q}^{(\alpha,\beta)}$ and polynomials $\widehat{D}_{n,q}^{(\alpha,\beta)}(x)$ of the second kind are introduced by the following expressions [6]:

$$\sum_{n=0}^{\infty} \widehat{D}_{n,q}^{(\alpha,\beta)} \frac{t^n}{n!} = (q-1)\frac{[\alpha]_q}{\alpha} \frac{\alpha-\beta-\frac{\log(1+t)}{\log q}}{q^{\alpha-2\beta-t-1}} (1+t)^1 = \int_{\mathbb{Z}_p} q^{-\beta y}(1+t)^{-y} d\mu_{q^\alpha}(y) \quad (6)$$

and

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$$\sum_{n=0}^{\infty} \widehat{D}_{n,q}^{(\alpha,\beta)}(x)\frac{t^n}{n!} = \int_{\mathbb{Z}_p} q^{-\beta y}(1+t)^{-(x+y)} d\mu_{q^\alpha}(y). \tag{7}$$

Remark 3. *When $\alpha = 1$, the polynomials (5) and (7) reduce to the generalised q-Daehee polynomials and generalised q-Daehee polynomials of the second kind [8].*
Remark 4. *Upon setting $\alpha = 1 = \beta$, the polynomials (5) and (7) turn into the modified q-Daehee polynomials and modified q-Daehee polynomials of the second kind [4].*
Remark 5. *In the case of $\alpha = 1$ and $\beta = 0$, the polynomials (5) and (7) reduce to the q-Daehee polynomials and q-Daehee polynomials of the second kind [12].*
Remark 6. *As q approaches 1, the polynomials (5) and (7) turn into the classical Daehee polynomials and classical Daehee polynomials of the second kind [14].*

The q-Daehee polynomials, along with numbers and various extensions for them, have been studied and developed by many authors [3, 6-8, 12, 14].

The falling factorial $(x)_n := x(x-1)(x-2)\cdots(x-n+1)$ satisfies the following identity:

$$(x)_n = \sum_{k=0}^{n} S_1(n,k)x^k, \tag{8}$$

where $S_1(n,k)$ denotes the Stirling number of the first kind [3, 6-8, 12, 14].

The p-adic gamma function is defined by a limit expression as follows:

$$\Gamma_p(x) = \lim_{n\to x}(-1)^n \prod_{\substack{j<n \\ (p,j)=1}} j \qquad (x \in \mathbb{Z}_p), \tag{9}$$

where *n* approaches *x* through positive integers. The p-adic Euler constant γ_p is given by the following formula:

$$\gamma_p := -\frac{\Gamma_p'(1)}{\Gamma_p(0)} = \Gamma_p'(1) = -\Gamma_p'(0). \tag{10}$$

The p-adic gamma function, as well as its several extensions, and the p-adic Euler constant have been developed by many physicists and mathematicians [3, 7, 9-11, 17, 18].

For $x \in \mathbb{Z}_p$, the symbol $\binom{x}{n}$ is given by $\binom{x}{0} = 1$ and $\binom{x}{n} = \frac{x(x-1)\cdots(x-n+1)}{n!}$ $(n \in \mathbb{N})$. The functions $x \to \binom{x}{n}$ form an orthonormal base of the space $C(\mathbb{Z}_p \to \mathbb{C}_p)$ with respect to the Euclidean norm $\|\cdot\|_\infty$. The aforementioned orthonormal base satisfies the following equality [6,15,18]:

$$\binom{x}{n}' = \sum_{j=0}^{n-1} \frac{(-1)^{n-j-1}}{n-j}\binom{x}{j}. \tag{11}$$

Mahler [15, 16] investigated a generalisation for continuous maps of p-adic variables by making use of the special polynomials as binomial coefficient polynomials. This means that for any $f \in C(\mathbb{Z}_p \to \mathbb{C}_p)$, there exist unique elements $a_1, a_2, a_3, \ldots$ of $\mathbb{C}_p$ such that

$$f(x) = \sum_{n=0}^{\infty} a_n \binom{x}{n} \quad (x \in \mathbb{Z}_p). \tag{12}$$

The base $\left\{\binom{*}{n} : n \in \mathbb{N}\right\}$ is named Mahler base of space $C(\mathbb{Z}_p \to \mathbb{C}_p)$, and the components $\{a_n : n \in \mathbb{N}\}$ in $f(x) = \sum_{n=0}^{\infty} a_n \binom{x}{n}$ are called Mahler coefficients of $f \in C(\mathbb{Z}_p \to \mathbb{C}_p)$. The Mahler expansion of the p-adic gamma function Γ_p and its Mahler coefficients are determined [18] as demonstrated below.

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Proposition 1. *For $x \in \mathbb{Z}_p$, let $\Gamma_p(x+1) = \sum_{n=0}^{\infty} a_n \binom{x}{n}$ be Mahler series Γ_p. Then its coefficients satisfy the following identity:*

$$\sum_{n\geq 0} (-1)^{n+1} a_n \frac{x^n}{n!} = \frac{1-x^p}{1-x} \exp\left(x + \frac{x^p}{p}\right). \tag{13}$$

The outline of this paper is as follows: the first part is an introduction, which provides the required information, notations, definitions and motivation. In the second part we attempt to discover some correlations including the p-adic gamma function and the weighted q-Daehee polynomials, by utilising the methods of the weighted p-adic q-integral and Mahler series expansion. In the last part we analyse the results obtained in this paper.

MAIN RESULTS

This section gives some properties, identities and correlations for the preceding function, the weighted q-Daehee polynomials and numbers, and the p-adic Euler constant. We start by providing the following theorem.

Theorem 1. *The following relation holds true for* $n \geq 0$:

$$\int_{\mathbb{Z}_p} q^{-\beta x} \binom{x}{n} d\mu_{q^\alpha}(x) = \frac{1-q}{1-q^{\alpha-2\beta}} \left(\frac{[\alpha]_q}{\alpha} \frac{(\alpha-\beta) q^{(\alpha-2\beta)n}}{(1-q^{\alpha-2\beta})^n} - \frac{[\alpha]_q}{\log q^\alpha} \sum_{k=0}^{n} \frac{(-1)^k q^{(\alpha-2\beta)(n-k)}}{k(1-q^{\alpha-2\beta})^{n-k}} \right). \tag{14}$$

Proof. From (4), we see that

$$\sum_{n\geq 0} \left(\int_{\mathbb{Z}_p} q^{-\beta x} \binom{x}{n} d\mu_{q^\alpha}(x) \right) t^n = \frac{1-q}{1-q^{\alpha-2\beta}} \frac{\frac{[\alpha]_q}{\alpha}(\alpha-\beta) + \frac{[\alpha]_q}{\alpha} \frac{1}{\log q} \log(1+t)}{1 - \frac{q^{\alpha-2\beta}}{1-q^{\alpha-2\beta}} t}$$

$$= \frac{1-q}{1-q^{\alpha-2\beta}} \sum_{n\geq 0} \frac{q^{(\alpha-2\beta)n}}{(1-q^{\alpha-2\beta})^n} t^n \left(\frac{[\alpha]_q}{\alpha}(\alpha-\beta) + \frac{[\alpha]_q}{\alpha} \frac{1}{\log q} \log(1+t) \right),$$

which yields the asserted result (14).

Remark 7. *In the special case $q \to 1$, the formula (14) reduces to Kim's result [14].*
Remark 8. *Setting $\alpha = 1$ and $\beta = 0$, the result (14) implies Kim's result [12].*

Here is the explicit formula for p-adic gamma function in the following theorem.

Theorem 2. *Let x be a p-adic integer and n be a natural number; then the following identity holds:*

$$\int_{\mathbb{Z}_p} q^{-\beta x} \Gamma_p(x+1) d\mu_{q^\alpha}(x) = \sum_{n=0}^{\infty} a_n \frac{1-q}{1-q^{\alpha-2\beta}} \left(\frac{[\alpha]_q}{\alpha} \frac{(\alpha-\beta) q^{(\alpha-2\beta)n}}{(1-q^{\alpha-2\beta})^n} \right.$$

$$-\frac{[\alpha]_q}{\log q^{\alpha}}\sum_{k=0}^{n}\frac{(-1)^k q^{(\alpha-2\beta)(n-k)}}{k(1-q^{\alpha-2\beta})^{n-k}}\Bigg),$$

where a_n is given by Proposition 1.

Proof. For $x, y \in \mathbb{Z}_p$, by Proposition 1, we get

$$\int_{\mathbb{Z}_p} q^{-\beta x}\Gamma_p(x+1)d\mu_{q^\alpha}(x) = \sum_{n=0}^{\infty} a_n \int_{\mathbb{Z}_p} q^{-\beta x}\binom{x}{n} d\mu_{q^\alpha}(x)$$

and by (14), we acquire

$$\int_{\mathbb{Z}_p} q^{-\beta x}\Gamma_p(x+1)d\mu_{q^\alpha}(x) = \sum_{n=0}^{\infty} a_n \frac{1-q}{1-q^{\alpha-2\beta}}\Bigg(\frac{[\alpha]_q}{\alpha}\frac{(\alpha-\beta)q^{(\alpha-2\beta)n}}{(1-q^{\alpha-2\beta})^n}$$

$$-\frac{[\alpha]_q}{\log q^{\alpha}}\sum_{k=0}^{n}\frac{(-1)^k q^{(\alpha-2\beta)(n-k)}}{k(1-q^{\alpha-2\beta})^{n-k}}\Bigg),$$

which implies the desired result.

We propose the following theorem.

Theorem 3. *Let $x, y \in \mathbb{Z}_p$. We have*

$$\int_{\mathbb{Z}_p} q^{-\beta x}\Gamma_p(x+y+1)d\mu_{q^\alpha}(x) = \sum_{n=0}^{\infty} a_n \frac{D_{n,q}^{(\alpha,\beta)}(y)}{n!}, \tag{15}$$

where a_n is given by Proposition 1.

Proof. For $x, y \in \mathbb{Z}_p$, by the relation $\binom{x+y}{n} = \frac{(x+y)_n}{n!}$ and Proposition 1, we get

$$\int_{\mathbb{Z}_p} q^{-\beta x}\Gamma_p(x+y+1)d\mu_{q^\alpha}(x) = \int_{\mathbb{Z}_p} q^{-\beta x}\sum_{n=0}^{\infty} a_n \frac{(x+y)_n}{n!} d\mu_{q^\alpha}(x)$$

$$= \sum_{n=0}^{\infty} a_n \frac{1}{n!}\int_{\mathbb{Z}_p} q^{-\beta x}(x+y)_n d\mu_{q^\alpha}(x),$$

which is the desired result (15) via Eq. (5).

We now examine a consequence of Theorem 3 as follows.

Corollary 1. *Upon setting $y = 0$ in Theorem 3, the following relation is given:*

$$\int_{\mathbb{Z}_p} q^{-\beta x}\Gamma_p(x+1)d\mu_{q^\alpha}(x) = \sum_{n=0}^{\infty} a_n \frac{D_{n,q}^{(\alpha,\beta)}}{n!}, \tag{16}$$

where a_n is presented by Proposition 1.

The weighted p-adic q-integral on $\mathbb{Z}_p$ of the derivative of the p-adic gamma function is now presented.

Theorem 4. *For $x, y \in \mathbb{Z}_p$, we have*

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$$\int_{\mathbb{Z}_p} q^{-\beta x}\Gamma_p'(x+y+1)d\mu_{q^\alpha}(x) = \sum_{n=0}^{\infty}\sum_{j=0}^{n-1} a_n \frac{(-1)^{n-j-1}D_{j,q}^{(\alpha,\beta)}(y)}{(n-j)j!}.$$

Proof. By utilising Proposition 1, we obtain

$$\int_{\mathbb{Z}_p} q^{-\beta x}\Gamma_p'(x+y+1)d\mu_{q^\alpha}(x) = \int_{\mathbb{Z}_p} q^{-\beta x}\sum_{n=0}^{\infty} a_n \binom{x+y}{n}' d\mu_{q^\alpha}(x)$$

$$= \sum_{n=0}^{\infty} a_n \int_{\mathbb{Z}_p} q^{-\beta x}\binom{x+y}{n}' d\mu_{q^\alpha}(x)$$

and using (11), we derive

$$\int_{\mathbb{Z}_p} q^{-\beta x}\Gamma_p'(x+y+1)d\mu_{q^\alpha}(x) = \sum_{n=0}^{\infty}\sum_{j=0}^{n-1} a_n \frac{(-1)^{n-j-1}}{n-j}\int_{\mathbb{Z}_p} q^{-\beta x}\binom{x+y}{j} d\mu_{q^\alpha}(x)$$

$$= \sum_{n=0}^{\infty}\sum_{j=0}^{n-1} a_n \frac{(-1)^{n-j-1}}{n-j}\frac{D_{j,q}^{(\alpha,\beta)}(y)}{j!}.$$

The immediate result of Theorem 4 is given as follows.

Corollary 2. *For $x \in \mathbb{Z}_p$, we have*

$$\int_{\mathbb{Z}_p} q^{-\beta x}\Gamma_p'(x+1)d\mu_{q^\alpha}(x) = \sum_{n=0}^{\infty}\sum_{j=0}^{n-1} a_n \frac{(-1)^{n-j-1}D_{j,q}^{(\alpha,\beta)}}{(n-j)j!}.$$

We now provide an interesting representation of the *p*-adic Euler constant by means of weighted *q*-Daehee polynomials and numbers.

Theorem 5. *We have*

$$\gamma_p = \frac{\alpha}{[\alpha]_q}\sum_{n=0}^{\infty}\sum_{j=0}^{n-1} a_n \frac{(-1)^{n-j-1}}{(n-j)j!}\frac{\left(q^{\alpha-\beta}D_{j,q}^{(\alpha,\beta)} - D_{j,q}^{(\alpha,\beta)}(-1)\right)}{(\alpha-\beta)(1-q)} + \frac{\Gamma_p^{(2)}(0)}{(\alpha-\beta)\log q}.$$

Proof. Taking $f(x) = \Gamma_p'(x)$ in Eq. (3) yields the following result:

$$q^{\alpha-\beta}\int_{\mathbb{Z}_p} q^{-\beta x}\Gamma_p'(x+1)d\mu_{q^\alpha}(x) - \int_{\mathbb{Z}_p} q^{-\beta x}\Gamma_p'(x)d\mu_{q^\alpha}(x)$$

$$= (q-1)\tfrac{[\alpha]_q}{\alpha}(\alpha-\beta)\Gamma_p'(0) + \tfrac{[\alpha]_q}{\alpha}\tfrac{q-1}{\log q}\Gamma_p^{(2)}(0), \tag{17}$$

where $\Gamma_p^{(2)}(0)$ is the second derivative of the *p*-adic gamma function at $x = 0$, and with some fundamental computations and using Theorem 4, we have

$$LHS = q^{\alpha-\beta}\int_{\mathbb{Z}_p} q^{-\beta x}\Gamma_p'(x+1)d\mu_{q^\alpha}(x) - \int_{\mathbb{Z}_p} q^{-\beta x}\Gamma_p'(x)d\mu_{q^\alpha}(x)$$

$$= q^{\alpha-\beta}\sum_{n=0}^{\infty}\sum_{j=0}^{n-1} a_n \frac{(-1)^{n-j-1}}{n-j}\frac{D_{j,q}^{(\alpha,\beta)}}{j!} - \sum_{n=0}^{\infty}\sum_{j=0}^{n-1} a_n \frac{(-1)^{n-j-1}}{n-j}\frac{D_{j,q}^{(\alpha,\beta)}(-1)}{j!}$$

$$= \sum_{n=0}^{\infty}\sum_{j=0}^{n-1} a_n \frac{(-1)^{n-j-1}}{n-j}\frac{\left(q^{\alpha-\beta}D_{j,q}^{(\alpha,\beta)} - D_{j,q}^{(\alpha,\beta)}(-1)\right)}{j!}$$

and

$$RHS = (\alpha-\beta)\frac{(1-q)[\alpha]_q}{\alpha}\gamma_p + \frac{[\alpha]_q}{\alpha}\frac{q-1}{\log q}\Gamma_p^{(2)}(0),$$

where LHS and RHS denote the left-hand side and right-hand side of (17) respectively. Consequently, the proof is found.

We state the following theorem.

Theorem 6. *For $x, y \in \mathbb{Z}_p$, we have*

$$\int_{\mathbb{Z}_p} q^{-\beta x}\Gamma_p(-x-y+1)d\mu_{q^\alpha}(x) = \sum_{n=0}^{\infty} a_n \frac{\widehat{D}_{n,q}^{(\alpha,\beta)}(y)}{n!},$$

where a_n is given by Proposition 1.

Proof. For $x, y \in \mathbb{Z}_p$, by the relation $\binom{-x-y}{n} = \frac{(-x-y)_n}{n!}$ and Proposition 1, we get

$$\int_{\mathbb{Z}_p} q^{-\beta x}\Gamma_p(-x-y+1)d\mu_{q^\alpha}(x) = \int_{\mathbb{Z}_p} q^{-\beta x}\sum_{n=0}^{\infty} a_n \frac{(-x-y)_n}{n!} d\mu_{q^\alpha}(x)$$

$$= \sum_{n=0}^{\infty} a_n \frac{1}{n!}\int_{\mathbb{Z}_p} q^{-\beta x}(-x-y)_n d\mu_{q^\alpha}(x),$$

which is the desired result for Eq. (7).

A consequence of Theorem 6 is given by the following corollary.

Corollary 3. *Setting $y = 0$ in Theorem 6 gives the following relation:*

$$\int_{\mathbb{Z}_p} q^{-\beta x}\Gamma_p(-x+1)d\mu_{q^\alpha}(x) = \sum_{n=0}^{\infty} a_n \frac{\widehat{D}_{n,q}^{(\alpha,\beta)}}{n!},$$

where a_n is given by Proposition 1.

CONCLUSIONS

In this paper we have discovered several weighted *p*-adic *q*-integrals of *p*-adic gamma function via its Mahler expansions. We have also computed the *q*-Volkenborn integral of the derivative of *p*-adic gamma function. Further, we have presented an interesting representation of the *p*-adic Euler constant with the help of the weighted *q*-Daehee polynomials and numbers.

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